AAPT UNITED STATES PHYSICS TEAM
AIP 2019

USA Physics Olympiad Exam

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Important Instructions for the Exam Supervisor

- This examination consists of two parts. Part A has three questions and is allowed 90 minutes. Part B also has three questions and is allowed 90 minutes.
- Divide the exam paper into 4 parts: the instructions (pages 2–3), Part A (pages 4–6), Part B (pages 8–10), and answer sheets for one of the questions in Part A (pages 12–13). The exam should be printed single-sided to facilitate dividing the test and scanning the answer sheets.
- Provide students with the instructions for the competition (pages 2–3). Students can keep the pages for both parts of the exam, as they contain a reference list of physical constants.
- Provide students with blank sheets of paper as scratch paper. Students are not allowed to bring their own papers.
- Then provide students with Part A and the associated answer sheets, and allow 90 minutes to complete Part A. Do not give students Part B during this time, even if they finish with time remaining. At the end of the 90 minutes, collect the solutions to Part A along with the answer sheets and questions.
- Students are allowed a 10 to 15 minute break between Parts A and B. Then allow 90 minutes to complete Part B. Do not let students go back to Part A.
- At the end of the exam, the supervisor *must* collect all papers, including the questions, the instructions, and any scratch paper used by the students. Students may *not* take the exam questions. The examination questions may be returned to the students after April 8th, 2019.
- Students are allowed calculators, but they may not use symbolic math, programming, or graphical features of these calculators. Calculators may not be shared and their memory must be cleared of data and programs. Cell phones, PDAs, or cameras may not be used during the exam or while the exam papers are present. Students may not use any tables, books, or collections of formulas.

We acknowledge the following people for their contributions to this year's exam (in alphabetical order):

JiaJia Dong, Abijith Krishnan, Brian Skinner, and Kevin Zhou.

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USA Physics Olympiad Exam

Instructions for the Student

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- At the beginning of the exam, you shall be provided with the instruction sheets, blank papers (both for your answers and scratch work), and the exam packet.
- Work on Part A first. You have 90 minutes to complete three problems. Each question is worth 25 points, but they are not necessarily of the same difficulty. Do not look at Part B during this time.
- After you have completed Part A you may take a break. You may consider checking your answers to Part A with the remaining time as you will not be allowed to return to Part A once you start Part B.
- Then work on Part B. You have 90 minutes to complete three problems. Each question is worth 25 points. Do not look at Part A during this time.
- Show your work and reasoning. Partial credit will be given if you make your reasoning clear. Do not write on the back of any page. Further guidance is given on the next page.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDAs or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- **In order to maintain exam security, do not communicate any information about the questions of this exam, or their solutions until after April 8th, 2019.**

Possibly useful information. You may use this sheet for both parts of the exam.

$g = 9.8$ N/kg	$G = 6.67 \times 10^{-11}$ N $\cdot$ m^2/kg^2
$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9$ N $\cdot$ m^2/C^2	$k_\mathrm{m} = \mu_0/4\pi = 10^{-7}$ T $\cdot$ m/A
$c = 3.00 \times 10^8$ m/s	$k_\mathrm{B} = 1.38 \times 10^{-23}$ J/K
$N_\mathrm{A} = 6.02 \times 10^{23}$ (mol)$^{-1}$	$R = N_\mathrm{A} k_\mathrm{B} = 8.31$ J/(mol $\cdot$ K)
$\sigma = 5.67 \times 10^{-8}$ J/(s $\cdot$ m^2 $\cdot$ K^4)	$e = 1.602 \times 10^{-19}$ C
1 eV $= 1.602 \times 10^{-19}$ J	$h = 6.63 \times 10^{-34}$ J $\cdot$ s $= 4.14 \times 10^{-15}$ eV $\cdot$ s
$m_e = 9.109 \times 10^{-31}$ kg $= 0.511$ MeV/c^2	$(1+x)^n \approx 1 + nx$ for $\lvert x\rvert \ll 1$
$\sin\theta \approx \theta - \theta^3/6$ for $\lvert\theta\rvert \ll 1$	$\cos\theta \approx 1 - \theta^2/2$ for $\lvert\theta\rvert \ll 1$

Following is some further guidance on formatting your solutions. Start each question on a new sheet of paper. Put your AAPT ID number, your proctor's AAPT ID number, the problem number and the page number and total number of pages for this problem, in the upper right hand corner of each page. As an example, the second page of your solution to B3 might look as follows.

student AAPT #29485
proctor AAPT #1038
B3: 2/2

Remember to also write the AAPT ID numbers on the provided answer sheets. Write single-sided to facilitate scanning. You may use either pencil or pen, but in either case, make sure to write sufficiently clearly so your work will be legible after scanning. To preserve anonymity of grading, do **not** write your name on any sheet.

End of Instructions for the Student

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

Part A

Question A1

Collision Course

Two blocks, A and B, of the same mass are on a fixed inclined plane, which makes a $30°$ angle with the horizontal. At time $t = 0$, A is a distance $\ell = 5\,\text{cm}$ along the incline above B, and both blocks are at rest. Suppose the coefficients of static and kinetic friction between the blocks and the incline are

$$\mu_A = \frac{\sqrt{3}}{6}, \qquad \mu_B = \frac{\sqrt{3}}{3},$$

and that the blocks collide perfectly elastically. Let $v_A(t)$ and $v_B(t)$ be the speeds of the blocks down the incline. For this problem, use $g = 10\,\text{m/s}^2$, assume both blocks stay on the incline for the entire time, and neglect the sizes of the blocks.

a. Graph the functions $v_A(t)$ and $v_B(t)$ for t from 0 to 1 second on the provided answer sheet, with a solid and dashed line respectively. Mark the times at which collisions occur.

b. Derive an expression for the total distance block A has moved from its original position right after its n^{th} collision, in terms of ℓ and n.

Now suppose that the coefficient of block B is instead $\mu_B = \sqrt{3}/2$, while $\mu_A = \sqrt{3}/6$ remains the same.

c. Again, graph the functions $v_A(t)$ and $v_B(t)$ for t from 0 to 1 second on the provided answer sheet, with a solid and dashed line respectively. Mark the times at which collisions occur.

d. At time $t = 1\,\text{s}$, how far has block A moved from its original position?

Question A2

Green Revolution[1]

In this problem, we will investigate a simple thermodynamic model for the conversion of solar energy into wind. Consider a planet of radius R, and assume that it rotates so that the same side always faces the Sun. The bright side facing the Sun has a constant uniform temperature T_1, while the dark side has a constant uniform temperature T_2. The orbit radius of the planet is R_0, the Sun has temperature T_s, and the radius of the Sun is R_s. Assume that outer space has zero temperature, and treat all objects as ideal blackbodies.

a. Find the solar power P received by the bright side of the planet. (Hint: the Stefan-Boltzmann law states that the power emitted by a blackbody with area A is $\sigma A T^4$.)

In order to keep both T_1 and T_2 constant, heat must be continually transferred from the bright side to the dark side. By viewing the two hemispheres as the two reservoirs of a reversible heat engine, work can be performed from this temperature difference, which appears in the form of wind power. For simplicity, we assume all of this power is immediately captured and stored by windmills.

b. The equilibrium temperature ratio T_2/T_1 depends on the heat transfer rate between the hemispheres. Find the minimum and maximum possible values of T_2/T_1. In each case, what is the wind power P_w produced?

c. Find the wind power P_w in terms of P and the temperature ratio T_2/T_1.

d. Estimate the maximum possible value of P_w as a fraction of P, to one significant figure. Briefly explain how you obtained this estimate.

[1] This question inspired by De Vos, Alexis, and Guust Flater, American Journal of Physics 59.8 (1991): 751-754.

Question A3

Electric Slide

Two large parallel plates of area A are placed at $x = 0$ and $x = d \ll \sqrt{A}$ in a semiconductor medium. The plate at $x = 0$ is grounded, and the plate at $x = d$ is at a fixed potential $-V_0$, where $V_0 > 0$. Particles of positive charge q flow between the two plates. You may neglect any dielectric effects of the medium.

a. For large V_0, the velocity of the positive charges is determined by a strong drag force, so that

$$v = \mu E$$

where E is the local electric field and μ is the charge mobility.

  i. In the steady state, there is a nonzero but time-independent density of charges between the two plates. Let the charge density at position x be $\rho(x)$. Use charge conservation to find a relationship between $\rho(x)$, $v(x)$, and their derivatives.

  ii. Let $V(x)$ be the electric potential at x. Derive an expression relating $\rho(x)$, $V(x)$, and their derivatives. (Hint: start by using Gauss's law to relate the charge density $\rho(x)$ to the derivative of the electric field $E(x)$.)

  iii. Suppose that in the steady state, conditions have been established so that $V(x)$ is proportional to x^b, where b is an exponent you must find, and the current is nonzero. Derive an expression for the current in terms of V_0 and the other given parameters.

b. For small V_0, the positive charges move by diffusion. The current due to diffusion is given by Fick's Law,

$$I = -AD\frac{\mathrm{d}\rho}{\mathrm{d}x}.$$

Here, D is the diffusion constant, which you can assume to be described by the Einstein relation

$$D = \frac{\mu k_B T}{q},$$

where T is the temperature of the system.

  i. Assume that in the steady state, conditions have been established so that a nonzero, steady current flows, and the electric potential again satisfies $V(x) \propto x^{b'}$, where b' is another exponent you must find. Derive an expression for the current in terms of V_0 and the other given parameters.

  ii. At roughly what voltage V_0 does the system transition from this regime to the high voltage regime of the previous part?

STOP: Do Not Continue to Part B

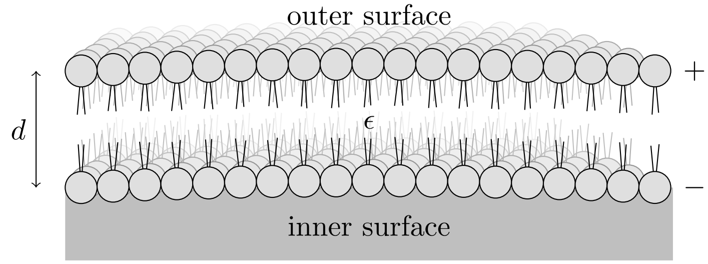
If there is still time remaining for Part A, you should review your work for Part A, but do not continue to Part B until instructed by your exam supervisor.

Once you start Part B, you will not be able to return to Part A.

Part B

Question B1

Strain in the Membrane[2]



The wall of a neuron is made from an elastic membrane, which resists compression in the same way as a spring. It has an effective spring constant k and an equilibrium thickness d_0. Assume that the membrane has a very large area A and negligible curvature.

The neuron has "ion pumps" that can move ions across the membrane. In the resulting charged state, positive and negative ionic charge is arranged uniformly along the outer and inner surfaces of the membrane, respectively. The permittivity of the membrane is ϵ.

a. Suppose that, after some amount of work is done by the ion pumps, the charges on the outer and inner surfaces are Q and $-Q$, respectively. What is the thickness d of the membrane?

b. Derive an expression for the voltage difference V between the outer and inner surfaces of the membrane in terms of Q and the other parameters given.

c. Suppose that the ion pumps are first turned on in the uncharged state, and the membrane is charged very slowly (quasistatically). The pumps will only turn off when the voltage difference across the membrane becomes larger than a particular value V_{th}. How large must the spring constant k be so that the ion pumps turn off before the membrane collapses?

d. How much work is done by the ion pumps in each of the following situations? Express your answers in terms of k and d_0.

   i. k is infinitesimally larger than the value derived in part (c).

   ii. k is infinitesimally smaller than the value derived in part (c).

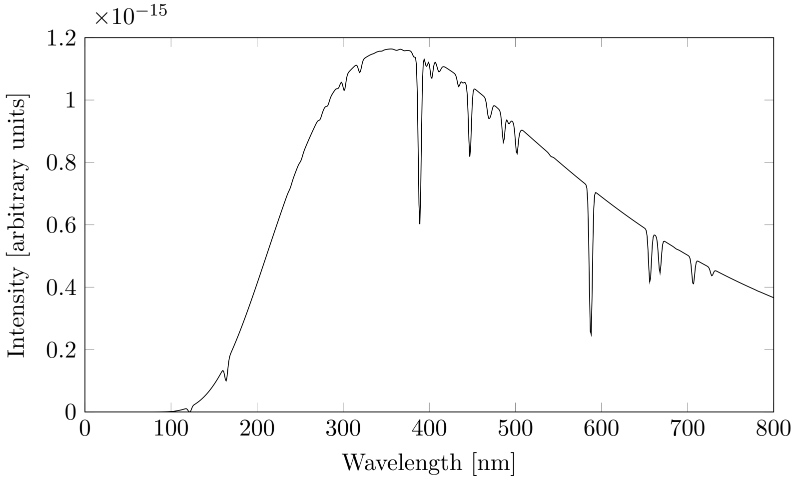
   Assume in each case that the membrane thickness d cannot become negative.

[2] This question inspired by Partenskii and Jordan, Physical Review E 80, 011112 (2009).

Question B2

Stellar Black Box

Scientists have recently detected a new star, the MAR-Kappa. The star is almost a perfect blackbody, and its measured light spectrum is shown below.



The total measured light intensity from MAR-Kappa is $I = 1.12 \times 10^{-8}\,\mathrm{W/m^2}$. The mass of MAR-Kappa is estimated to be $3.5 \times 10^{30}\,\mathrm{kg}$. It is stationary relative to the sun. You may find the Stefan-Boltzmann law useful, which states the power emitted by a blackbody with area A is $\sigma A T^4$.

a. The spectrum of wavelengths λ emitted from a blackbody only depends on h, c, k_B, λ, and T. Given that the sun has a surface temperature of $5778\,\mathrm{K}$ and peak emission at $500\,\mathrm{nm}$, what is the approximate surface temperature of MAR-Kappa?

b. The "lines" in the spectrum result from atoms in the star absorbing specific wavelengths of the emitted light. One contribution to the width of the spectral lines is the Doppler shift associated with the thermal motion of the atoms in the star. The spectral line at $\lambda = 389\,\mathrm{nm}$ is due to helium. Estimate to within an order of magnitude the thermal broadening $\Delta\lambda$ of this line. The mass of a helium atom is $6.65 \times 10^{-27}\,\mathrm{kg}$.

c. Over the course of a year, MAR-Kappa appears to oscillate between two positions in the background night sky, which are an angular distance of $1.6 \times 10^{-6}\,\mathrm{rad}$ apart. How far away is MAR-Kappa? Assume that MAR-Kappa lies in the same plane as the Earth's orbit, which is circular with radius $1.5 \times 10^{11}\,\mathrm{m}$.

d. What is the radius of MAR-Kappa?

Over the course of some time, you observe that the star's intensity periodically dips from I to $(1 - 10^{-5})I$ and then rises back to I, with period t. One possible explanation for this observation is that an exoplanet is orbiting the star and blocking the starlight for some time.

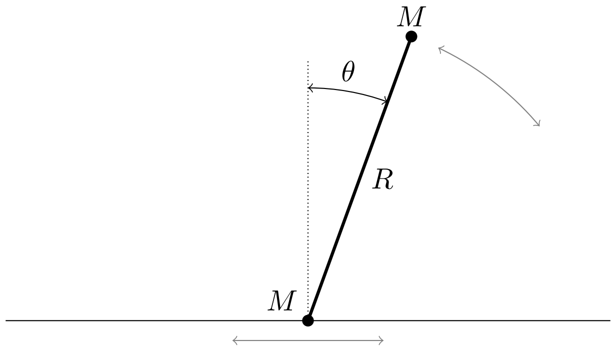
e. Estimate the exoplanet's radius, assuming that it is much closer to the star than to the Earth.

f. Assume the exoplanet is a blackbody with uniform temperature in a circular orbit around the star. What must t be so that the planet has a temperature of $250\,\mathrm{K}$? (If this were true, and the planet had an appropriate atmosphere, the temperature would increase enough to support life.)

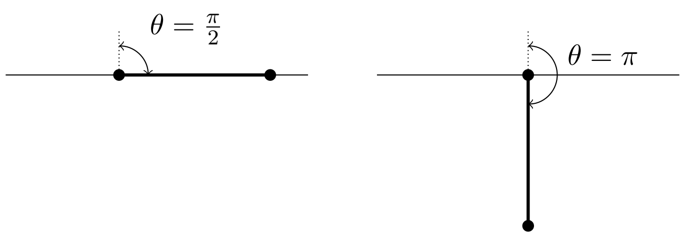
Question B3

Pitfall

A bead is placed on a horizontal rail, along which it can slide frictionlessly. It is attached to the end of a rigid, massless rod of length R. A ball is attached at the other end. Both the bead and the ball have mass M. The system is initially stationary, with the ball directly above the bead. The ball is then given an infinitesimal push, *parallel* to the rail.



Assume that the rod and ball are designed in such a way (not shown explicitly in the diagram) so that they can pass through the rail without hitting it. In other words, the rail only constrains the motion of the bead. Two subsequent states of the system are shown below.



a. Derive an expression for the force in the rod when it is horizontal, as shown at left above, and indicate whether it is tension or compression.

b. Derive an expression for the force in the rod when the ball is directly below the bead, as shown at right above, and indicate whether it is tension or compression.

c. Let θ be the angle the rod makes with the vertical, so that the rod begins at $\theta = 0$. Find the angular velocity $\omega = d\theta/dt$ as a function of θ.

Answer Sheets

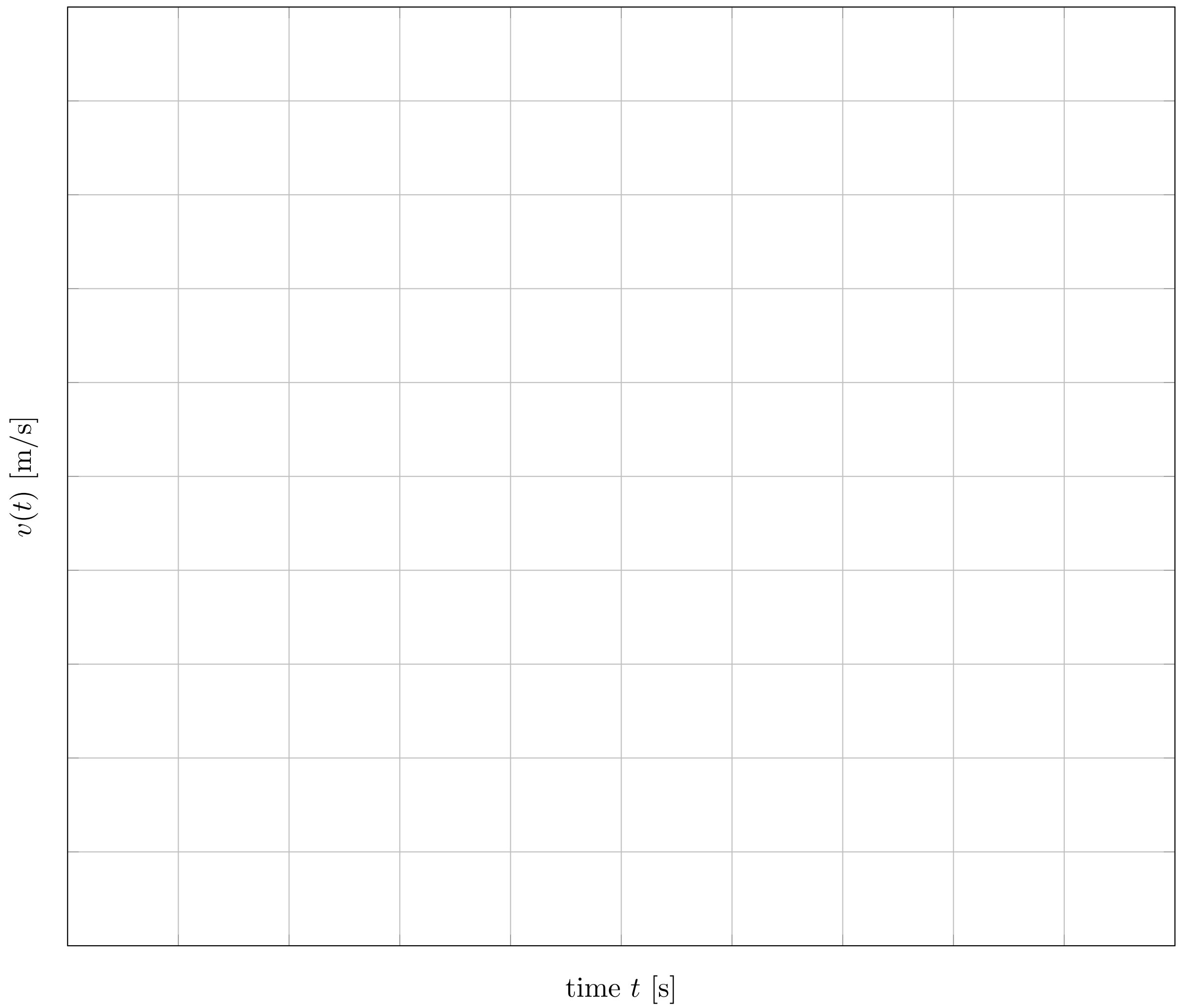
Following are answer sheets for the graphing portion of Problem A1.

Student AAPT ID #:

Proctor AAPT ID #:

A1: Collision Course

(a)



Student AAPT ID #:

Proctor AAPT ID #:

(c)