Physics Challenge for Teachers and Students

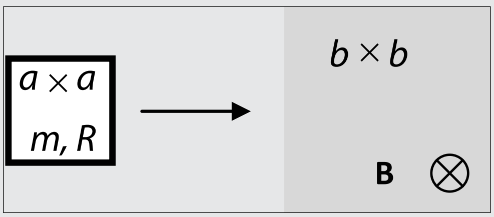
***Boris Korsunsky*, Column Editor**
Weston High School, Weston, MA 02493
korsunbo@post.harvard.edu

Solution to May 2016 *Challenge*

The mystery of May B

A uniform magnetic field B exists in a region of size $b \times b$. A square-shaped wire loop of dimensions $a \times a$ where $a < b$, mass m, and resistance R approaches this region. Vector **B** is perpendicular to the plane of the loop as shown. If the loop enters this magnetic region with the minimum speed allowing it to pass through, calculate the amount of time required for the entire loop to enter the region. Neglect gravitational effects.

(Submitted by José Ignacio Íñiguez de la Torre Bayo, Universidad de Salamanca, Salamanca, Spain)



Solution:

There is an induced current I as the loop enters or exits the field whose direction must, by energy conservation, result in a drag force $-IaB$ on the entering or exiting edge of the loop, respectively. The induced current is equal to the motional emf $Ba\upsilon$ generated in that edge divided by the resistance R of the loop, so that Newton's second law becomes

$$-\frac{Ba\upsilon}{R}aB = m\frac{d\upsilon}{dt}. \tag{1}$$

Using the chain rule we can rewrite this derivative as

$$\frac{d\upsilon}{dt} = \frac{d\upsilon}{dx}\frac{dx}{dt} = \frac{d\upsilon}{dx}\upsilon, \tag{2}$$

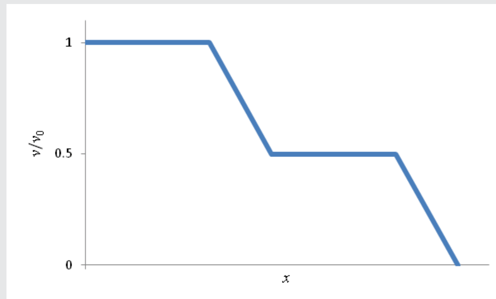
so that Eq. (1) becomes

$$\frac{d\upsilon}{dx} = -k \quad \text{where} \quad k \equiv \frac{B^2a^2}{mR}, \tag{3}$$

which integrates to give the spatial variation in the speed of the loop as it enters or exits the field,

$$\upsilon(x) = \upsilon_\text{i} - kx, \tag{4}$$

where υ_i is the initial speed as the loop before that happens. Consequently, the speed of the loop must vary with position as sketched below, where the loop initially has speed υ_0 say before it ever enters the field.



It just loses all of its speed during the second ramp region of the graph while it moves horizontally a distance of a with a constant slope of $-k$ according to Eq. (3). However, as it enters the field in the first ramp region, it also moves a distance of a and thus must lose the same amount of speed. Hence, the speed of the loop while it is entirely inside the field must be $\upsilon_0/2$. Incidentally, that implies the initial speed of the loop must be

$$\upsilon_0 = 2ka = \frac{2B^2a^3}{mR}. \tag{5}$$

Finally, if we separate variables and integrate Eq. (1) over the time interval T during which the loop enters the field, we find

$$\ln\tfrac{1}{2} = -kT \quad \Rightarrow \quad T = \boxed{\frac{mR}{B^2a^2}\ln 2}, \tag{6}$$

since the speed drops to half over that interval according to the preceding graph.

(Submitted by Carl E. Mungan, U. S. Naval Academy, Annapolis, MD)

We also recognize the following successful contributors:

Phil Cahill (The SI Organization, Inc., Rosemont, PA)
Don Easton (Lacombe, Alberta, Canada)
Supriyo Ghosh (Kolkata, India)
John F. Goehl, Jr. (Barry University, Miami Shores, FL)
Fredrick P. Gram (retired, Cleveland, OH)
Gerald E. Hite (TAMUG, Galveston, TX)
Art Hovey (Galvanized Jazz Band, Milford, CT)
Matthew W. Milligan (Farragut High School, Knoxville, TN)

Daniel Mixson (Naval Academy Preparatory School, Newport, RI)

Thomas Olsen (Dar Al Uloom University, College of Medicine; Riyadh, Saudi Arabia)

Pascal Renault (John Tyler Community College, Midlothian, VA)

Joseph Rizcallah (School of Education, Lebanese University, Beirut, Lebanon)

José Antonio Santiago Espinal, student (Escuela Politécnica Superior, University of Seville, Seville, Spain)

Robert Siddon (U. S. Naval Academy, Annapolis, MD)

Jason L. Smith (Richland Community College, Decatur, IL)

Evaristo Tejada Calvente, student (Escuela Politécnica Superior, University of Seville, Seville, Spain)

Guidelines for contributors:

- We ask that all solutions, preferably in Word format, be submitted to the dedicated email address *challenges@aapt.org*. Each message will receive an automatic acknowledgment.
- The subject line of each message should be the same as the name of the solution file.
- The deadline for submitting the solutions is the last day of the corresponding month.
- Each month, a representative selection of the successful solvers' names will be published in print and on the web.
- If your name is—for instance—Donald Duck, please name the file "**Duck16May**" (do not include your first initial) when submitting the May 2016 solution.
- If you have a message for the Column Editor, you may contact him at *korsunbo@post.harvard.edu*; however, please do not send your solutions to this address.

Many thanks to all contributors; we hope to hear from many more of you in the future.

We also hope to see more submissions of the original problems – thank you in advance!

Boris Korsunsky, Column Editor