THE MECHANICAL UNIVERSE

High School Adaptation

QUAD II

CONSERVATION LAWS AND FUNDAMENTAL FORCES

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THE MECHANICAL UNIVERSE

High School Adaptation

A co-production of the
California Institute of Technology
University of Dallas
and
Southern California Consortium

QUAD II

CONSERVATION LAWS AND FUNDAMENTAL FORCES

Conservation of Energy
Conservation of Momentum
Angular Momentum
The Fundamental Forces

An Annenberg/CPB Project

National Science Foundation

Materials Development Council

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FOREWORD

Today, scientific and educational leaders are seriously concerned about the quality of science and mathematics education in the United States. It is as though the problems have been rediscovered, 25 years after Sputnik! In addition to those problems which have repeated themselves, today many qualified science and mathematics teachers at the pre-college, college, and university levels are being lured from the classroom by higher-paying jobs in business and industry. Many classrooms, therefore, have become the responsibility of instructors with limited preparation in the subject matter they are called upon to teach. And yet, more than ever the nation's current economic, social, and political needs call for a technologically literate population.

The Mechanical Universe, which served as the basis for the high school materials, addresses one critical need in science education by providing video and print materials that can serve as the basis of a solid, introductory college-level physics course. The video offers an exciting array of audiovisual resources for classroom instruction: close-ups of complicated experiments; extensive computer animation sequences that make abstract concepts and mathematical processes understandable; historical reenact-ments that provide a philosophical fabric for the development of ideas of physics.

The Mechanical Universe, part of the Annenberg/CPB collection, has as its primary purpose the provision of a quality learning experience for those whose lives cannot fit into the traditional campus schedule. This 52-program introduction to physics also offers a partial answer to some of the current problems of science education, for it can be used to upgrade skills of secondary science teachers and to provide supplementary support in the college and university classes.

Through the sponsorship of the National Science Foundation, selected programs of **The Mechanical Universe** have been adapted for use in high school. These materials represent the same quality and innovation as the college series, but they are presented in shorter and less mathematically oriented tapes that can be used in a wide variety of high school curricula. Teachers who find themselves teaching high school physics in spite of limited preparation will discover that, by enrolling in **The Mechanical Universe** course and using the adaptations in their classes, they will enjoy the confident feeling that they are presenting their students with quality instruction.

INTRODUCING

THE MECHANICAL UNIVERSE

High School Adaptation

The adaptations of **The Mechanical Universe** were created by twelve outstanding high school physics teachers (the Materials Development Council) through the generous support of the National Science Foundation. The clear purpose of the Council and the entire staff was to produce quality materials that would be used to improve instruction in physics. No one was satisfied with the goal of producing materials that would simply motivate or fascinate students, or would provide a change of pace. From the start, the challenge was to create materials which could make wise use of the power of television in developing a sound and solid understanding of physics.

Herewith the fruit of these labors: sixteen modules each consisting of a video adaptation from **The Mechanical Universe** with written support materials. Each module stresses conceptual understanding of underlying physical principles. The written materials support the video dimension of the modules. These support materials provide the teacher with additional background information and mathematical derivations, pre-video and post-video questions, applications, demonstrations, and evaluation questions.

The Mechanical Universe was originally developed for lower-division college courses in physics. The materials from **The Mechanical Universe** that have been adapted for use in high schools were field tested in 1984-86 by over 100 high school physics teachers located in schools widely scattered across the county in both urban and rural communities that serve various socio-economic populations. As a result of the assessment of the field testing, the videos were re-edited and the written materials were focused more directly on the videos to provide the best support possible for teachers.

PREFACE

These materials are intended for all teachers of high school physics. Teachers new to the arena of physics will discover rigorous, conceptual video presentations of traditional and not-so-traditional topics in classical physics. We hope that each word of the written materials will be savored. They are your resources and we hope that you tap them to capture the excitement of **The Mechanical Universe**. Experienced teachers will find a different slant to classical physics in the space age: a humanizing, compelling, integrated approach to the greatest revolution in the history of Western civilization. These teachers, too, we hope, will find the written materials continually refreshing resources.

Although **The Mechanical Universe** is a calculus-based course, the excerpts for high school use were selected to focus on concepts. That is not to say that the videos for high school use are not rigorous; they present sound logic at every stage in the development. Mathematics is occasionally used in the high school materials as a language to relate ideas concisely. In many cases the original mathematical derivations have been modified to be appropriate to the high school level. Nonetheless, mathematical derivations go by quickly in the video and we hope that teachers will replay these sections for their students. The mathematical background sections of the modules, we expect, will be read by all teachers even though they may not necessarily present to their classes the same level of mathematics provided in the print materials. We hope that teachers as well as students will gain a better appreciation of the vital role of mathematics in physics.

No laboratory component is currently suggested. The reason is not because we judge a physics laboratory component to be unimportant or uninteresting. On the contrary, we believe that demonstrations and laboratories lie at the heart of a sound education in high school physics. Instead we concentrated on what we could offer best: instruction through television. There are dozens of laboratory manuals which can be appended easily to these materials and we expect that each teacher will decide how best to handle the laboratories. On the other hand, since many demonstrations and applications to everyday life are presented in the video, we identified simple, short, and effective demonstrations that tie into concepts in the video. We hope that all physics teachers will enjoy performing them.

Not all the topics covered in the modules are conventional to high school physics curricula. *Angular Momentum* and *Harmonic Motion*, effectively covered in the videos, are two topics which are not necessarily a part of every curriculum. *Navigating in Space*, on the other hand, represents an exciting application of Kepler's ellipses and Newton's gravity that is not covered in typical curriculum. Other topics, such as *The Fundamental Forces* and *Curved Space and Black Holes*, provide tantalizing looks at twentieth century physics from the perspective of classical physics.

The Mechanical Universe is the story of the Copernican revolution, why it was necessary, and how it unfolded in the work of Galileo, Kepler, and Newton. It is the story of the eventual wedding of the heavens with the earth through the synthesis of mechanics and astronomy. History is presented in the series, not for the sake of historical detail, but for a fuller sense of how scientific thought proceeded through the intellectual searches and triumphs of men who reshaped the society of their times. We hope the infectious spirit of **The Mechanical Universe** will inspire teachers and students and will contribute to a lifelong scientific interest in the workings of the universe.

ACKNOWLEDGEMENTS

The adaptations of these instructional materials for high school use would not have been possible without the assistance of a long list of people who aided through the dedicated use of their diverse and specialized skills.

Heading the list is Professor David L. Goodstein, of Caltech, whose inspiration and guiding force in the creation of **The Mechanical Universe** led to the development of these materials.

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Lynn Strech, Artist
Judy Sullivan, Southern California Consortium, Design and typesetting

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Materials Development Council
Irving, Texas
July 1987

STRUCTURE OF THE MATERIALS

The written materials are designed to support and extend the VIDEO presentation of each module. The format and content of the materials are designed to help the user (1) to integrate the concept(s) presented in the VIDEO with traditional high school materials, (2) to supplement and promote conceptual understanding of the phenomena presented in the VIDEO, and (3) to infuse the students with a new spirit of inquiry concerning the mechanics of physics.

Each module is composed of components of written materials. Each component is intended as a resource to promote active engagement of the learner in developing conceptual understanding of the physical phenomena. The five components of the print materials are:

TEACHER'S GUIDE

Pre-Video Activities*

Content and Use of the Video – describes what the VIDEO does and does not cover.

Terms Essential for Understanding the Video – includes the definitions of terms listed in the STUDENT'S GUIDE, discussion of critical elements or relationships.

What to Emphasize and How to do It – includes the objectives of the module, references to demonstrations, possible applications, and suggestions for correcting common misconceptions.

Points to Look for in the Video – includes common misconceptions when relevant; characteristics and questions concerning critical elements presented in the VIDEO. Answers to questions in the STUDENT'S GUIDE are included.

Post Video Activities*

Everyday Connections and Other Things to Discuss – suggests additional questions to promote student participation and discussion. An essential purpose of the questions is to engage students in review and clarification of the concepts.

Summary – reviews the key concepts that have been presented.

STUDENT'S GUIDE

(designed for duplication and distribution)

Introduction – a brief statement about the content or purpose of the VIDEO.

Terms Essential to Understanding the Video – includes terms or critical elements of the VIDEO, with definitions and explanations provided in the TEACHER'S GUIDE.

Points to Look for in the Video – includes common misconceptions when relevant; characteristics and questions concerning critical elements presented in the VIDEO along with figures representative of key points in the VIDEO.

TEACHER RESOURCES

Supportive Background Information –summarizes additional historical, physical, and mathematical information that relate to the topics and content presented in the VIDEO.

Additional Resources –includes demonstrations and applications the teacher may use to extend and enrich the treatment of the topic.

Evaluation Questions – provides ten multiple-choice questions dealing with the objectives of the module and two essay questions that require student's explanations of certain concepts related to the topics.

*The repeated showing of the video (in full and part) is essential to student understanding. The division of activities into prevideo and postvideo activities, therefore, is somewhat artificial. It is likely that most, if not all, prevideo activities will precede the initial showing of the video. Sections of the video will undoubtedly be sprinkled throughout the postvideo activities, with a full showing being used for closure where time permits.

QUAD II

CONSERVATION OF ENERGY

WHAT DOES CONSERVATION OF ENERGY MEAN? Galileo's fertile experiments with balls rolling down inclined planes indicated that some quantity was conserved. Not until the nineteenth century did the law of conservation of energy emerge from experiments exploring the relationships between heat, work, and energy. In this video, an overview of mechanical energy is presented. The connections between work, kinetic energy, potential energy, and heat energy are woven together in the law of conservation of energy.
Running time: 15:06

CONSERVATION OF MOMENTUM

IF THE UNIVERSE FOLLOWS PURELY MECHANICAL LAWS, WHAT KEEPS IT TICKING UNTIL THE END OF TIME? The idea of a mechanical universe was set forth by René Descartes who postulated that the total "quantity of motion" in the universe is constant. Newton later identified the quantity of motion as momentum. In this video, how Newton's laws of motion, taken together, lead to the fundamental law of conservation of momentum is illustrated. Not only does conservation of momentum aid in understanding what keeps the mechanical universe ticking, but also it provides a powerful principle for analyzing collisions, even at a local pool hall.
Running Time: 13:34

ANGULAR MOMENTUM

WHAT DO THE MOTIONS OF A SPINNING ICE SKATER AND OF AN ORBITING PLANET HAVE IN COMMON? Kepler's second law of planetary motion, the equal areas law, is rooted in a much deeper principle – the law of conservation of angular momentum. In this video, the ideas of angular momentum and torque are developed through applications to planetary motion, whirlpools, tornadoes, and spinning ice skaters. Along with the laws of conservation of energy and of momentum, conservation of angular momentum remains among the deepest mysteries of nature.
Running Time: 8:20.

THE FUNDAMENTAL FORCES

WHAT ARE THE FUNDAMENTAL FORCES OF NATURE? Newton's mechanics clarified the role of forces in shaping the motion of all things and focused attention on identifying forces at play in the universe. In this video, a survey of how all forces are manifestations of nature's fundamental forces – gravity, electricity, the weak nuclear force, and the strong nuclear force – is presented. The unification of these forces into a common description remains to this day a beguiling quest in physics.
Running Time: 16:18.

TEACHER'S GUIDE TO CONSERVATION OF ENERGY

CONTENT AND USE OF THE VIDEO - In presenting the law of conservation of energy, the video offers an excellent overview of mechanical energy. Work is defined and shown to be the process involved in transforming energy from one form to another. Both potential energy and kinetic energy are explored. The video stresses that total energy is always conserved within a closed system even though energy is sometimes transformed into heat.

This topic usually follows a study of forces and work in the traditional high school physics course. Some basic knowledge of work in a straight line, kinetic energy, and gravitational potential energy near the surface of the earth is assumed. The video also can be used as an introduction to work and energy. If, however, it follows the study of the separate topics of work, kinetic energy, and potential energy, it can then serve to integrate these concepts and to show their relationship to thermodynamics. This latter emphasis on relationships is more consistent with the fundamental focus of the video.

Note: There are different types of potential energy; within the context of this video, only gravitational potential energy is considered.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - Since the following terms are introduced in the video, it might be helpful if students are familiar with them prior to viewing:

work--a process in which energy is transformed from one form to another or transferred from one system to another; it is the product of the displacement and force in the direction of that displacement ($W = Fd$).

kinetic energy--the energy of an object by virtue of its motion; for an object, $K = 1/2mv^2$.

potential energy--the energy of an object by virtue of its position; for an object in a constant gravity field, $U = mgh$.

heat energy--random kinetic (thermal) energy. The energy transferred from a warm substance to a cooler substance.

conservation of energy--the total amount of energy within an isolated system remains constant.

friction--a name for those forces which convert ordered kinetic energy into disordered, chaotic kinetic energy called heat energy.

transformation--a change from one form to another, e.g., from potential energy to kinetic energy.

joule--a unit of energy equal to 1 kg m/s^2.

calorie--a unit of heat energy equivalent to 4.19 joules.

WHAT TO EMPHASIZE AND HOW TO DO IT - energy is one of the few topics in mechanics not handed down to us by Sir Isaac Newton. Galileo's experiments with inertia had suggested a conservation-of-energy principle, but it was not until about 1840 that James Prescott Joule experimentally established the relationships among heat, work, and energy that led to the law of conservation of energy. Taken together, the three conservation laws--energy, momentum, and angular momentum--constitute a firm foundation for understanding mechanics.

The video develops the concept that **the total amount of energy in the universe is constant although energy may change form.**

Objective 1: Describe the law of conservation of energy.

Since the video emphasizes changes in the form of energy, it is important to focus student attention on those processes prior to viewing. DEMONSTRATION #3 shows the transformation of energy within a pendulum system. Help students distinguish between gravitational potential energy and kinetic energy. Ask your students what has happened to the energy when the swinging bob comes to rest. Solicit opinion, and then instruct the students to look for a fuller explanation in the video.

Review the pendulum activity after students have seen the video. Be sure they understand that energy changes constantly from potential to kinetic, from kinetic to potential, and back again. In whatever form, however, mechanical energy (potential or kinetic) is eventually transformed into heat energy.

Objective 2: Recognize that work is done when energy is transferred from one system to another.

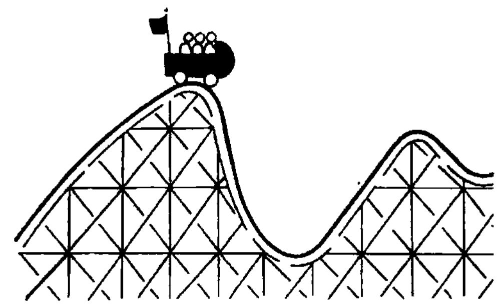
Work is a bookkeeping device to keep track of the transfer of energy from one object to another. When one body gains energy from another body, work is done on the body that gains energy. (The word "system" can be substituted for the word "body" in the previous sentence.) DEMONSTRATION #4 uses collision balls to illustrate the transfer of energy from one object to another.

In the video, energy is transferred from the muscles of the weightlifter to the steel of the barbell. Ask students to attend carefully to the weightlifting sequence in the video and to be able to discuss the activity in terms of energy transfer.

Objective 3: Distinguish between potential energy and kinetic energy.

Potential energy is dependent upon position and is thought of as "stored" energy. Kinetic energy, on the other hand, is related to motion. Any of the demonstrations in ADDITIONAL RESOURCES can be used to illustrate these forms of energy. DEMONSTRATION #1 can be easily related to the roller coaster rides with which most students are familiar.

At the top of the hill the roller coaster has all potential energy, $U = mgh$. As it rolls down the hill, work is done, and the potential energy is converted into kinetic energy: $\Delta U = -\Delta K$, $mg(h_f - h_i) = -1/2m(v_f^2 - v_i^2)$, where f stands for final and i for initial value. When it reaches the bottom, the energy is kinetic, $K = 1/2mv^2$.



Remind the students of the slow motion pole-vaulter in the video. Discuss the energy transformations from the time he starts his run to the time he completes his fall to the ground. The vaulter gains kinetic energy as he accelerates. When he bends the pole, he does work on it, giving it potential energy. The pole springs back, transferring its energy to the vaulter, lifting him up and giving him gravitational potential energy. Near the top, the vaulter pushes with his arms to raise his body even higher, giving it more potential energy. As he falls, the potential is transformed into kinetic energy.

Objective 4: Describe the transformation of potential energy or kinetic energy into other forms such as heat.

Several simple examples can help students visualize the transformation of energy into heat. Have your students rub their hands together rapidly. Ask these questions: What happens when you press harder? Where does the heat go?

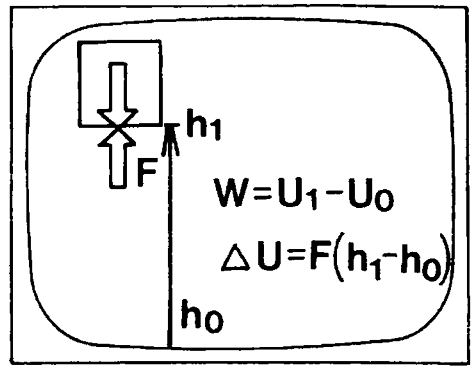
Have your students stretch a rubber band four of five times and then hold it against their lips to feel the heat energy. The mechanical energy involved in stretching is transformed into heat energy. Ask them what happens if the rubber band is stretched more tightly?

Suggest to students that they try to find examples in the video where energy is transformed into heat. What happens to the energy of a barbell if a weightlifter drops it? What happens to the energy of a pole-vaulter when he hits the earth after his vault?

POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with some suggested responses and frames from the video.

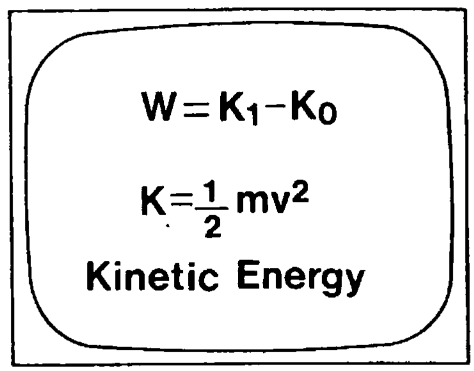
Will more energy be transformed to potential energy by an empty elevator in going from the fourth floor to the tenth or by one that has a load of twelve people?

Since gravitational potential energy depends directly on mass according to $U = mgh$, it follows that the greater mass that is raised the greater the increase in gravitational potential energy. Therefore, more electrical energy will be transformed into gravitational potential energy to produce this increase.



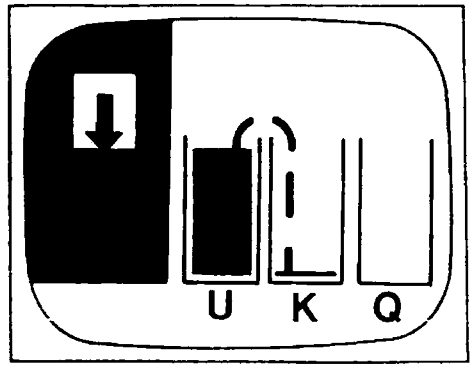
If the speed is increased to twice its original value, how much greater would be its kinetic energy? How much work is needed for this to occur?

Since $K_0 \propto v_0^2$, doubling the speed would produce four times the kinetic energy; i.e., $[K_1 \propto v_1^2 \propto (2v_0)^2 \propto 4v_0^2]$ $W = K_1 - K_0$. The work done produces the change in kinetic energy and is equal it.

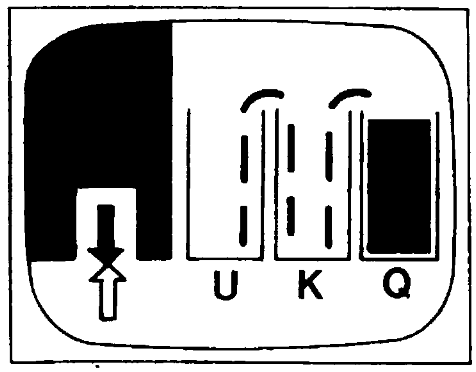


How do these two frames from the video differ in the way that they depict the law of conservation of energy?

The first frame shows the transformation of potential energy, $U = mgh$, of the raised block into kinetic energy, $K = 1/2mv^2$ when there are no friction forces.

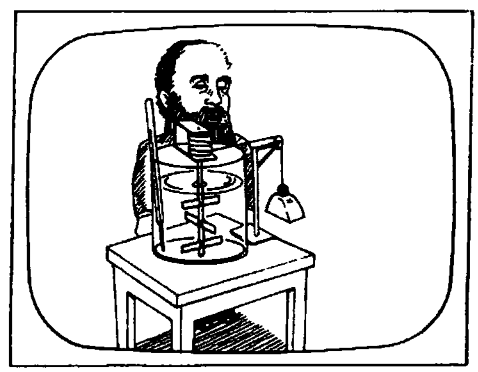


The second frame illustrates the dissipation of some of the kinetic energy upon impact into heat energy Q.



In the Joule experiment, what is the purpose of the falling weights?

As the weight falls, potential energy is converted into kinetic energy of the turning paddle wheels. The churning of the water by the paddle wheels, produces random thermal energy called heat.



EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in the video, you might pose the following questions to your students.

1. **What are the energy transformations involved in throwing a ball upward (ignore air resistance)?**

   While you throw the ball, work is done on it. During the upward and downward motions, the ball has kinetic energy. As the ball rises, the velocity decreases. Therefore, the kinetic energy decreases: but since the ball gets higher, its gravitational potential energy increases. At the top of the path, the ball has no kinetic energy. All the energy has been converted to potential energy. Energy is conserved, since the total energy is a constant. On the downward path the ball loses potential energy and gains kinetic energy. When the ball strikes the surface, it bounces back up, but not as high, because some energy has been converted to heat energy upon collision with the surface.

2. **What is the work-energy conversion which occurs when a crane lifts a heavy crate from the ground to the top of a three-story building?**

   The chemical energy of the crane's fuel is transformed into gravitational potential energy of the crate as its height increases. The crane does work on the crate.

3. **How can the energy of the water behind a dam be converted into useful work?**

   While water behind a dam is, for all practical purposes, at rest, it has gravitational potential energy. As the water is allowed to fall over the dam, the gravitational potential energy is converted into kinetic energy of moving water. The water moving through the generator assembly allows the kinetic energy of the water to be converted into kinetic energy of the turbine blades. The kinetic energy of the blades is transformed into electrical energy, which can be used to do useful work.

4. **What happens to the energy when a moving car hits a brick wall and comes to rest?**

   When the car is moving, it has kinetic energy because of the work done by the engine, using energy supplied by the fuel. After the car strikes the wall, its energy is converted into heat energy, making the metal of the car, the rubber of the tires, the soil of the earth, and the bricks of the wall a little warmer than before impact. Some energy is also expended making sound waves, and rearranging the structure of the car and the wall.

5. **What has happened to the energy of a ball rolling across the floor as it comes to a stop?**

   Work was done on the ball to start it moving. As it moves across the floor, its kinetic energy decreases due to friction. The decrease in kinetic energy appears in the form of heat energy. The total energy of the ball-and-floor system is conserved.

6. **How does bouncing on a trampoline illustrate kinetic and potential energy?**

   When you bounce on a trampoline, you have kinetic energy when moving up and down. However, at the very top of the bounce all your energy is in the form of potential energy. The trampoline material and springs are moving and therefore have kinetic energy. For a brief period at the bottom they are still, and so are you, so all the energy is in the form of elastic potential energy.

7. **Why do hand-powered machines allow you to do work more easily?**

   Many machines make work easier in the sense that a smaller force can be used. The smaller force, however, is applied through a larger distance, so the same amount of work is done. A steering wheel is a good example.

8. **A bicycle lamp operates by means of a small generator attached to the front tire. Why is it harder to pedal when operating the lamp-generator? What would you notice about the effort if the lamp burned out?**

   It is harder to pedal because you not only have to overcome friction between the generator and wheel, but also have to work to move electrical charges through the lamp. If the bulb burns out, it is easier to pedal – indicating that the increased work goes into moving charges in the circuit.

9. **Why does a car generally get poorer gas mileage when the air-conditioner is used? Will the mileage depend on whether the radio is used or not? Explain.**

   Poorer mileage results because the car's engine must also do work in operating the air-conditioner. It should be noted that having the windows up will reduce air friction and increase gas mileage. The engine must also do work to operate the radio, but the amount is very small in comparison to the work done to operate the air-conditioner. Therefore, the mileage is decreased very little.

10. **Why are most mountain roads designed so that they weave back and forth with a gradual slope rather than going straight up the mountain?**

    The same amount of work would be done in either case. In the first case, a small force is applied over a greater distance. In the second, a larger force is applied for a shorter distance. If the road were too steep, the car's engine might not be able to supply adequate force to move the car at a reasonable speed.

11. **Trace, as far as you can, the following forms of energy back to their sources: a spring wound wrist watch; a light bulb; a car engine; a hair dryer.**

    Students should be able to trace back some of the forms for each case. These forms include: mechanical (potential and kinetic), electrical, chemical, biological, and heat. The responses should indicate that the energy ultimately comes from the sun.

SUMMARY - Energy changes constantly from potential to kinetic, from kinetic to potential, and back again. In whatever form, mechanical energy (potential or kinetic) is eventually transformed into heat energy. Energy is never lost; it is always strictly conserved. When mechanical energy is transformed into heat energy, the energy becomes harder and harder to retrieve and almost impossible to make use of again. Work is a bookkeeping device to keep track of the transfer of energy from one object to another.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO CONSERVATION OF ENERGY

INTRODUCTION - The video describes two basic forms of energy, potential and kinetic. Work is done whenever energy is transferred from one body to another or transformed from one form to another. No matter what the process, the total energy in a closed system is constant.

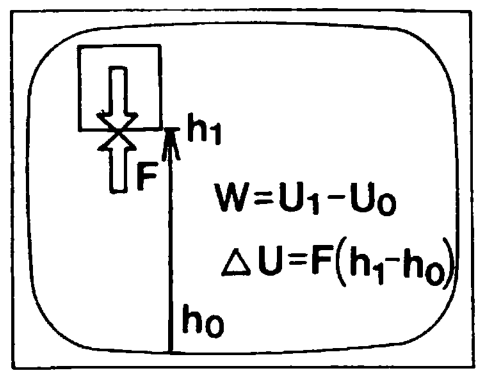
Terms Essential for Understanding the Video

work
kinetic energy
potential energy
heat energy
conservation
friction
transformation
joule
calorie

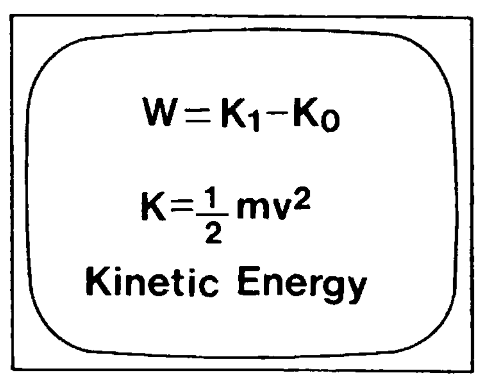
**** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ****

Points to Look for in the Video

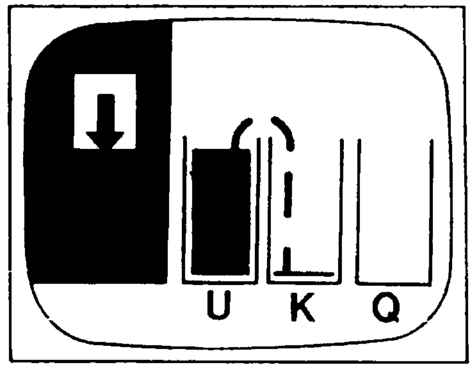
Will more energy be transformed to potential by an empty elevator in going from the fourth floor to the tenth or by one that has a load of twelve people?

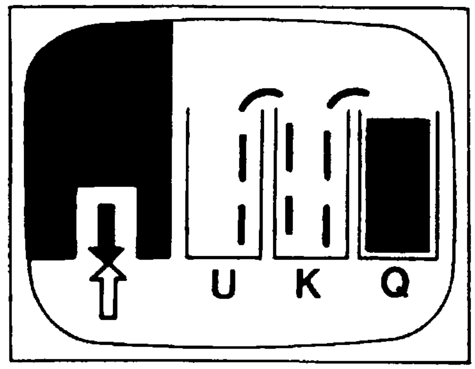


If the speed is increased to twice its original value, how much greater would be its kinetic energy? How much work is needed for this to occur?

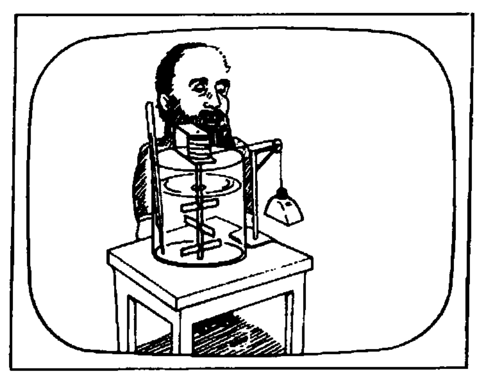


How do these two frames from the video differ in the way that they depict the law of conservation of energy?





In the Joule experiment, what is the purpose of the falling weights?



TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - Three of the great conservation laws which are fundamental to mechanics are conservation of energy, momentum, and angular momentum. The law of conservation of energy states that the total amount of energy in the universe is constant, although it may take different forms. But what is energy, and what does it mean to say that it is conserved?

Galileo's inclined-plane experiments demonstrate the concept of conservation of energy. It is easy to understand that the ball will roll down the first incline when released and, because of its inertia, will continue beyond the center and up the second incline. What is not obvious is why it always returns to the same height. Clearly something remains the same. We say that something has been conserved, and we give that conserved quantity the name energy. One aspect of quantum mechanics is similar: scientists study collisions of subatomic particles to see what is conserved, and then they name the conserved quantities. It is interesting to speculate whether energy would ever have been named if it were not a conserved quantity. Perhaps that is why, although its different forms are readily identified, the overall concept of energy is elusive, and not easily defined.

Energy that a body has **by virtue** of its position with respect to nearby other masses is called **gravitational potential energy**, while **energy of motion** is called **kinetic energy**. Work and heat involve the transfer of energy within and between systems. Their numerical measure is equal to the energy gained or transferred. Thus work may be thought of as a "bookkeeping device," useful for keeping track of the changes in energy. Sometimes energy seems to leave a system; we say that it has been "lost" to friction, a force, when we really mean that it has been transformed into heat energy, or has done work on something. It has changed form, but the books must always balance.

Work and energy have the same units; in SI these are called joules after James Prescott Joule. One joule is equal to one newton-meter. Between 1837 and 1847, Joule performed experiments to determine the "mechanical equivalent of heat," a conversion factor between work or energy units and heat units. It has been established that 4186 joules of work or energy is equivalent to 1 kilocalorie (food calorie) of heat. In 1847 Herman von Helmholtz published a mathematical proof showing that the law of conservation of energy follows directly from Newton's laws.

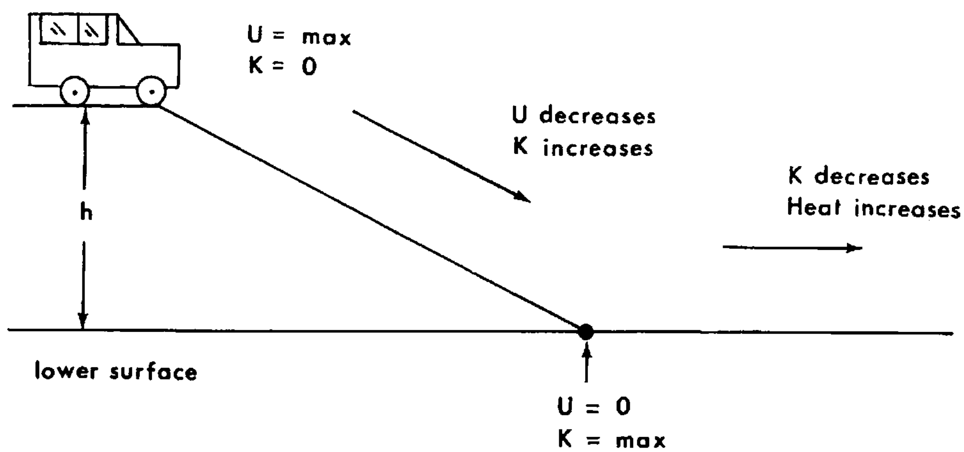
The law of conservation of energy is also known as the first law of thermodynamics. The second law of thermodynamics is a generalization of the studies of heat engines done by Sadi Carnot in the 1820s. It says that, although the total amount of energy in the universe is constant, it is becoming more dispersed and therefore less useful.

ADDITIONAL RESOURCES

Demonstration #1: Conservation of Energy (Inclined Plane)

Purpose: To show that energy is conserved although some is not useful.

Materials: Inclined plane; toy car or block of wood.



Procedure and Notes:

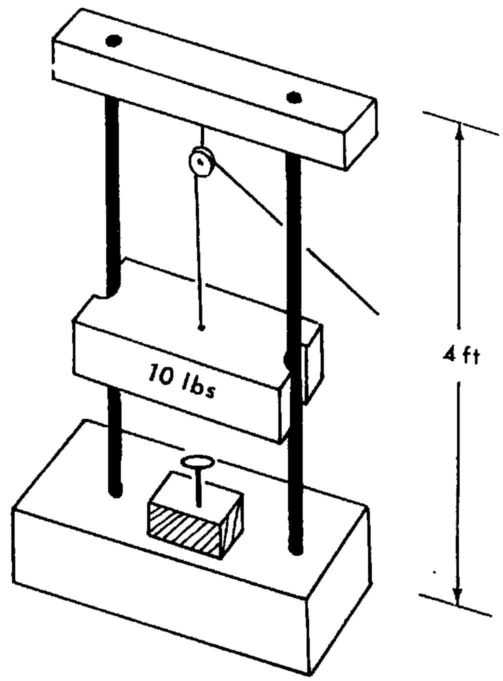
1. Set the car or block on the lower surface.
2. Lift the car vertically to the top of the inclined plane.
3. Release the car and allow it to roll down the plane and along the lower surface until it comes to a stop.

Explanation: Work is done on the car when it is placed at the top. This work is stored as gravitational potential energy. When the car is released and it moves down the incline, its velocity increases. Therefore, the kinetic energy increases. The height decreases and thus the gravitational potential energy decreases. At the instant the car reaches the lower surface, all the energy is kinetic. The car then moves along the horizontal surface and slows to a stop. The energy has not been lost. Instead, the kinetic energy has been transformed into heat energy by the frictional force that slows down the car.

Demonstration #2: The Pile Driver

Purpose: To demonstrate energy transformation.

Materials: Pile-driver apparatus, a block of wood, a nail.



Procedure and Notes:

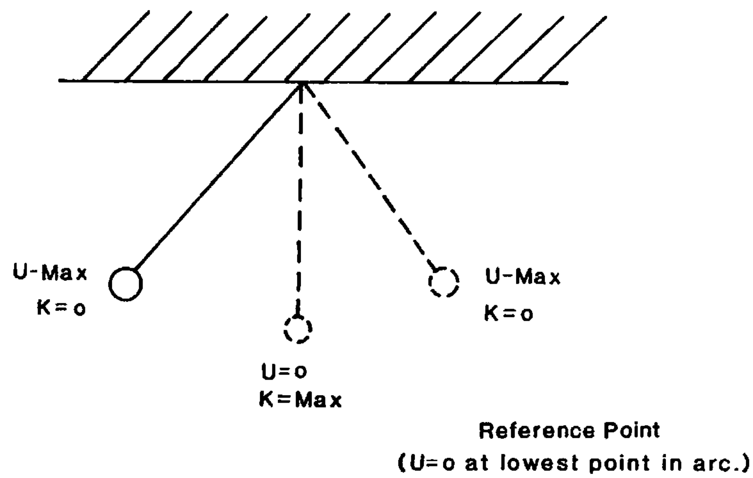
1. Under the driver, position a block of wood with a nail partially driven into the block.
2. Pull the driver all the way to the top of the rails.
3. Allow the driver to fall.

Explanation: Work is done on the system to raise the driver. The amount of work done is determined by measuring the force required to lift the driver and the height to which it is raised. The product of these two quantities is the gravitational potential energy that the driver has at the top of the rails. As the driver falls, its potential energy is converted to kinetic energy. This energy is used to place a large force on the nail and move it a short distance.

Demonstration #3: Conservation of Energy (Pendulum)

Purpose: To show the transformations of energy.

Materials: Simple pendulum.



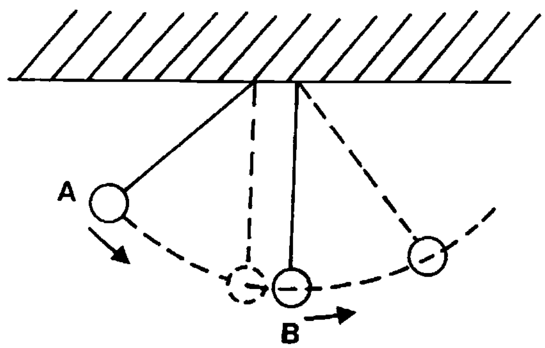
Procedure and Notes: Raise the bob to a given height and release. Have students describe the energy transformation.

Explanation: At the top of the arc, the pendulum momentarily stops; all the energy is potential. As the bob swings back down, the potential energy is transformed into kinetic energy. The kinetic energy is greatest at the bottom of the arc, and the potential energy is zero (note reference point). The pendulum gradually decreases in height, indicating the transformation into heat.

Demonstration #4: Energy Transfer

Purpose: To show the transfer of energy from one object to another.

Materials: Collision balls, billiard balls, or a Newtonian demonstrator.



Procedure: Raise one ball and let it collide into the stationary ball. Have students describe the transfer of energy.

Explanation: Most of the kinetic energy of Ball A is transferred to Ball B. This is evidenced by the fact that Ball A stops, and Ball B has about the same speed as Ball A had (at least by eye-ball observation).

Note that the kinetic energy given to Ball B is transformed to potential energy, making Ball B rise to nearly the same height as Ball A had originally. If this process were allowed to continue, the heights would gradually decrease showing energy transformed to other forms, namely heat and sound energy.

EVALUATION QUESTIONS

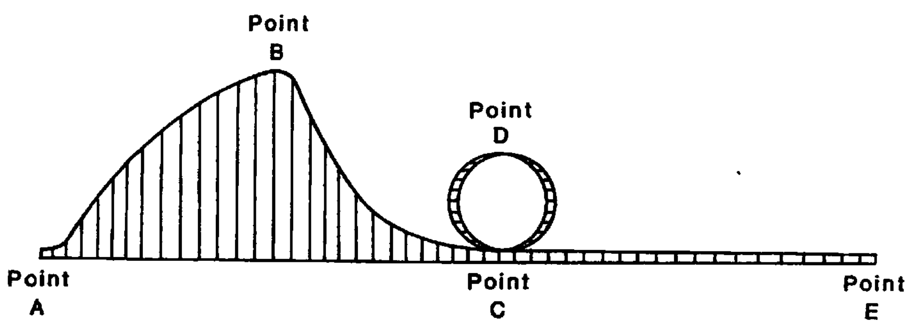
1. **A pendulum swings back and forth. At the bottom of its arc**

   A. U and K are maximum.
   B. U and K are minimum.
   C. U is maximum and K is minimum.
   D. U is minimum and K is maximum.

2. **Which of the following is *not* an example of kinetic energy?**

   A. A flying bird.
   B. A boy riding a tricycle.
   C. A bird sitting on a telephone wire.
   D. A rotating lawn sprinkler.

A roller coaster car is initially pulled up a hill from Point A, where it has no potential energy, to Point B. It then rolls down the hill, around the loop at Points C and D, and comes to rest at Point E. (Points A, C, and E are all at the same level.) Questions 3 through 8 refer to the motion of the car.



3. **The greater part of the work done on the roller coaster as the car is pulled from Point A to Point B goes into**

   A. speed of the car.
   B. kinetic energy.
   C. gravitational potential energy.
   D. air resistance.

4. **At which point does the roller coaster car have both kinetic and potential energy?**

   A. Point A
   B. Point B
   C. Point E
   D. Point D

5. **Since the roller coaster car starts out at rest at Point A and ends up at rest at Point E,**

   A. its energy is conserved from Point A to Point B.
   B. the work done initially pulling the car up the hill is eventually completely converted into heat energy.
   C. the potential energy of the car at Point A is greater than at Point E.
   D. the kinetic energy of the roller coaster car remains constant from Point A to Point D.

6. **Suppose that for an added thrill you wanted to have the roller coaster car moving at twice its present speed at the bottom of the hill. To accomplish this, you would have to make the hill**

   A. twice as high as it presently is.
   B. half as high as it presently is.
   C. four times as high as it presently is.
   D. three times as high as it presently is.

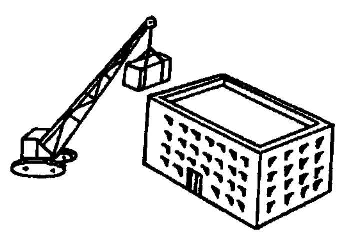
7. **The track from Point A to Point B is not as steep as from Point B to Point C. The roller coaster is designed this way so that**

   A. a smaller force is needed to get the roller coaster car up the hill.
   B. less work is done to get the roller coaster car up the hill.
   C. the roller coaster car will have a greater speed at Point C than it would if it had rolled down the hill from Point B to Point A.
   D. the roller coaster car gains more potential energy on the way up than it loses on the way down the hill.

8. **In which of the following is there no change in gravitational energy?**

   A. A person roller skating at a rink.
   B. A person skiing down a slope.
   C. A person on a roller coaster ride.
   D. A person swinging in a swing.

9. **A crane lifts a large mass (M) to the top of a two story building.**



   Suppose that just as the mass reaches the second floor the rope breaks and the mass falls to earth. Its energy just prior to striking the ground will be

   A. gravitational potential energy.
   B. kinetic energy.
   C. heat energy.
   D. primarily dissipated in the air.

10. **A stone is dropped from a height of 120 m above the ground. At what point in the fall is the kinetic energy equal to the potential energy assuming that is has zero potential energy at ground level?**

    A. 30 m.
    B. 40 m.
    C. 60 m.
    D. 80 m.

ESSAY QUESTIONS

11. **Describe the energy changes of a bouncing ball that does not return to its original height.**

12. **Trace the energy transformation for a skier who starts from rest at the top of a hill and then stops after she reaches the bottom.**

KEY

1. **D**
2. **C**
3. **C**
4. **D**
5. **B**
6. **C**
7. **A**
8. **A**
9. **B**
10. **C**

SUGGESTED ESSAY RESPONSES

11. As the ball falls, its potential energy is being transformed into kinetic energy. Just before striking the ground, the energy is all kinetic. As the ball strikes the ground, the kinetic energy is transformed into elastic potential energy, which is returned to the ball as kinetic energy. The ball is sent upwards giving it potential energy once again. Because the ball is not at the original height, some energy must have been lost.

12. At the top of the hill the skier has all potential energy, $U = mgh$. As she skis down the hill, work is done on her by the gravitational force of the earth, and her potential energy is converted into kinetic energy. At the bottom of the hill, her energy is entirely kinetic, $K = 1/2mv^2$. When she comes to a stop, all of her kinetic energy has been transformed into heat energy by friction acting on her and her skis.

TEACHER'S GUIDE TO CONSERVATION OF MOMENTUM

CONTENT AND USE OF THE VIDEO - This video should be shown after a study of Newton's laws. If energy has not yet been studied, students should be exposed to a brief treatment of conservation of energy that covers the definition of kinetic energy and its conservation in elastic collisions. The video is intended as an introduction to conservation of momentum and to the topic of collisions. It should prime students for further work with elastic collisions of unequal masses, non-elastic collisions, and impulses.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - The following terms are used throughout the video. It is advisable to introduce and/or review them before viewing the video.

momentum--the product of an object's mass and its velocity; a vector quantity, usually denoted **p**. Newton originally used the term "quantity of motion" where momentum is now used.

center of mass--the point where all the mass of an extended body can be imagined as being concentrated in order to analyze the translational motion of the body.

kinetic energy--the energy an object possesses simply because it is moving; mathematically, this energy is expressed as half the product of an object's mass and the square of its speed.

conservation of momentum--a statement representing the fact that the sum total of momenta of all parts of a system remains constant if no outside forces act on the system.

elastic collision--the type of collision in which, during the interaction, no energy is transformed into heat or other types of stored energy; kinetic energy, and certainly momentum, are conserved in elastic collisions.

closed system--a system of bodies upon which no external forces act or upon which any forces that act are assumed small enough to be ignored.

WHAT TO EMPHASIZE AND HOW TO DO IT - The concept that **the total momentum of a system is constant when no external forces act on that system** is developed in the video. The law of conservation of momentum flows directly from Newton's laws of mechanics. Discussion of the momentum of a compound body focuses on the motion of the center of mass. The video further develops the concept that **a collision in which both momentum and kinetic energy are conserved is an elastic collision.**

Prior to viewing the video, point out to students that block letters represent vector quantities while script letters represent scalars.

Objective 1: Recognize the relationship between Newton's laws of mechanics and the law of conservation of momentum.

Students can be led to a formulation of the law of conservation of momentum by restating Newton's laws in terms of momentum. The SUPPORTIVE BACKGROUND INFORMATION focuses on this translation. DEMONSTRATIONS #3, #4, and #5 provide opportunities to discuss conservation of momentum from several different perspectives. A number of the EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS challenge students to consider application of the law.

Objective 2: Describe the motion of the center of mass of a compound system.

Explain that the center of mass is not necessarily at the geometric center of an object; it might not even be *in* the object. Cite doughnuts, boomerangs, and hammers as examples. To familiarize students with the concept of center of mass, you might perform DEMONSTRATIONS #1 and #2. DEMONSTRATION #5 shows the motion of the center of mass during an "explosion," which can be related to fireworks and the exploding-billiard-ball segment in the video. Remind students that the center of mass is the point to focus on when considering the velocity and acceleration of a compound body. If you stop the video at various places during the exploding-billiard-ball segment, you can trace the motion of the center of mass on the screen with your finger. Point out that the motion of the center of mass remains the same before and after the explosion.

Objective 3: Identify the conditions necessary for an elastic collision.

The billiard-ball collisions shown in the video are assumed to be elastic collisions, where kinetic energy is conserved. Momentum is conserved in all collisions, whether elastic or inelastic. In a glancing collision between two objects of equal masses, such as billiard balls, the balls rebound at right angles, as illustrated in the video; DEMONSTRATION #6 reinforces this idea.

You might ask students why head-on collisions are advantageous in atom smashers. They should realize that the "struck" particle is more likely to be shattered since there is a total transfer of momentum in a head-on collision.

Objective 4: Recognize that the total momentum of a closed system is constant even though the individual momenta of any number of bodies within the system may change.

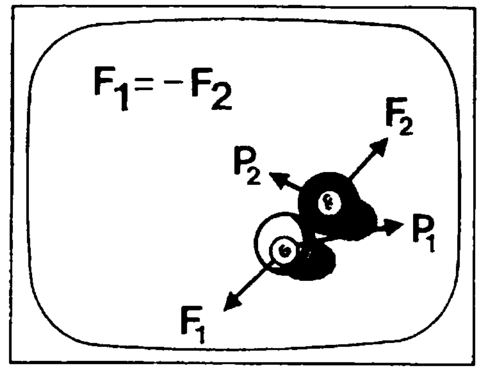
Be sure that students understand the definition of a closed system. Discuss what influences, if any, are being ignored by considering the following as closed systems: (a) the billiard balls depicted in the video, where friction is ignored; (b) the earth-sun system where other gravitating objects are ignored; and (c) the universe, where all influences are considered.

DEMONSTRATIONS #3 and #4 illustrate that the total momentum in a closed system is conserved even though momenta of the individual carts (or balls) change. Discuss the relationship of the momenta of the carts to their velocity and mass.

POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and frames from the video.

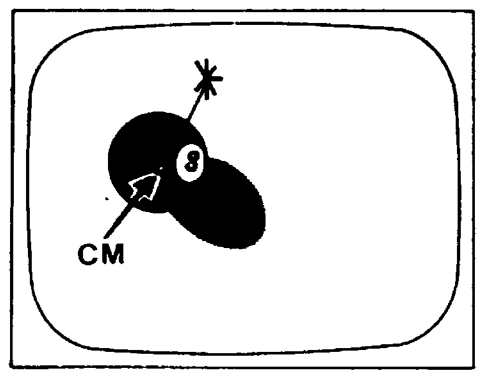
What forces are acting on the billiard balls in the frame?

A body with no forces acting on it will continue moving with the same momentum, i.e., at the same speed in a straight line. This is the law of inertia, Newton's first law. But when billiard balls collide, each one applies a momentary force on the other, causing the momentum to change. Here Newton's third law gets into the action.



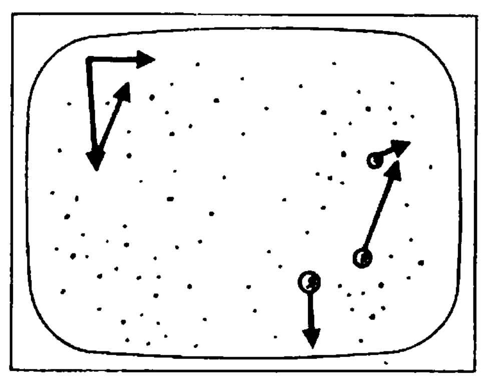
What happens to the motion of the center of mass of the moving 8 ball if it explodes?

The center of mass continues to move at constant speed in a straight line.

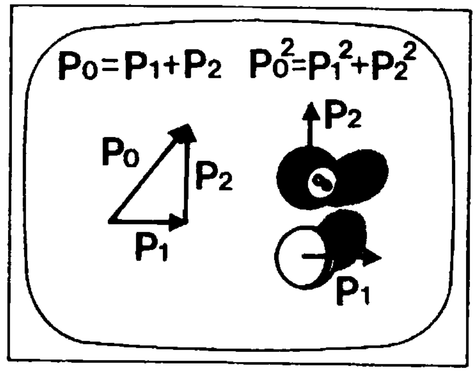


What does this frame represent?

It shows that the sum of the three bodies' momentum vectors is equal to the momentum vector of the center of mass.

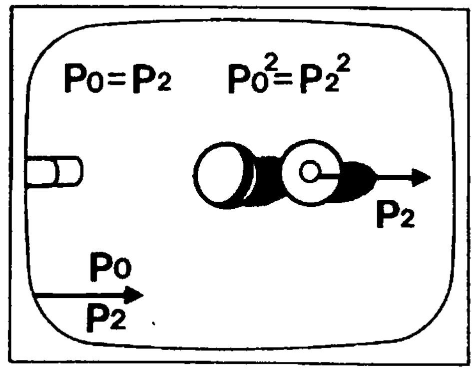


When one billiard ball strikes another, initially at rest, how do they rebound?



In this special case of an object of equal mass colliding with one initially at rest, they rebound with their momenta at right angles. Note that the equation on the left is a statement of conservation of momentum. The equation on the right (obtained by the Pythagorean theorem) is a statement of conservation of energy. Divide both sides by 2m ($p^2/2m = 1/2\,mv^2$).

If the ball is hit just right, that is, exactly head-on, both laws, conservation of momentum and conservation of energy, can still be satisfied in another way. What is that way?



One ball can stop completely, transferring all its momentum and thus energy as well to the other ball.

EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in the video, you might pose the following questions to your students.

1. **What characterizes an elastic collision?**

   When both momentum and kinetic energy are conserved, the collision is said to be an elastic collision. In inelastic collisions only momentum is conserved.

2. **Why is it better to hold a rifle tightly against your arm while firing it rather than holding it loosely away from your body?**

   The force on the gun equals the force on the bullet. Holding the gun tight to you would increase the mass and lessen the recoil velocity.

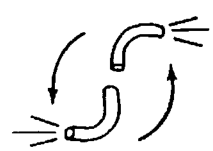
3. **What is "center of mass" and how does it relate to momentum conservation?**

   The point of a body where we can imagine all its mass as being concentrated is called the center of mass. The motion (or momentum) of the center of mass of a system is constant when no net outside force acts on it.

4. **Everyone has experienced conservation of momentum on countless occasions. The following are some common experiences and applications with which students may be familiar.**

   (a) Two persons pushing each other away on roller skates or ice skates.
   (b) The recoil of a gun or cannon when shot.
   (c) The path of a fireworks rocket before and after the explosion.

   (d) The recoil of a raft in a lake when a person dives off one end.
   (e) The recoil of a skateboard when a person jumps off it.
   (f) The propulsion of rockets and airplanes.
   (g) The motion of billiard balls when struck by the cue ball.
   (h) The playing of various games such as air hockey.
   (i) The turning of a lawn sprinkler.



5. **How do high-jumpers and pole-vaulters use center of mass?**

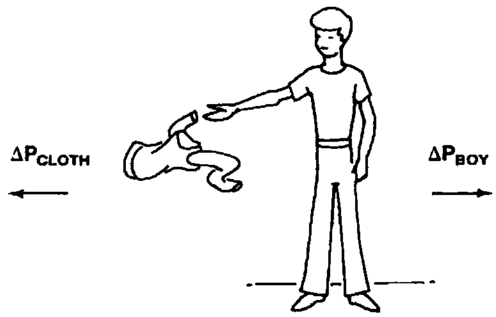
   The jumpers arch themselves so that their bodies can pass above the bar, even though their center of mass passes below the level of the bar.

6. **A watermelon is dropped and strikes the ground without bouncing. What becomes of its momentum?**

   The watermelon represents a small mass traveling at a measurable velocity. When its momentum is transferred to the earth, which has a large mass, the earth recoils with such a small velocity that it is unmeasurable.

7. **On a cold day a person is at rest in the middle of a frictionless ice pond. How can the person get to shore?**

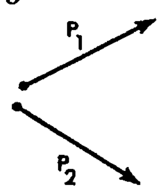
Throw off an article of clothing. The change in momentum of the clothing is equal and opposite to that of the person so the person recoils in the opposite direction that the clothing is tossed.



8. **While you are driving, a bug splatters on your car windshield. Compared to the change in momentum of the bug, how much does your car's momentum change?**

The change in momentum of the bug is equal and opposite to that of the car. Because the bug's mass is much smaller than the car's, the resulting change in velocity of the car is not noticeable, even though the change in the bug's velocity is large. And since the bug's structure is weaker than the car's, the collision has a devastating effect on the bug.

9. **A certain object is at rest. It suddenly "explodes." Two particles are detected shooting off as shown. Are these the only particles given off? Explain.**

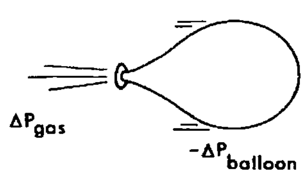


No. At least one other particle, whose momentum is $-(P_1 + P_2)$ *must be ejected. The total momentum of the system after the explosion must be equal to the total momentum of the system before the explosion (which equalled zero).*

NOTE: The antineutrino was discovered in this way. A free neutron "decays" into a proton, an electron (beta particle), and an antineutrino.

10. **Release an inflated balloon. Ask students to think of an explanation for the motion.**

The escaping gases have a change in momentum that is equal to but opposite to the change in momentum of the balloon: $\Delta P_{gas} = -\Delta P_{balloon}$. *Consequently, the balloon moves in the direction opposite to the escaping gases.*



11. **Each of the following situations is an example of the law of conservation of momentum. Discuss them in terms of Newton's laws of mechanics.**

    (a) A cannon recoils when fired.
    (b) An ice skater glides along a nearly frictionless surface with no change in her momentum.
    (c) A bowling ball hits the pins, scattering them.

    Answers should be discussed in terms of Newton's three laws of mechanics and should contain elements of each law:

    First Law: *An object will remain at rest or continue in motion in a straight line at a constant speed unless acted upon by some external force.*

    Second Law: *Force equals mass times acceleration.*

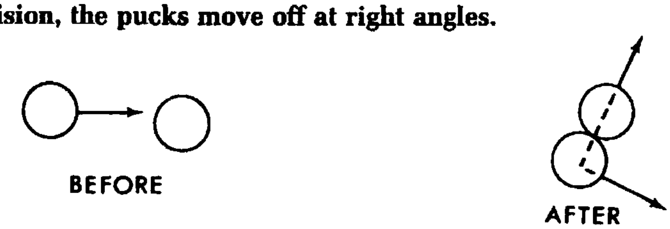
    Third Law: *For every action, there is an equal and opposite reaction.*

12. **Which of the following are examples of elastic collisions?**

    (a) Object A hits a stationary object (B) of equal mass. After colliding, A is found to be still and B has moved off with the original velocity of A.

    (b) A blob of putty of mass *m* is thrown against a cart of mass 2*m*, where it sticks. The putty-cart combination moves off together with one-third the original velocity.

    (c) An air hockey puck collides with an identical stationary puck in a non-head-on collision. After the collision, the pucks move off at right angles.



    Both (a) and (c) are elastic collisions because they satisfy conservation of momentum and of kinetic energy. In (b), kinetic energy is not conserved.

13. **Any good saloon brawler knows that, to avoid getting your jaw broken with a punch, keep your mouth closed (literally and figuratively). How does conservation of momentum shed light on this advice?**

    When your mouth is closed, your jaw becomes part of your head's mass. The increased mass requires a greater velocity to displace it, and so your jaw is less likely to be shattered.

14. A projectile explodes while in flight. Fragments are blown in all directions. What can you say about the center of mass of the system after the explosion?

The center of mass continues to move along a parabolic path.

SUMMARY - Momentum is defined as the product of a body's mass and velocity. Newton's three laws of motion, stated in terms of momentum are really a statement of the law of conservation of momentum. In a system of interacting particles, momenta of the individual particles may change but the total momentum of a system is constant when no external forces act on the system. If the interaction takes the form of elastic collisions, not only is momentum conserved but kinetic energy is also conserved. Application of the law of conservation of momentum to collision interactions is an important study in modern physics.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO CONSERVATION OF MOMENTUM

INTRODUCTION - This video defines momentum and explores the concept of conservation of momentum by examining colliding billiard balls.

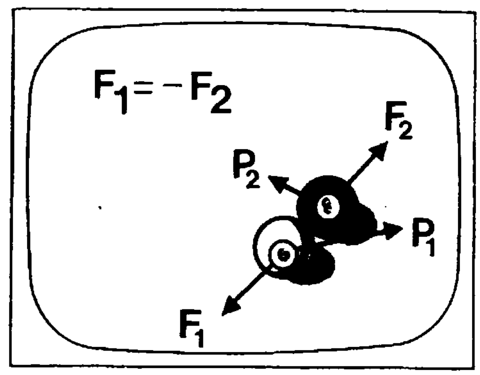
Terms Essential for Understanding the Video

momentum
kinetic energy
elastic collision
center of mass
conservation of momentum
closed system

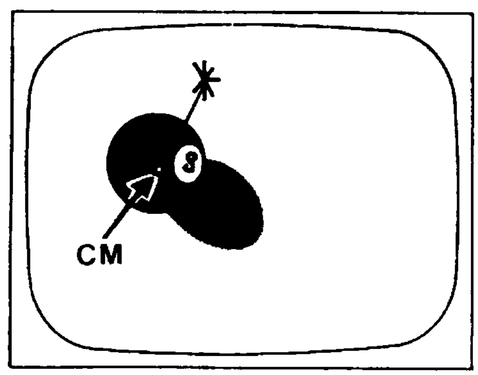
**** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ****

Points to Look for in the Video

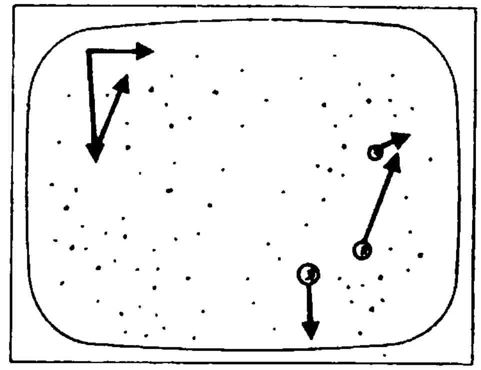
What forces are acting on the billiard balls in the frame?



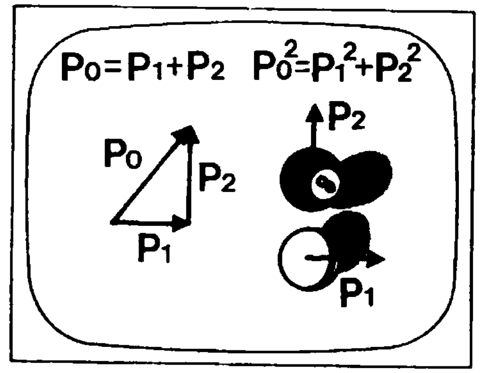
What happens to the motion of the center of mass of the moving 8 ball if it explodes?



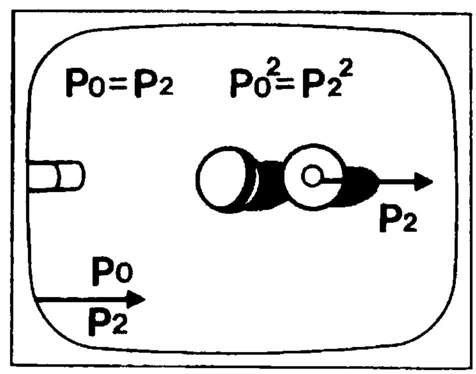
What does this frame represent?



When one billiard ball strikes another initially at rest, how do they rebound?



If the ball is hit just right, that is, exactly head-on, both laws, conservation of momentum and conservation of energy, can still be satisfied in another way. What is that way?



TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - The seventeenth century was the age when science assumed its modern form and the scientific spirit infected Europe. One of the most influential figures of that century was René Descartes. He sought new foundations for knowledge that could underpin certainty in our knowledge of nature. He strove to identify mathematics as the language of physics. He clarified Galileo's principle of inertia by stating that, in the absence of forces, a body would continue to move in a straight line. In creating a system of the universe as a whole, Descartes envisioned the universe as a clockwork machine inexorably following purely mechanical laws. He proposed that the total "quantity of motion" in the universe is constant. Isaac Newton refined the idea of quantity of motion, identifying it as mass times velocity for a body – a quantity now called momentum. His three laws of motion, which irrevocably altered physics and mankind's perception of the universe, embody an even deeper principle – the law of conservation of momentum. Today we call Descartes' "quantity of motion" momentum – the vector quantity $\mathbf{p}$ that is the product of the mass of an object times its velocity:

$$\mathbf{p} = m\mathbf{v} .$$

In SI units, momentum is measured in kg · m/s.

Descartes' idea of conservation of momentum follows from Newton's laws of motion. Consider Newton's second law:

$$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{\Delta(m\mathbf{v})}{\Delta t} = \frac{m\Delta \mathbf{v}}{\Delta t} = m\mathbf{a} .$$

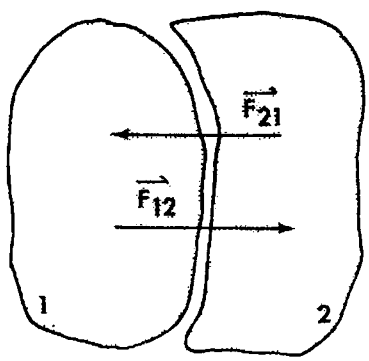
Note that in going from $\Delta(m\mathbf{v})$ to $m\Delta\mathbf{v}$ we are assuming that the mass is not changing. Newton's second law, stated in terms of momentum, is more general.

If there are no net unbalanced forces acting on a mass m, then

$$\frac{\Delta \mathbf{p}}{\Delta t} = 0 ,$$

which implies $\Delta\mathbf{p} = 0$. But if $\Delta\mathbf{p}$ is zero, then $\mathbf{p}$ is unchanging: $\mathbf{p}$ = constant. This is Newton's first law stated in terms of momentum.

Imagine a compound body composed of two separate particles. If no net external or outside force acts on the compound body, the internal forces (gravitational, electric, nuclear) must add up to zero. If $\mathbf{F}_{12}$ is the force exerted on 2 by 1 and $\mathbf{F}_{21}$ is the force exerted on 1 by 2, then



$$\mathbf{F}_{12} + \mathbf{F}_{21} = 0 ,$$

$$\mathbf{F}_{12} = -\mathbf{F}_{21} .$$

By Newton's second law this indicates

$$\frac{\Delta \mathbf{P}_{12}}{\Delta t} = - \frac{\Delta \mathbf{P}_{21}}{\Delta t} ,$$

$$\Delta \mathbf{P}_{12} = - \Delta \mathbf{P}_{21} .$$

The change in momentum which 1 experiences due to the influence of 2 is equal and opposite to the change in momentum which 2 experiences due to 1. This is Newton's third law stated in terms of momentum.

Taken together, Newton's laws of motion are really a statement of the **law of conservation of momentum**. If no outside forces act on a closed system of n objects, the total momentum is a constant:

$$\mathbf{P}_{total} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \cdots + \mathbf{p}_n = \text{constant.}$$

The ultimate closed system is, of course, the universe. Frequently, smaller "closed" systems are used in problems where outside influences are small enough to be ignored. The video considers the earth-sun system as a closed system in this context. If friction is ignored, billiard balls constitute another closed system.

The video refers to the concept of **center of mass** – the point of a body where we can think of all the mass as being concentrated. It can be, but need not be, at the geometric center of an object. It may even lie outside the object, as in the case of a doughnut or a roll of tape. Newton's laws describe the motion of the center of mass of a compound body.

The calculations done on the Newtonia sequence in the video are:

$$\text{total momentum before firing} = 0\,,$$
$$\text{momentum after firing} = 0\,.$$

Therefore $\mathbf{p} + \mathbf{P} = 0\,,$

or $m\mathbf{v} + M\mathbf{V} = 0\,,$

where the lowercase letters refer to the astroshell and the capitals refer to the Newtonia. Since the recoil velocity $\mathbf{V}$ of the Newtonia is directed opposite to the astroshell velocity $\mathbf{v}$, the last vector equation can be changed to an algebraic one:

$$mv - MV = 0\,,$$

so that $V = \dfrac{m}{M}\,v\,.$

From this we see that the more massive the Newtonia, the less it recoils.

The billiard-ball collisions shown in the video are assumed to be **elastic collisions**. In elastic collisions, not only is momentum conserved, but also kinetic energy is conserved. Remember that the **total** energy is always a conserved quantity in the universe. Since no transformations of energy into heat or potential energy occur through an elastic collision, we need consider only the total kinetic energy before and after the collision.

The connection between kinetic energy and momentum is easily established:

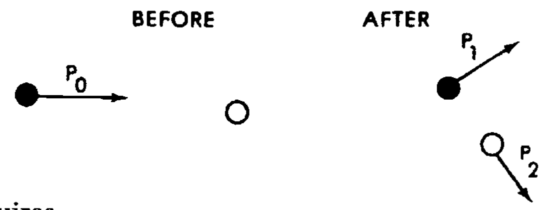
$$K = 1/2mv^2 = 1/2\,\frac{m}{m}\,mv^2\,,$$

$$K = 1/2\,\frac{(mv)^2}{m}\,,$$

and since $p = mv$, we get

$$K = \frac{p^2}{2m} .$$

This connection leads to the interesting result that, in the non-head-on elastic collision of two equal masses where one is initially at rest, the two masses will fly off at right angles. Here's why: Suppose the initial momentum of the incident ball is $\mathbf{P}_0$ and its momentum after the collision is $\mathbf{P}_1$. The other (target) ball has zero momentum before the collision and $\mathbf{P}_2$ after. The balls collide and move off as shown below.



Conservation of energy requires

$$K_{before} = K_{after} , \text{ which implies}$$

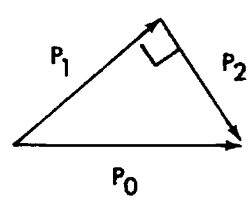
$$\frac{P_0^2}{2m} = \frac{P_1^2}{2m} + \frac{P_2^2}{2m} ,$$

$$P_0^2 = P_1^2 + P_2^2 .$$

Conservation of momentum implies

$$\mathbf{P}_0 = \mathbf{P}_1 + \mathbf{P}_2 .$$

Geometrically, this means that the vectors form a closed triangle, as shown below:



Because the Pythagorean relation $P_0^2 = P_1^2 + P_2^2$ holds, it follows that $\mathbf{P}_1 \perp \mathbf{P}_2$. In any elastic glancing collision of two equal masses, the objects move off at right angles to each other. If the elastic collision between equal masses is head on, one mass will stop completely and give all its motion to the other mass. This can be shown by invoking both momentum and kinetic energy conservation:

$$\frac{P_0^2}{2m} = \frac{P_1^2}{2m} + \frac{P_2^2}{2m} ,$$

and $$\mathbf{P}_0 = \mathbf{P}_1 + \mathbf{P}_2 .$$

Eliminating $\mathbf{P}_0$ we get

$$(\mathbf{P}_1 + \mathbf{P}_2)^2 = \mathbf{P}_1^2 + \mathbf{P}_2^2 ,$$

which can only be true if either $\mathbf{P}_1$ or $\mathbf{P}_2$ is zero, but not both. If $\mathbf{P}_2 = 0$, then no collision occurred (the target ball remained at rest). Therefore, $\mathbf{P}_1 = 0$ which means that the first ball comes to rest and the target ball leaves with the same initial momentum as the incident ball.

The processes and ideas dealt with in this video are extremely important in physics. Collisions shape the real world. The very information we acquire through sound and light may be described ultimately in terms of collisions, and our only source of information about the subatomic world comes from collisions. Consequently, experiments are conducted in colossal particle accelerators ("atom smashers") where particles, such as protons, are smashed together to investigate the structure of the particles and the forces at play. The debris of these collisions can be seen as tracks of tiny bubbles in chambers of liquid hydrogen. From such tracks, momenta can be measured and particles identified as shown in the video.

Newton's laws, stated in terms of force, break down in describing the quantum-mechanical interactions of the world around us. In fact, it is often said that quantum mechanics replaces the classical Newtonian physics. However, Newton's laws imply the conservation of momentum. And this principle, along with conservation of energy, remains unaltered in quantum mechanics.

ADDITIONAL RESOURCES

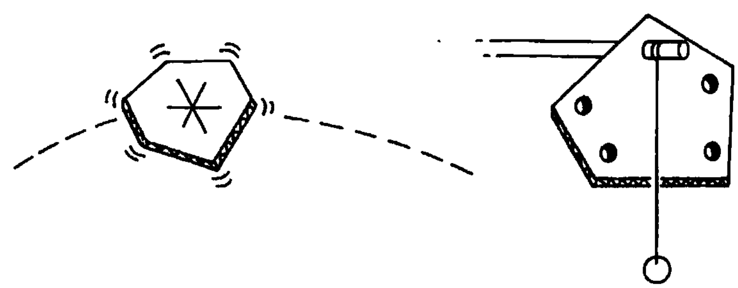
Demonstration #1: Finding the Center of Mass

Purpose: To determine the center of mass of an irregular flat shape. To show that an object will rotate about its center of mass.

Materials: Cardboard, nail, plumb line, clamp, and pole.

Procedure and Notes:

1. Punch four or five holes around the edge of the irregular piece of cardboard. Hang the object from these points; each time using the plumb line to mark the line from the support point perpendicular to the earth's surface. The intersection of these lines will be at the center of mass.
2. After determining the center of mass, place the cardboard on your fingertip at the center of mass. This point is also called the center of gravity because the object is stable when supported there.
3. Sail the cardboard, giving it a rotation, across the room. Students should try to watch the center of mass.
4. Place a mark (⋆) on the opposite side of the cardboard at a position other than the center of mass. Sail the cardboard once again and have the students observe the mark.



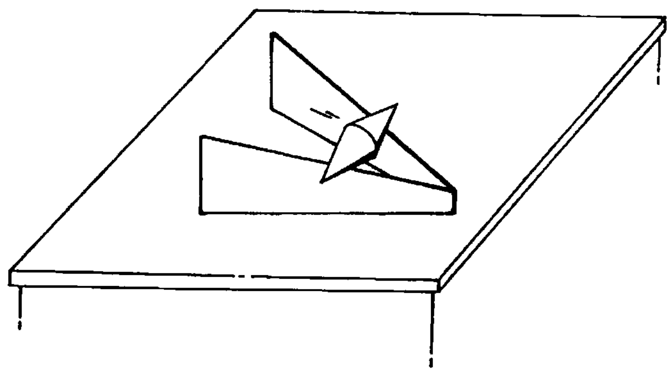
Explanation: The object will always orient itself so that it has a minimum potential energy; hence the center of mass will always be directly below the point of support. In the first case, the cardboard will rotate about the center of mass, which will take the path of a projectile. In the second case, the center of mass follows a parabolic path as before, but the mark rotates about the center of mass.

Demonstration #2: Double Cone

Purpose: To demonstrate the fact that as a body rolls downhill its center of gravity is lowered.

Materials: Double cone and ramp.

Procedure and Notes: Place the center of the double cone near the bottom of the ramp. When released, it *appears* to roll uphill.



Explanation: Although the cone rolls on the ramp, its center of gravity actually rolls *down* the ramp. This occurs because the rails are not parallel and so the cone makes contact with the rail farther and farther from its center. As this occurs, the center of gravity becomes lower and lower. You may wish to measure the height of the center of mass of the cone from the table at the start and the finish. Students will then see the center of mass is lower at the finish.

Demonstration #3: Conservation of Momentum

Purpose: To demonstrate the law of conservation of momentum.

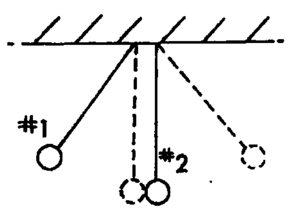
Materials: Two collision balls or two momentum (spring) carts.

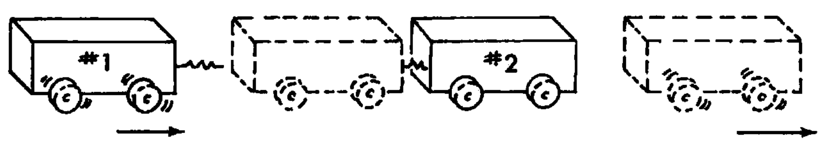
Procedure and Notes: Raise and release one ball and let it collide with the second, or let a moving cart collide with one at rest. Have students note the velocity of each throughout the event. Since, in each case, the masses are the same, the velocities represent the momenta:

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$
$$m_1v_1 + 0 = 0 + m_2v_2'$$
$$m_1v_1 = m_2v_2'$$

Since $m_1 = m_2$,
$v_1 = v_2'$.

Note: If the collision balls are used, they must be confined to swing in a single plane.





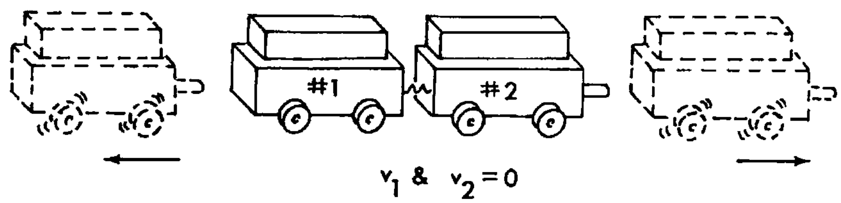
Explanation: Since the masses are equal, the fact that #1 stopped and #2 gained nearly the same velocity as #1 tells us that the momentum was about the same before and after the collision. Of course, friction is ignored in the collision to make the two balls or carts a closed system.

Demonstration #4: Conservation of Momentum

Purpose: To demonstrate the law of conservation of momentum.

Materials: Two momentum (spring) carts and several bricks.

Procedure and Notes: Set two momentum carts end to end with the spring of one compressed. Load each with a brick. Release the spring and observe the velocities. Repeat using unequal numbers of bricks on the carts.



Explanation: In the case where the masses are nearly equal, the carts fly apart with nearly the same speed but in *opposite* directions. This means that the total momentum of the system before and after the explosion is the same. For the unequal masses, the only way momentum could possibly be conserved is for the more massive cart to have a smaller velocity than the less massive cart.

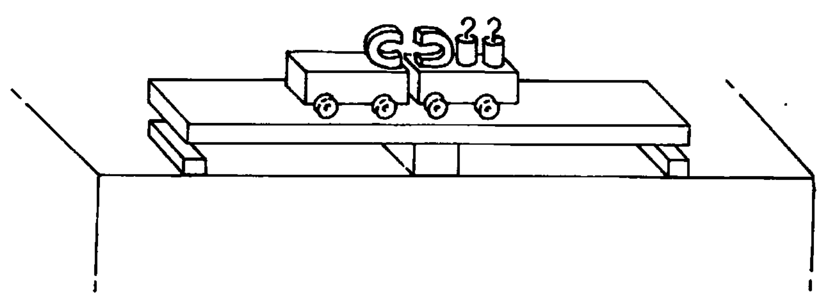
Demonstration #5: Conservation of Momentum and the Motion of the Center of Mass

Purpose: To demonstrate the law of conservation of momentum and to observe the motion of the center of mass during an "explosion."

Materials: Two momentum carts, two small magnets, bricks or masses, and special track.

Procedure and Notes: See Demonstration #3 for the basic idea of the conservation of momentum. Another method of creating an explosion is to use two large (magnetron) magnets. One is attached to each cart in such a way that their poles repel. A track can easily be made to constrain the motion of the carts to a single dimension. Obtain a 6 foot, 1 × 6-inch board; attach two thin strips of wood or metal so that the wheels of the cart run between the strips:

The movement of the center of mass can be demonstrated if one balances the track with the carts in the ready-to-explode position. Release the carts and observe that the system stays balanced as the carts move to the far ends of the track.



Explanation: The two carts become an isolated system because there are no outside forces acting on the system in the horizontal direction. Friction has about the same effect in slowing each cart. If there are no outside forces acting on any system, the center-of-mass motion remains the same. In this case, the center of mass was at rest before the explosion. It remained at rest after the explosion, because no outside forces acted upon it.

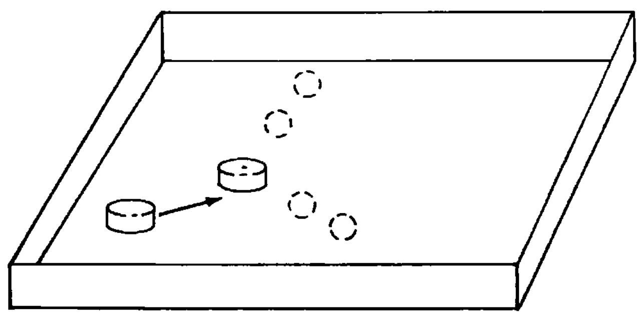
Demonstration #6: Collision in Two Dimensions

Purpose: To demonstrate that colliding objects which do not strike head-on move apart at right angles to one another.

Materials: Two objects (pucks, marbles, etc.) and a box lined with carbon paper.

Procedure and Notes:

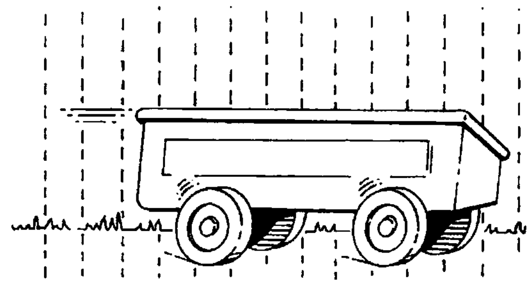
1. Place an object in a stationary position in the box. Mark the position.
2. Collide a second object with the first object, but not head-on.



3. Note the spots where the two objects struck the walls of the box.
4. Draw lines connecting the initial position of the stationary object with the positions where the objects struck the walls.
5. Measure the angle between the lines.

Explanation: Colliding objects which do not strike head-on move apart at right angles to one another. Therefore, the measure of the angle between the lines should approximate 90 degrees.

EVALUATION QUESTIONS



An open wagon is rolling along a level road in a downpoor in which the raindrops are falling vertically with respect to the ground. As the wagon moves, a substantial amount of water falls into it and remains there. The following questions refer to this situation.

1. **As the falling raindrops hit the wagon they exert force on the wagon, which is equal in magnitude to the force of the wagon on the drops according to**

   A. Newton's first law.
   B. Newton's second law.
   C. Newton's third law.
   D. The law of inertia.

2. **The force of the wagon on the falling drops causes the drops**

   A. only to stop falling vertically.
   B. only to move horizontally with the wagon.
   C. to stop falling vertically and to move horizontally at first.
   D. to come to rest.

3. **As the water accumulates, the speed of the wagon and water will**

   A. increase.
   B. decrease.
   C. not change.
   D. change, but in a way that can't be specified.

4. **As the water accumulates, the momentum of the wagon and water will**

   A. increase.
   B. decrease.
   C. not change.
   D. change, but in a way that can't be determined with the information provided.

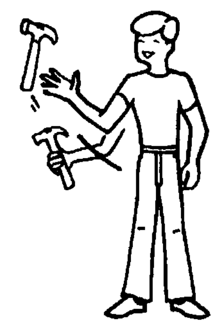
5. **If the rain stopped, ignoring all frictional effects, the wagon would**

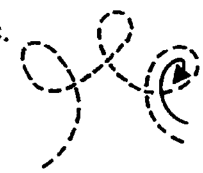
   A. continue to move with a constant speed.
   B. slow down.
   C. speed up.
   D. come to a halt.

6. **A light and a heavy body have equal momenta. How do their speeds compare?**

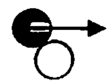
   A. They are the same.
   B. The heavier body has a lesser speed.
   C. The heavier body has a greater speed.
   D. It cannot be determined with the information provided.

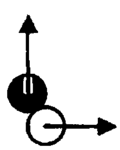
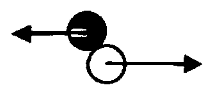
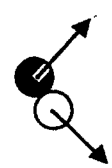
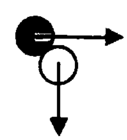
7. **A hammer is thrown upward at an angle with respect to the horizontal as shown. Which of the following depicts the path of the center of mass of the hammer?**



a.  b.  c.  d. 

8. **Two equal-mass billiard balls undergo an off-center, elastic collision, one of them initially at rest, as shown to the right. Which of the following vector diagrams best describes the possible momentum vectors of the balls after the collision?**



a.  b.  c.  d. 

9. **Object A, mass *m*, speed v, collides with object B at rest. After the collision, A moves with speed 1/2 v and B moves with speed 1/2 v. What was the mass of object B if this was an elastic collision?**

   A. 4*m*
   B. 3*m*
   C. 2*m*
   D. 1*m*

10. **A bug crashes into the windshield of a moving car. Which of the following is true?**

   A. Both bug and car undergo the same change in momentum.
   B. The car undergoes a greater change in momentum than the bug does.
   C. The bug undergoes a greater change in momentum than the car does.
   D. The bug's collision is elastic.

ESSAY QUESTIONS

11. **How may it be said that Newton's laws of motion embody the law of conservation of momentum?**

12. **A coconut falls from a tree and barely bounces. What happens to its momentum?**

KEY

1. C
2. C
3. B
4. C
5. A
6. B
7. D
8. C
9. B
10. A

SUGGESTED ESSAY RESPONSES

11. Newton's laws may be stated in terms of momentum. In an open system an unbalanced force causes a change in momentum for the particles in the system (second law). In a closed system the unbalanced force is zero and momentum is constant (first law). In a closed system all forces are internal so that any changes in momentum experienced by particles within the system are counteracted by equal and opposite changes in momentum by the remaining particles (third law).

12. The coconut's momentum transfers to the earth. Since it has a large mass, the earth recoils with an immeasurably small velocity.

TEACHER'S GUIDE TO ANGULAR MOMENTUM

CONTENT AND USE OF THE VIDEO - Although conservation of angular momentum is one of the fundamental principles of classical mechanics, it is not often taught in high school physics courses. When it is taught, it is usually done so with a focus on torque. The video approaches conservation of angular momentum from the perspective of planetary motion and within the context of Kepler's three laws. Torque is explained, but not stressed.

Students should have a thorough grounding in conservation of linear momentum and conservation of energy prior to viewing the video. If Kepler's laws are studied, the video could be introduced at that time; if not, the video could be used at any time that seems appropriate. Since an understanding of angular momentum is so dependent upon the idea of the vector cross product, a concept not always covered in high school mathematics, a brief discussion of the essentials of the cross products is presented in the SUPPORTIVE BACKGROUND INFORMATION section. To maximize student understanding of the concepts, it is recommended that the cross product be discussed.

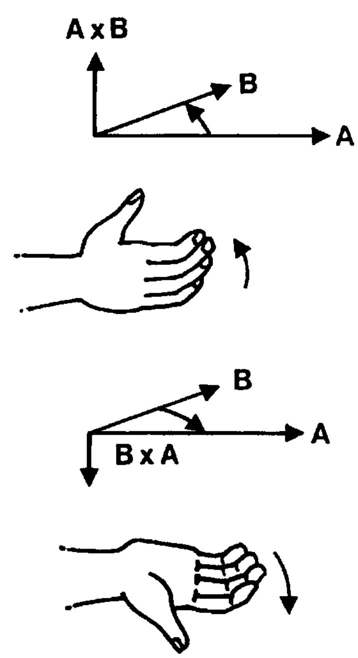
TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - Prior to viewing the video, the following terms should be discussed:

Kepler's law of equal areas--the law which states that, as a planet moves in its orbit about the sun, the line drawn from the sun to the planet will sweep out equal areas in equal times.

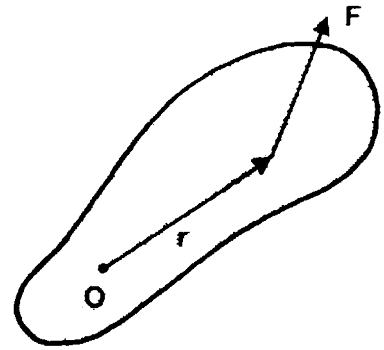
radial--in the direction of the radius (e.g., sun to planet orbiting the sun).

cross product--the product vectors **A** and **B** whose magnitude is given by **A** x **B** = |**A**| |**B**| sin θ. The direction of the resulting vector is perpendicular to the plane containing **A** and **B**, and its "sense" is given by the right hand rule.

right hand rule--the technique for determining the direction of the final vector when taking the cross product **A** x **B**. The vectors **A** and **B** will form a plane. With your right hand fingers extended and thumb perpendicular to your fingers put your right hand along the first vector (**A**) and curl your fingers toward the direction of the second vector (**B**). Your thumb, which will be perpendicular to the plane of **A** and **B**, gives the direction of the cross product. Note that **B** x **A** will reverse the direction of your thumb. Thus **B** x **A** = -**A** x **B**.



torque--something which causes a twist; a force can cause an object to rotate only if there is a component of the force perpendicular to the line drawn from the point of application of the force to the vector (r) drawn from the axis of rotation (O) to the point of application of the force. In the video, torque is defined with the following equation: $\tau = \mathbf{r} \times \mathbf{F}$, where x symbolizes a cross product multiplication.



vortex--the rotational motion of a fluid as it moves toward a common point; for angular momentum to be conserved, the velocity of the fluid becomes larger as the radius becomes smaller.

linear momentum or "momentum"--the product of a body's mass and its velocity, i.e., $\mathbf{p} = m\mathbf{v}$.

angular momentum--the cross product of the displacement vector of a moving object from some reference point and its linear momentum; angular momentum can be given by the equation: $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v}$.

conservation of angular momentum--a basic principle of physics which states that the total angular momentum in a closed system remains constant.

WHAT TO EMPHASIZE AND HOW TO DO IT - All three conservation laws--energy, linear momentum, and angular momentum--came from Newton's laws of motion and are useful bookkeeping devices for applying Newton's laws in complicated situations. For example, to write down and to solve Newton's laws for the motion of water in a bowl as it spirals down and out through an opening in the bottom would be exceedingly difficult. The principle of why a hole develops in the middle of the water, however, is not difficult to understand, once we understand why angular momentum is conserved. Nevertheless, the conservation laws are more significant than simply being special ways of applying Newton's laws. The video develops the concept that, in the absence of external torques, angular momentum is conserved.

Objective 1: Understand the concepts of torques and angular momentum and how they are related.

Angular momentum and torque are basic concepts which relate to how things rotate. Just as force causes linear momentum to change, so does torque cause angular momentum to change. It might be a good idea to review with students the conditions necessary for linear momentum to be changed, i.e., that an external force must be applied. The effects of torque, just as the effects of force, can be demonstrated in static situations. DEMONSTRATION #2 is a basic torque demonstration. It allows students to see how a force causes a twist in a static situation and illustrates the important parameters of torque. After the demonstration, you might discuss why a worker may remove a stubborn nut more easily with a large wrench. They should understand that the greater the distance from the point of rotation to the application of the force, the greater the torque.

Objective 2: Describe changes in angular momentum (a) when no torque acts on the body and (b) when a torque acts on the body.

When no torque acts on a rotating body, the angular momentum goes unchanged. When a torque acts on a rotating body, the angular momentum is changed. To emphasize further the relationship of angular momentum and torque, DEMONSTRATION #4 is appropriate here. This demonstration involves a more complex analysis since it shows how the vector nature of torque and angular momentum are related.

Objective 3: Recognize that conservation of angular momentum is a basic conservation law of physics.

Kepler's study of planetary motion revealed that the planets move about the sun in elliptical paths and sweep out equal areas in equal times. In order for the planet to sweep out equal areas in equal times, it is required that it move faster when it is close to the sun than it does when it is at a greater distance. The law of equal areas in equal times is equivalent to the statement that the angular momentum of the planet is constant. In a similar way, angular momentum is conserved when the velocity of an object varies inversely with its distance from the axis of rotation. DEMONSTRATIONS #1 and #3 illustrate this fundamental law of conservation of angular momentum. You might follow the demonstrations with a discussion of how satellites orbit the earth. Since satellites are being acting upon by gravity--a centrally directed force--they experience no external torques and therefore, must conserve angular momentum. Kepler's law of equal areas must result.

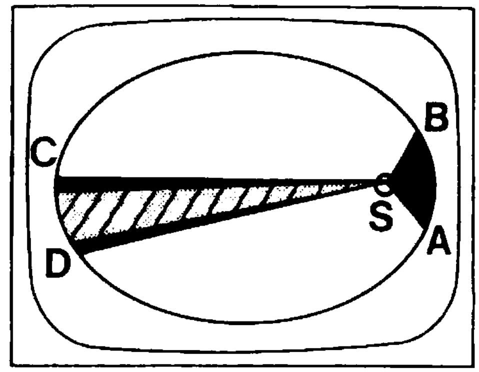
Objective 4: Recognize applications of angular momentum.

Brainstorm with students applications where conservation of angular momentum helps explain what happens. Examples might include: formation of galaxies, satellites descending to earth, hurricanes, a child walking from the outside to the inside of a merry-go-round, a spinning ice skater, a diver going off a board, and water going down a drain. The section on EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS provides a discussion of some of these and other applications.

POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and frames from the video.

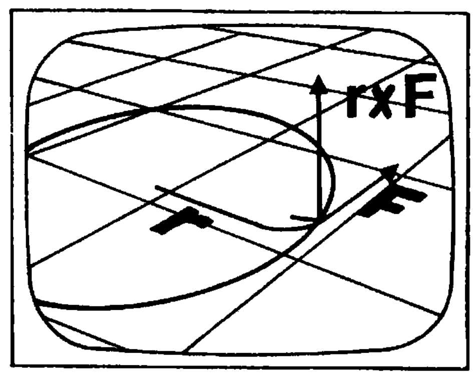
Does a planet move faster or slower when it is near the sun?

According to Kepler's second law, a planet moves faster when nearer the sun. The planet moves from A to B in same amount of time as it moves from C to D so that area ABS equals area CDS.



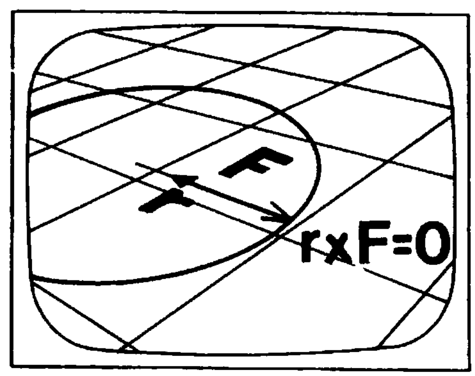
Whenever a force acts to cause a twist, we say it exerts a torque. The amount of torque depends upon three things: the size of the force, F; the distance, r; and the angle between F and r. Torque is a vector and is given by the cross product. Torque equals r cross F. Does this twisting force exist for a planet in orbit or for water flowing down a drain?

No. Neither the gravitational force of the sun on the planet nor pressure in a fluid can twist about the center of the motion. The force in each case is along the radius vector.

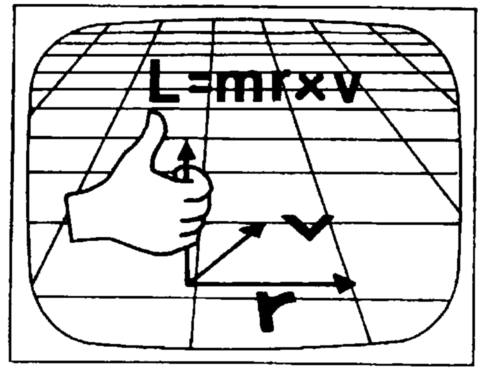


A spinning body rotates in the same direction unless a twist, or torque, is applied. This is the essence of the law of conservation of angular momentum. If the force shown in the illustration were to act on an object, would its angular momentum be changed?

No. Since the force causes no torque, angular momentum will be conserved.



The angular momentum, L, is a vector following the right-hand rule. For circular motion, when v and r are perpendicular, the size of L is mrv. What does the direction of the thumb mean in the illustration?



The thumb will point in the direction of the cross product. Remember, the fingers will curl in the order of the operation, r x v. The thumb points in the direction of the angular momentum vector, L.

EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To

reinforce further the concepts presented in the video, you might pose the following questions to your students.

1. **Why do gymnasts and springboard divers contract their limbs into a tight position when doing flips rather than having their limbs extended out? Why do ice skaters or ballet dancers spin faster when their arms are held nearer their bodies? How do children on a playground merry-go-round make it go faster by changing the position of their bodies?**

   Because angular momentum is conserved in these cases, L = mrv = constant. Consequently, more rotations (greater v) are possible with a given initial amount of angular momentum if the mass is moved closer to the axis of rotation, i.e., r is reduced.

2. **Explain the motion of a helicopter as it begins to rise from the ground.**

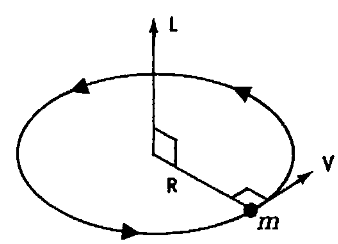
   If there were no rear rotor, the helicopter body would rotate in the opposite direction to the main rotor blade to conserve angular momentum. The rear rotor provides a counter torque which prevents the rotation of the body of the helicopter. You may notice that many times the rear of the helicopter will move sideways upon liftoff until the right speed of the rear rotor is achieved for stability of the body.

3. **What does Kepler's second law imply about the speed of a planet in a circle? In an elliptical orbit?**

   In a circle, the speed would remain constant. In an elliptical orbit, the speed would be greatest when the planet is closest to the sun and least when its distance from the sun is greatest.

4. **An object of mass *m* moves in a circle of radius *R* with a constant speed *v*. What is its angular momentum?**

   At any instant the velocity **v** *of the object is perpendicular to the radius vector* **R***; therefore, the magnitude of the angular momentum is L = mvR sin 90° = mvR.*



5. **Many galaxies share the basic shape of a flat disk around a globular center. Why is this disk shape so pervasive throughout the universe?**

   A galaxy starts out as a massive cloud of matter, all of which condenses toward a point in the center. If the cloud is rotating about some axis, then as the particles move inward, they pick up speed and, rather than falling to the center, continue forever in orbit. Thus angular momentum is conserved, and the galaxy takes on a pancake shape.

6. **How is angular momentum different from linear momentum?**

   Linear momentum involves straight-line motion, whereas angular momentum involves curved motion; therefore, the axis of rotation must be considered in calculating angular momentum.

7. **An unbalanced force is required to change linear momentum. What is required to change angular momentum?**

   An unbalanced torque.

8. **In which of the following cases do you think the angular momentum of the object will change?**

   (a) **Water goes down the drain. Does the angular momentum of the circulating water change?** *Answer: No.*

   (b) **The Space Shuttle fires retro-rockets and moves to a lower orbit. Does the angular momentum of the Space Shuttle with respect to the earth change during the firing of the retros?** *Answer: Yes.*

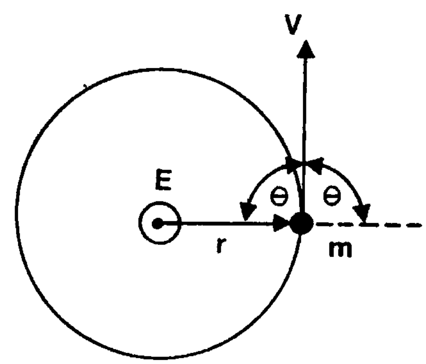
   (c) **Air moves into a low pressure area and forms a cyclone. Does the angular momentum of the air change?** *Answer: No.*

   (d) **A yo-yo rolls down a string. Does the angular momentum of the yo-yo change?** *Answer: Yes.*

   (e) **A bicycle accelerates. Does the angular momentum of the bicycle wheel change?** *Answer: Yes.*

   (f) **Two ice skaters are moving toward each other and grab hands. They immediately begin rotating about one another. Does their angular momentum change?** *Answer: No.*

For Questions 9 through 12, consider a satellite in a nearly circular orbit around the earth. The radius of the satellite's orbit is r, its mass is m, its velocity is v, and the angle between the radius vector and the velocity vector is θ. Ignore the effects of air friction. (See figure below.)



9. **The angular momentum of the satellite is**

   (a) 0.
   (b) $\mathbf{r} \times \mathbf{F}$.
   (c) $\mathbf{r} \times m\mathbf{v}$.
   (d) $|\mathbf{r}|\ |m\mathbf{v}| \cos\theta$.
   (e) $|\mathbf{r}|\ |m\mathbf{v}|$. *Answer: (c)*

10. **The direction of the angular momentum vector is**

    (a) into the plane of the paper.
    (b) out of the plane of the paper.
    (c) to the left.
    (d) to the right.
    (e) in the direction of the linear momentum. *Answer: (b)*

11. **The torque on the satellite is**

    (a) 0.
    (b) $\mathbf{r} \times m\mathbf{v}$.
    (c) $|\mathbf{r}|\ |\mathbf{F}| \cos\theta$.
    (d) the change in angular momentum.
    (e) $\mathbf{F} \times \mathbf{r}$. *Answer: (c)*

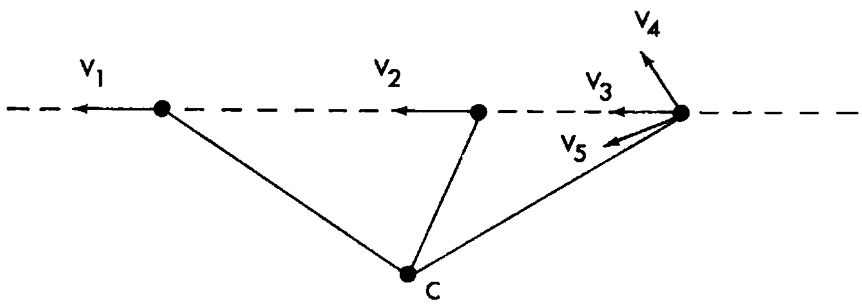
12. **A rocket is fired off the satellite in the same direction as the velocity vector of the satellite. The angular momentum of the satellite will**

    (a) remain unchanged.
    (b) increase.
    (c) decrease.
    (d) be zero.
    (e) be equal to the applied torque. *Answer: (c)*

13. **An object moves in the directions indicated in the illustration. In all cases the speed is the same.**

Which of the velocities will have the greatest angular momentum about C? *Answer:* V_4

Which of the velocities will have the least angular momentum about C? *Answer:* V_5



SUMMARY - Kepler's discovery of the law of equal areas was an important step in our understanding of the behavior of objects in the heavens. However, a more detailed examination of the law revealed that all objects, whether in the heavens or on the earth, would obey this law provided that the force acting on the moving object was always directed toward a single point. A force which always acts in the direction of a single point is said to produce no torque about that point. If a complex system of moving objects interacts in the absence of external torques, the total angular momentum of this "torque isolated system" will always remain constant. This is the essence of the conservation of angular momentum principle; it forms one of the most basic laws in all of physics.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO ANGULAR MOMENTUM

INTRODUCTION - In this video the concepts of torque and angular momentum are introduced. The law of conservation of momentum is discussed in reference to planetary motion.

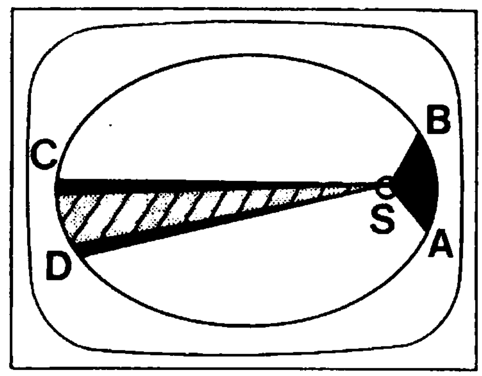
Terms Essential for Understanding the Video

Kepler's law of equal area
radial
cross product
right hand rule
torque
vortex
linear momentum
angular momentum
conservation of angular momentum

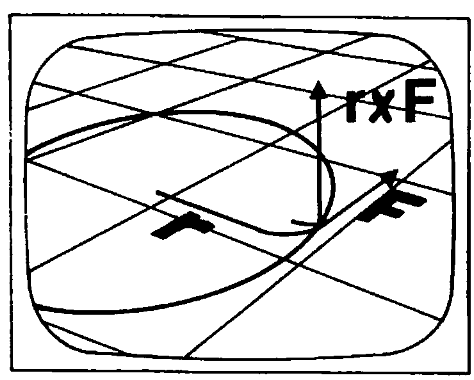
**** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ****

Points to Look for in the Video

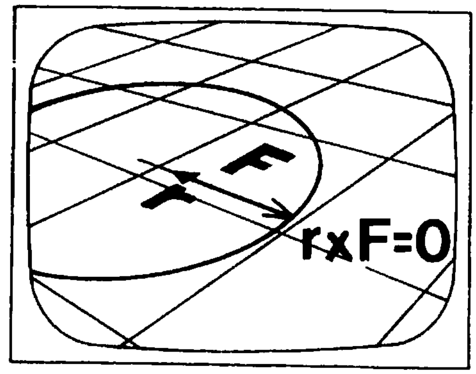
Does a planet move faster or slower when it is near the sun?



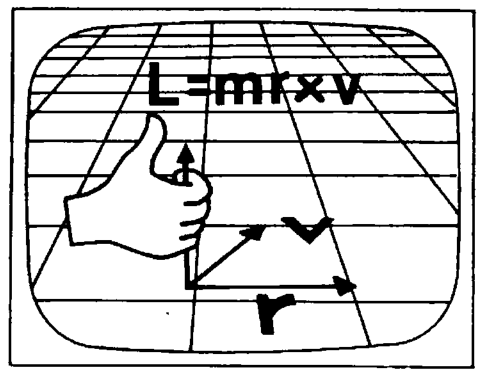
Whenever a force acts to cause a twist, we say it exerts a torque. The amount of torque depends upon three things: the size of the force, F; the distance, r; and the angle between F and r. Torque is a vector and is given by the cross product. Torque equals r cross F. Does this twisting force exist for a planet in orbit or for water flowing down a drain?



A spinning body rotates in the same direction unless a twist, or torque, is applied. This is the essence of the law of conservation of angular momentum. If the force shown in the illustration were to act on an object, would its angular momentum be changed?



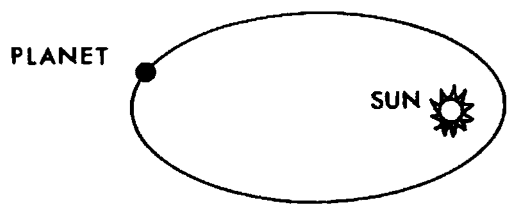
The angular momentum, L, is a vector following the right-hand rule. For circular motion, when v and r are perpendicular, the size of L is mrv. What does the direction of the thumb mean in the illustration?



TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - Kepler's three laws are the pillars on which Isaac Newton built his universe. The laws themselves were the consequence of Kepler's efforts to describe heavenly motion in geometrical terms and to assign a physical cause to it. He asked questions about the motions of the planets, their number and their distribution, which no one had ever raised. Kepler's work received considerable impetus from the astronomical observations of Tycho Brahe. Brahe was a tenacious observer whose major contributions were his systematic and precise approach to astronomical measurement and the wealth of data acquired during his twenty years of careful observation. A year before his death, Brahe hired Kepler to work out the mathematical details of his system. The experience, combined with data inherited from Brahe, helped launch Kepler on one of the most profound and influential efforts in the history of human intellect.

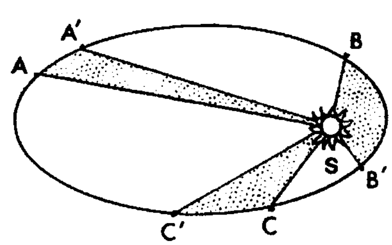
Kepler directed his work toward describing planetary motion within a sun-centered system. Through his book, *Mysterium Cosmographicum*, he was the first professional astronomer in the fifty years since the death of Copernicus to give public support to the Copernican system of the universe. After years of false starts and drudgery in the attempt to find a model which would use only perfect circles, Kepler finally found the courage to abandon that dictum of antiquity. He suggested that the planets moved in some kind of path other than perfect circles. Two thousand years of circular thinking had finally given way to the ellipse. In Kepler's system the planets move in ellipses with the sun at one focus; this became known as Kepler's first law.



Kepler's law of ellipses.
(Here exaggerated greatly in shape, for actual orbits are much closer to circles.)

Kepler's observations indicated that the planets do not move at a constant speed in their orbits. He found that each planet moves faster when it is close to the sun than when it is far away. In seeking to explain this observation Kepler found that, if a line is drawn from the sun to a planet, this line will sweep out equal areas in equal times; this is Kepler's second law.

Time for planet to move between pairs of points is the same for all three cases.
$t_{AA'} = t_{BB'} = t_{CC'}$



Areas ASA', BSB', CSC' are equal

Kepler's law of equal areas.

It was another nine years before Kepler discovered his third law. He was particularly fond of this law, since it gave a harmony to his interpretation of the universe. He found the planets dance about the sun in such a way that the time to orbit the sun once (the period) is related to the planet's distance from the sun. This "harmonic" third law states: The square of a planet's period is proportional to the cube of its mean distance from the sun.

Nearly one hundred years after Kepler described the behavior of the planetary system, Isaac Newton formulated his laws of mechanics which demonstrate why the system has this behavior. In this guide, we will concentrate on those aspects of Newtonian mechanics which led to the development of one of the major conservation laws of physics: the conservation of angular momentum. It will be seen that Kepler's law of equal areas can be linked through the definition of the cross product to the idea of angular momentum. Since the area swept out by a planet in a given time is constant, it follows that the angular momentum of the planet will also remain constant. Today we regard conservation of angular momentum as the fundamental principle and Kepler's law as a consequence.

Essentially every high school physics course discusses the product of a scalar and a vector. Although dot and cross products are not used as frequently, an understanding of the cross product is essential to this video.

The dot product is very useful in defining the concept of work. The dot product of two vectors at an angle θ is a scalar whose magnitude is given by

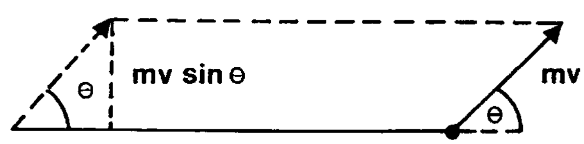
$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \cos \theta.$$

The cross product of two vectors, **A** and **B**, results in a vector whose magnitude is given by

$$|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}|\,|\mathbf{B}| \sin \theta,$$

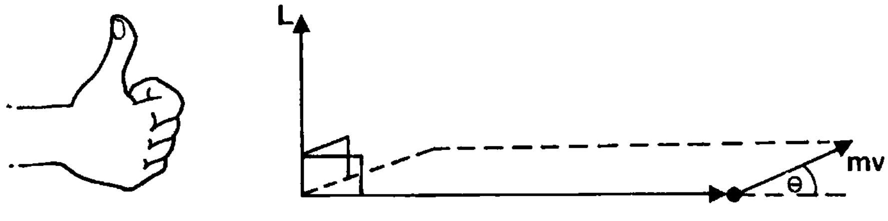
where θ is the angle between **A** and **B**. The direction of the resulting vector is perpendicular to the plane of **A** and **B** and is given by the right-hand rule.

In the video, the cross product is used to represent the angular momentum vector. The angular momentum **L** of an object of mass m, velocity **v**, and at position **r** from an axis, is defined as $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$. The magnitude of the angular momentum is given by $L = mrv \sin \theta$, which is the area of the parallelogram, as shown below.



As illustrated in the figure, $mv \sin \theta$ is the component of $m\mathbf{v}$ perpendicular to the radius vector **r**. When **v** and **r** are perpendicular, the object is moving in a circle, $\sin \theta = \sin 90° = 1$, and, therefore, the magnitude of the angular momentum is simply given by $L = mvr$.

The direction of the angular momentum is found from the right-hand rule: Point the fingers of your right hand in the direction of **r**, then curve them toward the direction of m**v** through the smallest angle. Your thumb then will point in the direction of **L**, which is perpendicular to the plane containing **r** and m**v**, as illustrated below:



The torque vector, τ = **r** × **F** follows similar rules for determining the magnitude ($\tau = rF\sin\theta$) and direction. You can decide the extent to which the mathematics of cross products should be carried. It should be appreciated, however, that the mathematics of cross products is isomorphic to the physics of torque and angular momentum. Furthermore, it should be appreciated that torque and change in angular momentum are always in the same direction. In addition, the torque vector is always perpendicular to the force vector, and the angular momentum vector is always perpendicular to the velocity vector. A more detailed derivation of the connections between Kepler's second law, angular momentum, and torque is presented in the ADDITIONAL RESOURCES section.

In the video, the formations of vortices in draining water (whirlpools) and upwelling air (hurricanes) are illustrated as examples of the law of conservation of angular momentum. It may be helpful to the students to review the underlying physics of these applications. Suppose you have a huge bowl (a bathtub, for example) that is full of water. When the plug is pulled, the water at first flows straight out through the hole. However, because of some initial rotation of the water, the rotation of the earth, and other factors, the water moves in a circle, and after a while, the water forms a whirlpool. Here's why: Aside from a small amount of friction from the walls of the bowl, no torques act on the water; consequently, the water conserves angular momentum as it flows out.

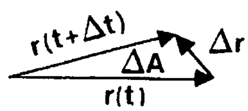
The angular momentum of a small portion of water, at any instant, as it spirals toward the center of the bowl is **L** = m**r** × **v**, where **r** is the radius vector to the bit of water, and **v** its velocity. Since **r** and **v** are perpendicular, the magnitude of the angular momentum is simply $L = mrv$ = constant. As each small bit of water moves down toward the center, it conserves angular momentum. Since r becomes smaller, the speed becomes greater according to $v = L/(mr)$.

Now when the water is moving in very small circles, it is moving very rapidly. Of course, some force must keep each bit of water moving in a circle; that force is the tensile strength of the water – the ability of the water to keep itself together. When the required force to keep a bit of water moving rapidly in a small circle exceeds the tensile strength of the water, the surface ruptures and forms a hole. The hole in the center of the whirlpool is known as a vortex.

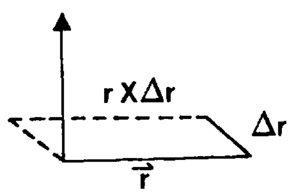
The same dynamics occurs in other instances on much larger scales. If there is an upwelling of air, such as that above warm water, and if there is any circulatory motion, a vortex can form. In this case it is called a hurricane. Hurricanes can be long-lived, persistent, and very destructive. The longest-lived hurricane we know of is on the planet Jupiter: the great Red Spot.

ADDITIONAL RESOURCES

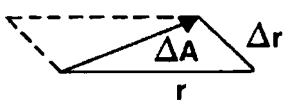
The following discussion outlines the connection between angular momentum and Kepler's second law:



As the radius vector moves, it sweeps out an area ΔA, and the change in the position of the tip of the vector is represented Δ**r**.



Here the cross product of **r** and Δ**r** is illustrated as the area of the parallelogram formed by **r** and Δ**r**.



Half the area of the parallelogram is the area of the triangle illustrated in the first diagram:

$$\Delta A = 1/2\ \mathbf{r} \times \Delta\mathbf{r}.$$

$$\frac{\Delta A}{\Delta t} = 1/2\ \mathbf{r} \times \frac{\Delta \mathbf{r}}{\Delta t}$$

Divide both sides by the change in time.

$$\frac{\Delta A}{\Delta t} = 1/2\ \mathbf{r} \times \mathbf{v}$$

Observe that $\mathbf{v} = \Delta\mathbf{r}/\Delta t$.

$$\frac{2\Delta A}{\Delta t} = \mathbf{r} \times \mathbf{v}$$

Multiply both sides by two.

$$\frac{m2\Delta A}{\Delta t} = m\mathbf{r} \times \mathbf{v}$$

Multiply both sides by m.

By Kepler's second law, $\Delta A/\Delta t$ is constant, and since m is a constant, the right-hand side also is a constant:

$$\text{constant} = m\mathbf{r} \times \mathbf{v}.$$

$$\mathbf{L} = m\mathbf{r} \times \mathbf{v}$$

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v}$$

This constant is known as the angular momentum. It is a vector, and its direction is determined through the definition of the cross product.

The relationship between the rate of change of angular momentum and torque is as follows:

$\Delta \mathbf{L} = \mathbf{r} \times m\Delta \mathbf{v}$	Consider the change in angular momentum, $\Delta \mathbf{L}$, and the change in velocity, $\Delta \mathbf{v}$, in the previous equation.
$\frac{\Delta \mathbf{L}}{\Delta t} = \mathbf{r} \times m\frac{\Delta \mathbf{v}}{\Delta t}$	Divide both sides by Δt.
$\frac{\Delta \mathbf{L}}{\Delta t} = \mathbf{r} \times \mathbf{F}$	By Newton's second law $\mathbf{F} = m(\Delta \mathbf{v}/\Delta t)$.
$\tau = \mathbf{r} \times \mathbf{F}$ $\tau = \frac{\Delta \mathbf{L}}{\Delta t}$	But the right-hand side of the above equation is the definition of the torque. Hence, we can see that the torque equals the rate of change in angular momentum, or $\tau = \Delta \mathbf{L}/\Delta t$.

Conservation of angular momentum is one of the basic principles of physics. In the absence of external torques, angular momentum will always remain constant. For a system on which the net external force is centrally directed, no torque is exerted on the system, and consequently, angular momentum is conserved. Newton's law of universal gravitation precisely describes that the force exerted on each planet by the sun is directed toward the center of the sun. Hence the angular momentum of each planet is conserved as the planet orbits the sun; that's the physical reason why Kepler's second law holds.

Note the similarity of Newton's second law with the law relating torque and angular momentum.

$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}$	linear force causes changes in linear momentum.
$\tau = \frac{\Delta \mathbf{L}}{\Delta t}$	rotational force causes changes in angular momentum.

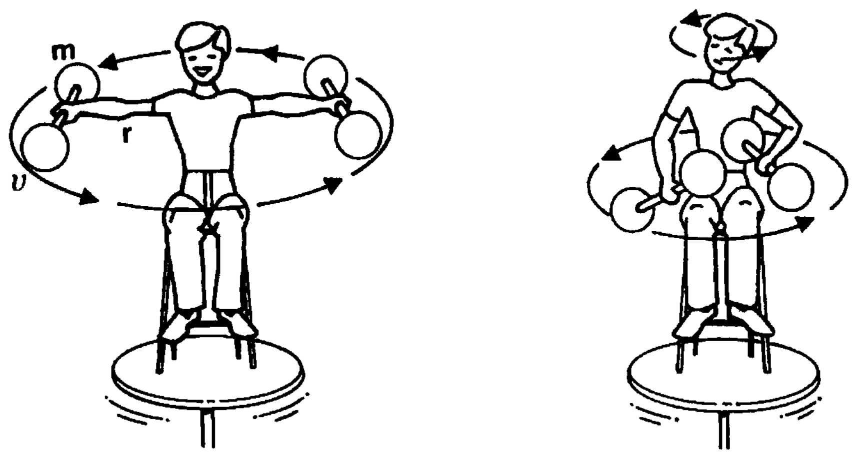
If the net linear force is zero, linear momentum is conserved. If the net rotational force is zero, angular momentum is conserved.

Demonstration #1: The Dumbbell Demonstration

Purpose: To show how, in the absence of external torques, angular momentum is conserved if mass remains constant and radius changes.

Materials: Teacher or student, two dumbbells or suitable masses, and a rotating stool or platform.

Procedure and Notes: Standing or sitting on a rotating platform with arms outstretched as illustrated, the demonstrator is rotated to a safe speed. The demonstrator pulls the masses inward, decreasing their radii, and so the velocities increase. When the masses are then returned to their original outstretched position, the system returns to nearly the same lower velocity. This experiment is a joy to perform and to watch; all students should experience it. However, be aware of the dizzying consequences of too much enthusiasm.

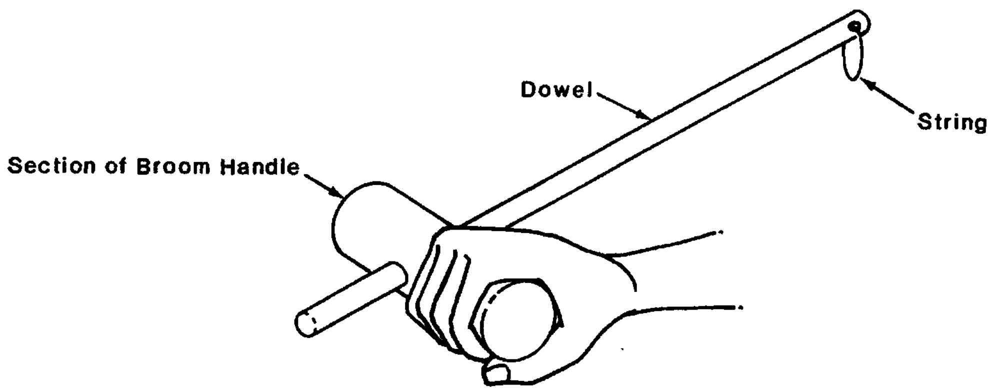


Explanation: From $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$, it is easily seen that as $\mathbf{r}$ changes $\mathbf{v}$ must adjust accordingly to keep angular momentum constant. The torque caused by the friction of the bearings will eventually transfer the angular momentum back to the earth.

Demonstration #2: Basic Torque Demonstration

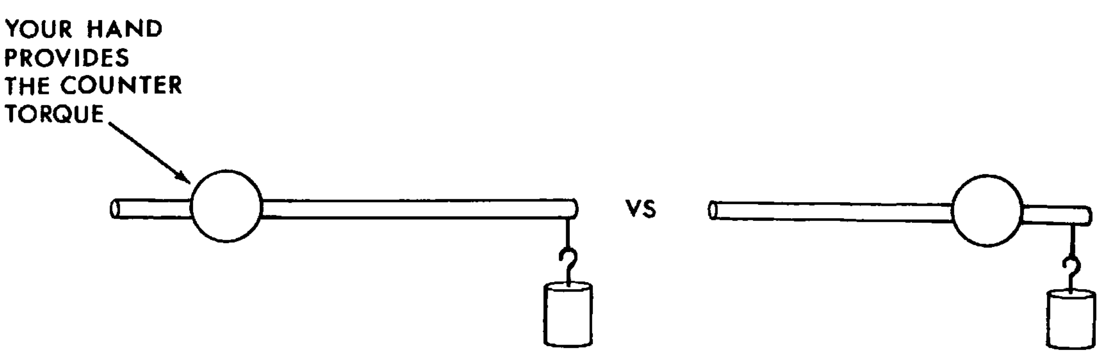
Purpose: To show how the definition of torque actually describes the effectiveness of a force in causing a twist.

Materials: A simple device for illustrating the essential idea of torque can be constructed from a short section of broom handle, approximately two feet of quarter-inch dowel, and a small piece of string. Drill a quarter-inch hole through the broom handle as illustrated so that the dowel can be slid through the hole with some friction. Pass a loop of string through a small hole drilled through the end of the dowel. Any small mass can be hung from this string to provide the force in the following demonstration.

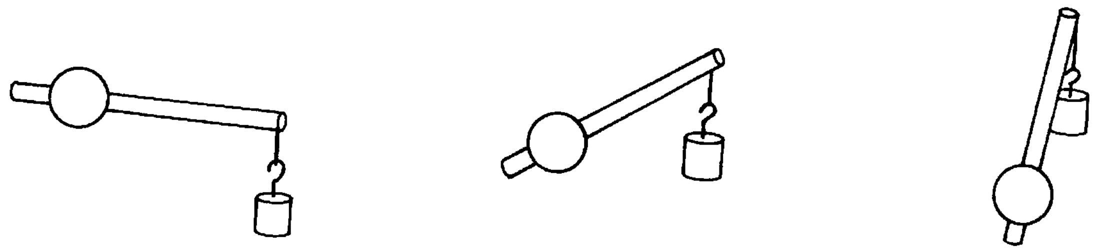


Procedure and Notes:

1. Hold the dowel horizontal, and demonstrate the effect produced by the weight hanging from the end as the distance from the point of application of the weight and the point of rotation is changed. Both the bending of the dowel and the effort of your hand twisting on the broom handle should illustrate the effect of the distance between the point of application of a force and the torque it causes.



2. Now rotate the dowel upward so that the weight no longer hangs at right angles to the dowel. The decrease in torque required by your hand on the broom handle should be obvious, as the force becomes more nearly parallel to the dowel, as shown below.



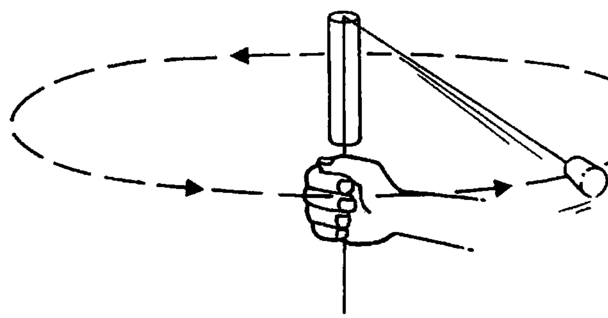
Explanation:

The twist provided by your hand on the broom handle to counter the effect of the weight of the mass when it acts at different distances and angles is completely explained by using the definition of torque: $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$.

Demonstration #3: Rubber Stopper and Glass Tube

Purpose: To show how angular momentum is conserved when a small mass is acted upon by a centrally directed force.

Materials: A piece of nylon string attached to a rubber stopper and a section of fire-polished glass tubing. (This is the same apparatus that is frequently used in an experiment on centripetal force.)



Procedure and Notes:

1. Rotate the stopper in a horizontal circle with a fairly large radius. Pull the string downward, decreasing the stopper's radius and increasing its angular velocity.

2. Try to increase and decrease the force while the stopper revolves in a single revolution. Perhaps attempt to force the stopper into a nearly elliptical orbit.

3. It might be possible to photograph the motion of the stopper from above while it is being illuminated by a strobe light. Care should be taken to get slightly less than a single revolution to avoid overlapping. Such a photograph should reveal that equal areas are swept out by the string in each time interval.

Explanation: Since the force always acts toward a single point, the stopper experiences no torque. In the absence of external torques, angular momentum is conserved. Because $L = mrv$ is constant, when the radius r is decreased, the speed of the stopper v must increase.

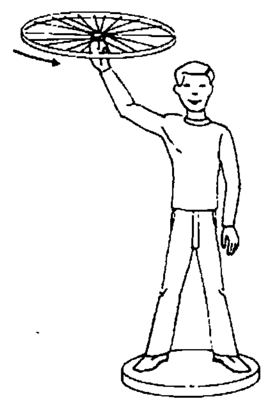
Demonstration #4: Weighted Bicycle Wheel and Rotating Platform

Purpose: To demonstrate the vector nature of torque and angular momentum and to show how they are related.

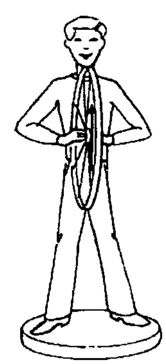
Materials: A rotating platform or stool and a weighted bicycle wheel. (Such wheels can be obtained from Cenco, Welch, etc. However, with a little ingenuity a teacher could convert a regular bicycle wheel for this purpose.) See *Demonstration Experiments in Physics* by R. M. Sutton, "M-178 Bicycle Wheel and Rotating Stand."

Procedure and Notes: There are many interesting demonstrations which can be done with this apparatus. The following are strongly recommended:

1. The demonstrator stands on the platform with wheel initially at rest and with the axis of the wheel parallel to the axis of the platform. With a free hand, the demonstrator suddenly gives the wheel a large angular velocity, with the result that the rest of the system rotates in the opposite direction. A brief time later, the wheel is stopped and the rest of the system stops. The operation can result in a net angular displacement but little or no final angular velocity.



2. Repeat the above, except now begin by holding the axis of the wheel perpendicular to the axis of the platform. If done carefully, no motion will result (since the platform is not free to rotate in this direction). However, after the wheel is given a large angular velocity, demonstrate the effect of rotating the axis of the wheel into a position parallel to the axis of the stool.



3. The demonstrator is motionless on the platform and is handed a rotating wheel with its axis parallel to the axis of the platform. The demonstrator flips the axis of the wheel, thereby acquiring an angular speed. If the demonstrator and his assistant are well coordinated, the wheel can be handed back, inverted by the assistant and again handed to the demonstrator. Depending upon the way the assistant rotates the wheel, the demonstrator's angular speed will increase or decrease. See Sutton, "M-79 Quanta of Angular Momentum."

4. Many similar demonstrations involving applying torques to the spinning wheel while on the rotating platform can be performed. All such demonstrations can lead to many interesting and/or confusing discussions. It is strongly recommended that students be allowed to play with this equipment. Subsequent discussions of their experiences may correct misconceptions and aid in reinforcing the concept of conservation of angular momentum.

Explanation:

The explanation of these demonstrations will require a clear understanding of the vector nature of torque and angular momentum and how one relates to the other.

Demonstration #5: Batter Up

Purpose: To demonstrate the conservation of angular momentum in an isolated system.

Materials: Rotating platform or stool and baseball bat.

Procedure and Notes: While standing on the platform or sitting on the stool, swing the bat in one direction only. Notice that the platform and person go in the other direction. Return the bat slowly to the original position and repeat.

Explanation: The angular momentum of the system is zero before the swing; hence it must be zero after the swing. Rapid swinging and a slow return will probably result in a net rotation which can be explained by considering the torque in the bearing of the rotating platform. A very slow movement will involve torques which are not sufficient to overcome the torque of friction.

EVALUATION QUESTIONS

1. **A block slides across a table and comes to rest. A physics student asserts that since the block came to rest, its momentum was "lost" and the conservation of linear momentum principle has been violated. His confusion could probably be best explained by**

   A. reminding him that momentum is a vector.
   B. pointing out that momentum conservation only works when there is friction.
   C. reminding him that momentum is the product of the scalar, mass, and the vector, velocity.
   D. pointing out that momentum conservation only applies in the absence of external forces.

2. **A NASA satellite has an elliptical orbit with a maximum distance from the surface of the earth of 200 km and a minimum distance of 50 km. At its maximum distance the velocity of the satellite is**

   A. greater than when at minimum distance.
   B. less than when at minimum distance.
   C. the same as when at minimum distance.
   D. has no relationship to its distance from the earth.

3. **A woman pushes on a door with a given force for a specified length of time. In order to get the door moving with the largest angular momentum, she should push**

   A. close to the hinge.
   B. in the middle of the door.
   C. far from the hinge.
   D. anywhere since it is just the force that counts.

4. **A boy is sitting on a rotating platform which has frictionless bearings. He has two weights in his hands and is spinning with his arms extended. If he lets go of the weights which of the following will happen?**

   A. He will speed up.
   B. He will slow down.
   C. He will retain the same speed.
   D. He will immediately stop.

5. **The total angular momentum of a system of particles**

   A. remains constant under all circumstances.
   B. remains constant only if there is no friction.
   C. changes when a net external torque acts on the system.
   D. may or may not change when a net external torque acts upon the system depending on the direction of the torque.

6. **Amusement parks often have large freely rotating turntables. You sit in the middle of the table. As it turns, you gradually move to the outer edge. How does the rate of rotation of the turntable change?**

   A. It increases.
   B. It decreases.
   C. It remains the same.
   D. It becomes zero.

7. **Several objects interact with one another in a region of space which is completely isolated from all external forces. We can apply the conservation of linear momentum to this situation because**

   A. momentum conservation applies when there are external forces.
   B. momentum conservation applies when there are no external forces.
   C. the conservation of momentum principle applies in all systems.
   D. conservation of momentum works only when there is no friction.

8. **A rubber stopper attached to a string is whirled overhead and then the string is pulled inward. As a result the stopper whirls faster. From the definition of angular momentum and the fact that it is conserved, the increased speed of the stopper can be explained as follows:**

   A. The increasing inward force on the stopper speeds it up.
   B. The linear momentum of the stopper is conserved so it must move faster as it is pulled into a smaller circle.
   C. The product of force and velocity must be constant so the decreased force must be accompanied by an increased velocity.
   D. The product of radius and velocity must be constant so the decreased radius must be accompanied by an increased velocity.

9. **The conservation of angular momentum principle can be applied to the motion of a diver as she twists through the air because**

   A. while in the air, there is little external torque.
   B. air friction is small compared to her weight.
   C. all freely falling objects experience a uniform acceleration.
   D. the torque received as she leaves the board must be conserved.

10. **A spinning object interacts with several other spinning objects on the same axis of rotation. If the total angular momentum of the spinning objects before the interactions is compared with the total angular momentum after the interactions, it will be found that**

    A. the angular momentum will be the same if there are no friction losses.
    B. the angular momentum will increase if the objects move closer together.
    C. the angular momentum will increase if the linear momentum decreases.
    D. the angular momentum will be the same.

ESSAY QUESTIONS

11. Sketch an illustration of how a planet moves in a very elliptical path. Use Kepler's law of area to show how the planet moves when it is near the sun compared to how it moves when it is a great distance from the sun. Discuss how this motion is an example of conservation of angular momentum.

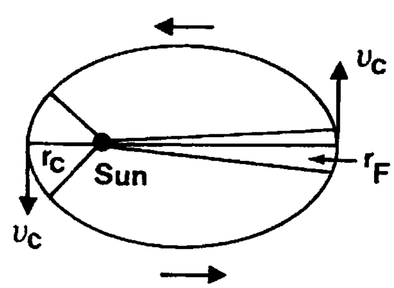
12. If the icecaps at the North and South poles were to melt and the resulting water be added to the oceans, why would you expect the length of the day to change?

KEY

1. D
2. B
3. C
4. C
5. C
6. B
7. B
8. D
9. A
10. D

SUGGESTED ESSAY RESPONSES

11.



In order to sweep out equal areas in equal times, the planet will have to move faster when it is close to the sun r_c than it does when it is at a great distance from the sun r_f. Since angular momentum is the product of the radius vector and the velocity vector, the planet's radius vector decreases as it approaches the sun. Hence, its velocity must increase in order to keep $L = r \times mv$ constant.

12. If water from the icecaps leaves the vicinity of the earth's axis and becomes distributed farther away from it, the earth's rotational velocity must decrease in order that angular momentum be conserved. Thus the length of the day would increase.

TEACHER'S GUIDE TO FUNDAMENTAL FORCES

CONTENT AND USE OF THE VIDEO - The video presents a survey of nature's fundamental forces; it is not intended as a comprehensive study of each force. Prior to viewing the video, students should have an understanding of Newton's law of universal gravitation and his laws of motion. Also, it is important that students have an understanding of the attraction and repulsion of electric charges.

A particularly effective time to use the video is in conjunction with a presentation of Coulomb's law. Another option is to present the video at the end of the year as a summary of the forces studied in physics. Students can then begin to appreciate the connections between gravity and electricity.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - This video deals with some of the most fundamental ideas in physics. The terms that are used to describe these ideas must, therefore, be so basic as almost to defy definition. However, students must at least be familiar with the following terms and how they differ from one another.

force--that push or pull that is exerted on one body by another body; one important property of a force is that it always has a physical source (an agent).

fundamental forces--all forces in nature seem to be a result of one or more of four fundamental forces:

(a) strong nuclear force--results from the interaction of particles in the nucleus and is considered to be the force that holds the nucleus together.
(b) weak nuclear force--results from the interaction of particles and is considered to be the cause of certain types of radioactive decay of the nucleus.
(c) electrical force--results from the interaction of electrically charged particles.
(d) gravitational force--results from the interaction of masses.

mass--a measure of inertia, which is the property of matter that resists changes in its state of motion. This is known as inertial mass. When mass is considered as the source of the gravitational force, it is known as gravitational mass.

electric charge--basic property of nature which is the origin of the electric force.

nucleon--a collective name for any particle in the nucleus.

WHAT TO EMPHASIZE AND HOW TO DO IT - The concept of fundamental forces is at the very heart of physics. Newton's mechanics clarified the role of forces in determining the motion of things. Since $F = ma$ is the keystone to understanding nature, it becomes imperative to know what is meant by F and what is meant by m: What are the fundamental forces of nature? And what does matter ultimately consist of? These are the questions upon which Newton's second law focuses attention. They remain, to this day, the central questions in physics.

The video develops the concept that all forces are the manifestations of one or more of the four fundamental forces. These fundamental forces are the gravitational force, the electrical force, the weak nuclear force, and the strong nuclear force.

Objective 1: Recognize examples of fundamental forces.

Students tend to think about forces in terms of the specific forces that they have studied, e.g., weight, friction, spring, etc. All of those forces originate in one or more of the four fundamental forces.

Explanations of the relationships among the four fundamental forces and their manifestations in everyday phenomena such as friction are complex and frequently confusing to students. The section on EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS discusses the action of a spring as a manifestation of the electrical force. This discussion deserves serious attention since it points out the subtlety and sophistication of the relationships among forces.

Objective 2: Describe the method Cavendish used to determine the universal gravitational constant *G*.

A demonstration of the torsion balance is presented in the SUPPORTIVE BACKGROUND INFORMATION. Following the video it may be helpful to show students the demonstration. If students still have difficulty understanding the reason for the extreme sensitivity of the balance and how it is used, the video can be replayed and stopped at appropriate places for further clarification.

Objective 3: Compare and contrast gravitational and electrical forces.

The relationships that describe electrical and gravitational forces are the same mathematically. Electricity can be repulsive as well as attractive, but gravity is only attractive. The section of the video showing these equations is powerful and demonstrative; therefore, it would be helpful to show it several times. Although the force between magnetic poles is mathematically identical to gravity and electricity, magnetism is only a manifestation of the electrical force. This offers an opportunity to remind students that there are four fundamental forces and that all other forces are manifestations of these four.

Objective 4: Distinguish the workings of the nuclear forces from those of electrical and gravitational forces.

The nature of the nuclear forces is not understood nearly as well as the nature of the gravitational and electrical forces. Students will probably feel that the range of the nuclear forces goes beyond the nucleus. This is not true. The strong nuclear force holds the nucleus together. The weak nuclear force is considered to be responsible for certain forms of radioactive decay. The SUPPORTIVE BACKGROUND INFORMATION provides a more detailed discussion of these forces.

Objective 5: Recognize the attempts of physicists to reduce all forces to one all inclusive force.

Physicists are working to unite the four fundamental forces into one. Early man used many different gods to explain the workings of the universe. Eventually, however, connections were established between seemingly unrelated events. Thunder was connected to lightning; the tides were related to the moon. In their attempts to unify the forces of nature, physicists are continuing in the quest for unification. The video attempts to communicate to students some of the challenges involved in their quest.

The SUPPORTIVE BACKGROUND INFORMATION provides a discussion of the search for a unified theory of nature's fundamental forces. Such a discussion is important, not only for the historical development which it affords, but also because it helps students understand how physics proceeds in its struggle to understand how the world works. Progress can be seen in the nature of the questions that physicists ask. Discuss with students how the questions of the twentieth century differ from those of the eighteenth and nineteenth centuries. Clarify how the emergence of more fundamental questions has led to the discovery of increasingly fundamental forces.

POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and frames from the video. The color coding used in the video animation is as follows:

Red = positive charge,
Blue = negative charge,
White = neutral.

The video presents gravitational and electrical forces as vector equations using unit vectors. A unit vector can be defined as any vector divided by its magnitude; hence, a unit vector has a magnitude of 1 while retaining the direction of the original vector. For example, the unit vector in the radial direction is written $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$. A negative sign with $\hat{\mathbf{r}}$ indicates attraction. The gravitational force is an example:

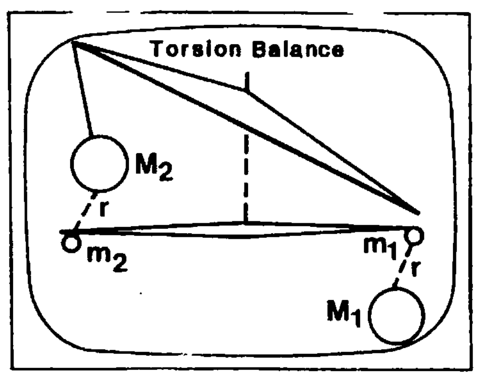
$$\mathbf{F}_g = -\frac{Gm_1m_2}{r^2}\,\hat{\mathbf{r}}\,.$$

Two unlike electrical charges produce a negative sign in the electrical force law (Coulomb's law) and, hence, an attractive force:

$$\mathbf{F}_E = -\frac{K_e q_1 q_2}{r^2}\,\hat{\mathbf{r}}\,.$$

This is analogous to the gravitational force. However, two like electrical charges produce a positive sign in the force equation and, hence, a repulsive force. Note that *all* vector quantities appear in bold face.

How could you weigh the earth?



Cavendish used a torsion balance to measure the universal gravitational constant G. He could also independently find the weight and mass of any common object on the surface of the earth. By substituting the mass of the object, its weight, and the distance to the center of the earth into the gravitational equation

$$F_g = \frac{GMm}{r^2}\,,$$

and solving for the mass of the earth

$$M_e = \frac{F_g r^2}{GM}\,,$$

Cavendish was able to "weigh" the earth. (He actually determined the mass of the earth.)

How is the electrical force similar to the gravitational force? How are the two different?

The gravitational and electrical forces are similar in that they are both inverse square laws for distance.

The gravitational force depends on mass and is always attractive.

The electrical force depends on charge and can be either attractive or repulsive, depending on the kinds of charges involved.

Between elementary particles such as protons or electrons, the electrical force is much stronger than the gravitational force ($F_E/F_g \sim 10^{37}$).

$$F_e = K_e \frac{q_1 q_2}{r^2} \hat{r}$$

$$F_g = -G \frac{m_1 m_2}{r^2} \hat{r}$$

The following table summarizes the four fundamental forces of nature--the strong, electromagnetic, weak, and gravitational--their respective ranges and importance.

The comparison of relative strengths is made considering the ratio of the electric force to the gravitational force between an electron and a proton at some fixed distance. The nuclear forces have very limited range and are important only inside the nucleus. These values for relative strength do not come from simple calculations.

Characteristics of the Four Fundamental Forces

Force	Relative Strength	Range	Example
Strong nuclear force	10^{39}	10^{-15} m (only within nucleus)	Holds nucleus together
Weak nuclear force	10^{34}	10^{-15} m (only within nucleus)	Nuclear radioactive decay
Electrical force	10^{37}	Usually neutral over large distances; predominant over distances of a few meters down to the size of an atom.	Everyday forces (friction, tension, contact, chemical)
Gravitational force	1	Can act over any distance, but is most significant when considering large astronomical distances and large masses.	Organizes the universe on a large scale

NOTE: Consult the SUPPORTIVE BACKGROUND INFORMATION for more information on the relative strength column.

EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in the video, you might pose the following questions to your students.

1. **Which of the fundamental forces govern the world of common experience? Where do the other fundamental forces operate?**

   The electrical force is responsible for the everyday contact forces with which we are familiar: friction, surface tension, spring forces, etc. The electrical force also shapes molecules. It is the source of the vast energy found in lightning as well as the source of energy for our electrical society.

   The gravitational force is the other familiar force. It holds objects to the surface of the earth and holds the moon and satellites in orbit. It causes the tides and is responsible for the formation of stars out of the gas and dust in space.

   The strong nuclear force binds the neutrons and protons together in the nucleus of the atom. It is also responsible for the energy released in nuclear reactions within the sun and in nuclear explosions.

   The weak nuclear force is the force involved in radioactive decay and is the "catch-all" force thought to be responsible for many reactions within the nucleus.

2. **How have the notions of positive and negative charge changed since Benjamin Franklin's introduction of the terms?**

   Franklin thought that electricity was a kind of continuous fluid contained in each body. If the body held too much fluid, Franklin said it was "positively charged." Today charge is not thought to be a continuous fluid but is known to exist in discrete uniform amounts; charge of +1e for every proton and -1e for every electron, where $e = 1.6 \times 10^{-19}$*c.*

   Franklin believed that charge could be poured into or out of a body. When he arbitrarily defined positive and negative electricity, the proton and electron had not yet been discovered. Today we know that charges are a necessary aspect of a body. Each body is composed of charged particles--protons and electrons (as well as neutral neutrons).

3. **How is the *action* of a spring a manifestation of the electrical force?**

   The spring wire is composed of metal ions which are positively charged and "glued" together by electrons which are negatively charged. Stretching the metal pulls the ions apart, but the electrons force them back together again.

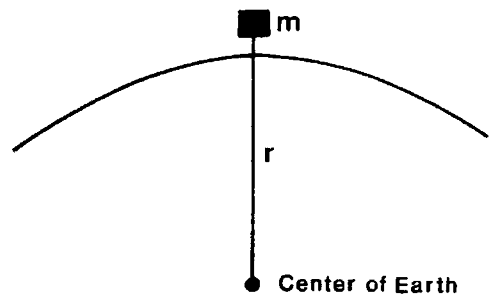
4. **Why is the *tension* in rope an electrical force?**

   The electrical forces tend to keep the atoms in the rope in equilibrium. When you pull on one end, each atom electrically tugs on its neighbors; but unlike a spring, the neighboring atoms don't move much, so the rope doesn't stretch. The pull is transmitted to the other end of the rope, usually undiminished in force.

5. How could you weigh the earth?

Cavendish used a torsion balance to measure the universal gravitational constant G. He could find the weight of any common object at the surface of the earth. By substituting the mass of the object, its weight, and the distance to the center of the earth into the gravitational equation

$$F_g = \frac{GM_eM}{r^2},$$



and solving for the mass of the earth

$$M_e = \frac{F_g r^2}{GM},$$

Cavendish was able to "weigh" the earth. (He actually determined the mass of the earth.)

6. How do the gravitational force and electrical force between two electrons compare? If the distance between the electrons were doubled, how would the forces compare then?

The gravitational force is much weaker (10^{-37} times) than the electrical force. Both forces follow an inverse square law. Therefore, if the distance between the particles doubles, both forces will be $1/2^2$ or 1/4 as strong as they were originally. The ratio remains the same.

7. What force slows the rate of expansion of the universe? Why?

The gravitational force acts on all matter in the universe. This force causes a negative acceleration of the outward expansion of the universe, thus slowing it down.

8. Protons, which have a positive charge, are bound within the nucleus. How must the nuclear force between them compare to the electrical force between them?

The electrical force on the protons would tend to make them repel. Since they are bound close together in the nucleus, the nuclear force must be stronger.

9. Since there is a lot of empty space in the atoms that make up the floor, why don't you fall right through it?

Although the atom is mostly empty space, the electrons on the outside of the atom are bound to the nucleus by electrical forces. Each atom is also bound to other atoms by chemical bonds which are manifestations of the electrical force. When people or other objects stand on the floor, the negative electrons in their bodies are repelled by the negative electrons in the floor and thus they are kept from falling through.

SUMMARY - Physicists believe that all actions in the universe are the result of one or more of four fundamental forces of nature: gravitational, electrical, strong or weak nuclear forces. An effort is being made to reduce this number to one or two forces. The gravitational force dominates over large distances. The electrical force is usually insignificant over large distances because of the balance of electric charges. However, the electrical force may become very important over distances of a few meters when there is an unbalance of electric charge. This force normally becomes the predominant force when the separation of bodies is small (less than a meter down to the size of an atom). Inside the nucleus the strong and the weak nuclear forces are dominant.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO FUNDAMENTAL FORCES

INTRODUCTION - The video develops the concept that all forces are a manifestation of one or more of the four fundamental forces. These fundamental forces are the gravitational force, the electric force, the weak nuclear force, and the strong nuclear force.

Terms Essential for Understanding the Video

force
fundamental forces
strong nuclear force
weak nuclear force
gravitational force
mass
electric charge
nucleon
electric force

**** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ****

Points to Look for in the Video

The color coding used in the video animation is as follows:

Red = positive charge,
Blue = negative charge,
White = neutral.

The video presents gravitational and electrical forces as vector equations using unit vectors. A unit vector can be defined as any vector divided by its magnitude; hence, a unit vector has a magnitude of 1 while retaining the direction of the original vector. For example, the unit vector in the radial direction is written $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$. A negative sign with $\hat{\mathbf{r}}$ indicates attraction. The gravitational force is an example:

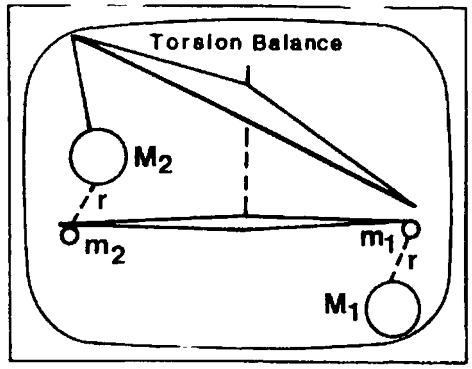
$$\mathbf{F}_g = -\frac{Gm_1m_2}{r^2}\,\hat{\mathbf{r}}\,.$$

Two unlike electrical charges produce a negative sign in the electrical force law (Coulomb's law) and, hence, an attractive force:

$$\mathbf{F}_E = -\frac{K_e q_1 q_2}{r^2}\,\hat{\mathbf{r}}\,.$$

This is analogous to the gravitational force. However, two like electrical charges produce a positive sign in the force equation and, hence, a repulsive force. Note that *all* vector quantities appear in bold face.

How could you weigh the earth?



How is the electrical force similar to the gravitational force? How are the two different?

$$F_e = K_e \frac{q_1 q_2}{r^2}\hat{r}$$

$$F_g = -G \frac{m_1 m_2}{r^2}\hat{r}$$

List examples of each force in the last column of the table below:

Characteristics of the Four Fundamental Forces

Force	Relative Strength	Range	Example
Strong nuclear force	10^{39}	10^{-15} m (only within nucleus)	
Weak nuclear force	10^{34}	10^{-15} m (only within nucleus)	
Electrical force	10^{37}	Usually neutral over large distances; predominant over distances of a few meters down to the size of an atom.	
Gravitational force	1	Can act over any distance, but is most significant when considering large astronomical distances and large masses.	

TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - In searching for a way to describe how bodies fall, Galileo cleverly used crude water clocks to time balls rolling down inclined planes. His breathtaking feat was a mathematical description constructed from observations of how all bodies fall. Galileo's work forged a path for science to follow: careful, quantitative experiments to uncover underlying principles of how the world works. Within half a century, Galileo's law of falling bodies was superseded by a deeper, more fundamental insight into nature – Newton's universal law of gravity. And in uncovering one of the fundamental forces of nature, Newton united the physics of the heavens with that of the earth.

Inspired by Newton, physicists in the eighteenth century sought to identify, classify, and mathematically describe the numerous forces observed in nature. Why? According to Newton's second law, $\mathbf{F} = m\mathbf{a}$, knowledge of these forces provides physics with a certain predictive power; forces shape the motion of all things. Through painstaking experiments, these physicists reached empirical descriptions of forces in the world about them: everyday pushes or pulls, friction, electric and magnetic forces, viscous forces, tension, compression, spring forces, chemical actions, and so on. Today physics books are filled with equations describing these forces, such as $F = -kx$, $F = \mu N$, $F = mg$, $F = -bv$, $F = -cv^2$. So many different names and equations, all labeled force, often lead to confusion. The question that confronted physicists of the eighteenth and nineteenth centuries still beguiles physicists today: Are all these forces fundamental, or can they be reduced to more basic, universally applicable forces?

Newton uncovered **gravity**, in the form of his universal law of gravitation,

$$\mathbf{F}_g = -G \frac{M_1 M_2}{r^2} \hat{\mathbf{r}} ,$$

as a fundamental force of nature and, with it, was able to explain how an apple falls and how the moon orbits the earth. Although its strength diminishes with increasing distance, the effects of gravity are nevertheless felt across the far reaches of the universe; nothing escapes the attractive force of gravity. Gravity causes the tides, holds together planets and stars, organizes solar systems and galaxies; it orders the universe.

Not until late in the eighteenth century did another force emerge out of the maze of empirical forces as being fundamental – the **electric** force. The French engineer Charles Augustin Coulomb first assumed that, analogous to gravity, the electric force between two charges is proportional to the product of the charges. Then, experimentally, he found that the analogy to gravity extended further: the electric force decreases as the square of the distance between the charges. Coulomb's law may be stated mathematically as

$$\mathbf{F}_e = K_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} .$$

Just as G is a universal constant for gravity, K_e is a universal constant for electricity. Unlike gravity, the electrical force can be either attractive or repulsive since charge can be positive or negative: opposite charges attract, whereas like charges repel. On the atomic level, the electrical force binds together electrons to nuclei and holds together molecules. Electricity is the fundamental force that governs the nature of the world around us. On distances large compared to the earth, gravity dominates because ordinary matter is electrically neutral. But in the world of matter as such, the electric force dominates when objects are in contact.

Magnetism had also been identified as a fundamental force of nature until the second half of the nineteenth century, when James Clerk Maxwell succeeded in unifying electricity and magnetism into electromagnetism. Maxwell's unification was expressed by a set of equations which interrelate electric and magnetic phenomena. Soon tensions, spring forces, friction, viscosity, chemical actions, and even light were recognized as arising fundamentally from the electromagnetic force. Based on Maxwell's success, the search for a common mathematical description, or unification of forces, had begun.

The story of twentieth-century physics is the story of the search for the ultimate structure of nature, ushered in by the discovery of radioactivity, the probing of atoms, and the subsequent realization that more than just gravity and electromagnetism would be needed to explain this new world. The pioneering investigations of Ernest Rutherford led to a microscopic planetary model of the atom in which a compact, positively charged nucleus is surrounded by orbiting negative electrons. The electrons were held in their orbit by the electric force. Although this model was a tremendous advance in man's understanding, it introduced new and more baffling questions.

One question was: How can the nucleus contain all the positive charge in an atom since positive charges repel? Rutherford's work clearly indicated that all the positive charge in a gold atom (79 elementary charges) was confined within a volume whose diameter was approximately 10^{-15} m. Clearly there had to be some force in nature, hitherto unknown, which dominated the electric force when the separation of charges was exceedingly small. Aptly named the "strong force," it overcomes the electric repulsion between protons and holds the nucleus together. Unlike gravity and electricity, the strong force does not extend to distant corners of the universe; it has a limited range – the size of a nucleus. Outside this range, the strong force has no effect; if it did, the universe would be one very dense lump of subatomic particles.

Another question could not be explained by any of the known forces – strong, electromagnetic, or gravitational – and was related to natural radioactivity. Experiments indicated that there were three kinds of radioactivity: alpha (α), beta (β), and gamma (γ). The alpha particles were found to be positively charged particles, beta particles were negatively charged, and gamma particles were neutral, all coming from the recently discovered nucleus. Other experiments revealed gamma radiation to be electromagnetic waves. But the greater puzzle was presented by beta decay. How could negatively charged electrons come from the nucleus?

This dilemma was not solved until the early 1930's, when James Chadwick showed the existence in the nucleus of neutrally charged particles of approximately the same mass as the proton, called neutrons. The strong force now had to account for the binding of proton to proton, proton to neutron, and neutron to neutron. Within a few years Enrico Fermi, following a suggestion by Wolfgang Pauli, proposed a theory of beta decay. Central to this theory was the ability of a neutron within the nucleus to decay into an electron, a proton, and an antineutrino. Thus, Fermi postulated the formation of electrons inside nuclei, which was needed to explain beta decay. The force that causes the decay of the neutron is neither the strong nuclear force, nor the electrical force; it came to be known as the weak nuclear force.

Over the past 50 years the study of nuclear physics has become the hunting ground for the ultimate structure of matter as physicists still try to understand m in $\mathbf{F} = m\mathbf{a}$. Bigger and bigger particle accelerators have been constructed, each in its turn seeming to promise the final answer. Today, over 200 particles have been discovered in this nuclear research. the proton and neutron are believed to be made of smaller particles called quarks. These quarks are currently believed to be among nature's basic building blocks.

The following table summarizes the four fundamental forces of nature – the strong, electrical, weak, and gravitational – their strengths and their respective ranges. The behavior of each of these forces is reasonably understood, but nobody knows why there should be four of them. Albert Einstein spent the last twenty years of his life unsuccessfully searching for a way to unify two of the forces – gravity and electromagnetism.

Characteristics of the Four Fundamental Forces

Force	Relative Strength	Range	Example
Strong nuclear force	10^{39}	10^{-15} m (only within nucleus)	Holds nucleus together
Weak nuclear force	10^{34}	10^{-15} m (only within nucleus)	Nuclear radioactive decay
Electrical force	10^{37}	Usually neutral over large distances; predominant over distances of a few meters down to the size of an atom.	Everyday forces (friction, tension, contact, chemical)
Gravitational force	1	Can act over any distance, but is most significant when considering large astronomical distances and large masses.	Organizes the universe on a large scale

Twentieth-century physicists have attempted to explain all the complexities of physics as aspects of simpler systems. Emerging are unified and grand unified theories that present coherent accounts of how these fundamental forces may have evolved from simpler laws in the infancy of the universe. The early universe ultimately may be the only experimental test for such theories.

But what if physicists had one central, grand equation which united all the forces? Then would they have understood all the mysteries of the universe? If they did have such an equation, it would be terribly complicated; yet a computer working for years might be able to solve the equation. However, the universe already contains, and therefore exceeds in complexity, many millions of pages of computer printout. So, what are physicists searching for in their attempts to unify the four fundamental forces of nature? The answer is something more subtle. What they are searching for is an intuitive understanding of how the universe works. That was the quest of Galileo and the legacy of Newton, and it remains so today.

Contact Forces

Another aspect of the fundamental forces is that they are all action-at-a-distance forces: their effects can be explained when particles are not in contact. On the other hand, many everyday forces arise from the contact of two objects. For example, the force of a table pushing on a book resting on it (normal force) exists only when the table and book are touching each other. Known as contact forces, these forces are **not** fundamental forces; instead they are macroscopic forces that arise from the microscopic electrical force between particles.

Contact forces abound in and shape our everyday world. Normal forces, spring forces, tensions, and friction are a few examples. Despite their apparent differences, **all** these forces arise from the fundamental electrical force acting in complicated ways. So the electrical force is the dominant force in the world.

To understand the connection between a contact force and the electrical force, consider the spring force. Inside a spring are metal atoms that are bound together by electrical forces. These electrical forces keep the metal atoms a certain distance apart, called the equilibrium distance. When a spring is stretched, each atom is pulled slightly away from the equilibrium distance. The electrical forces try to pull the atoms back and restore equilibrium. The net result of all the electrical forces acting on the atoms is what causes the end of the spring to exert a macroscopic force.

Trying to describe how a spring works by examining the electrical forces acting between the atoms is impossible because of the sheer numbers of atoms to consider – typically 10^{23}. Rather than attempting to describe all these complicated interactions in terms of a fundamental force, physicists describe them by empirical rules, which are experimental summaries of the net result of all the complications. Most of these empirical descriptions were deduced by eighteenth century scientists.

The empirical law for a spring, for example, is simple: the force exerted by a spring (on an object) is proportional to the change in the length of the spring. Hooke's law, $\mathbf{F} = -kx$, is the empirical law describing the vastly complex interactions of the electrical force in a spring.

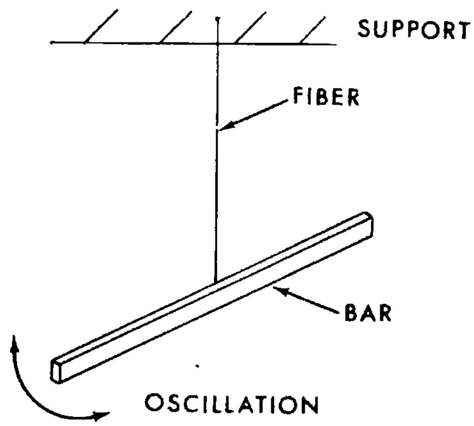
Friction, too, is an inescapable example of electrical forces. The empirical law of sliding friction states that $\mathbf{F} = \mu\mathbf{N}$, where $\mathbf{N}$ is the normal force and μ is the coefficient of sliding friction. This equation was derived to explain experimental results, and serves well for that purpose. With it the force of friction on a sliding body can be calculated. On another level one might ask, "What is the source of friction?" It turns out that this is a very difficult question to answer. On a microscopic level the two surfaces are revealed to be very irregular; there are many points of contact where atoms of the two materials cling together or where momentary welds or bonds are formed. This attraction of molecules in one body for those in the other body is due to electrical forces. As one body slides over the other, these bonds are snapped apart and atomic vibrations ensue. Vibrations take energy, and this energy is dissipated in a warming of the surface.

ADDITIONAL RESOURCES

The Torsion Balance

The torsion balance is an extremely sensitive, yet simple, instrument. Henry Cavendish used one in 1798 to measure the universal gravitational constant G and thus to measure the mass of the earth. The device was so sensitive that with it he could measure a force of 10^{-7} newton (this is equivalent to the weight of an object that has a mass of 0.00001 grams). The torsion balance had been invented some ten years earlier by Charles Coulomb, who used it to investigate the electrical force.

In essence, the torsion balance consists of a bar suspended at its center by a thin fiber. The fiber acts to create a restoring torque on the bar so, if twisted by an external torque, the bar oscillates. This motion is simple harmonic motion, and its analysis is identical to that of a vibrating spring. The frequency of vibration of a spring is proportional to the square root of the spring constant. This constant is a measure of the stiffness of the spring. Similarly, the frequency of vibration of a torsion balance is proportional to the square root of the torsional constant, which is a measure of the stiffness of the fiber to rotation. By measuring the frequency of vibration one may find the torsion constant for the fiber. Therefore, the restoring torque of the balance can be evaluated. When measuring a very small force (either electrical or gravitational), the unknown force creates a torque which causes the balance to rotate, and this rotation is countered by the fiber.



The torsion balance is an extremely sensitive device and is typically housed in a box to protect it from air currents in the room. This historic type of instrument has been used for very sensitive measurements for almost 200 years.

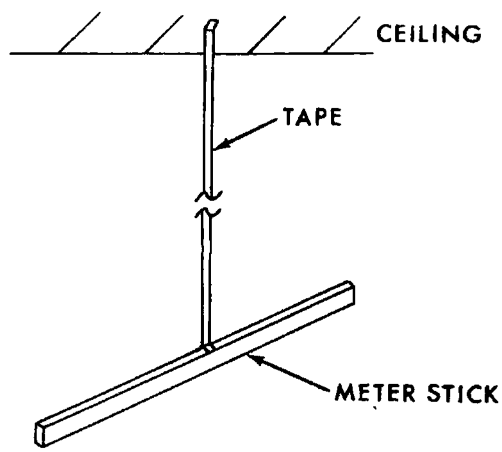
Demonstration of a Torsion Balance

Purpose: To illustrate the sensitivity of a torsion balance.

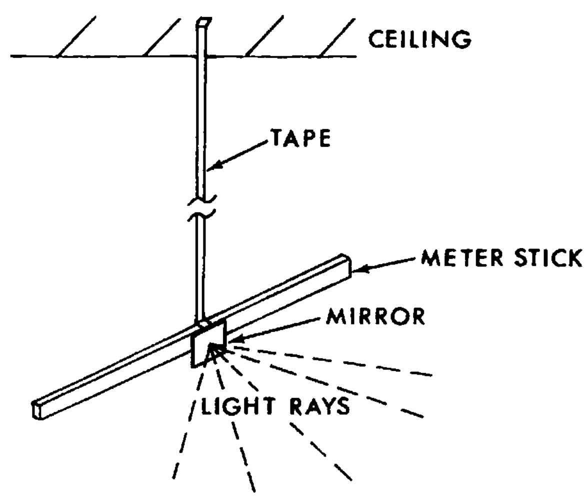
Materials: Meter stick with a 2-m length of tape (computer tape, ticker tape, or any similar material) attached at the midpoint to form a torsion balance; small mirror; charged rod; source of a beam of light (flashlight, laser).

Procedure and Notes: Steps 3 and 4 are optional, but add depth to the demonstration.

1. Arrange the balance as shown in the figure below.

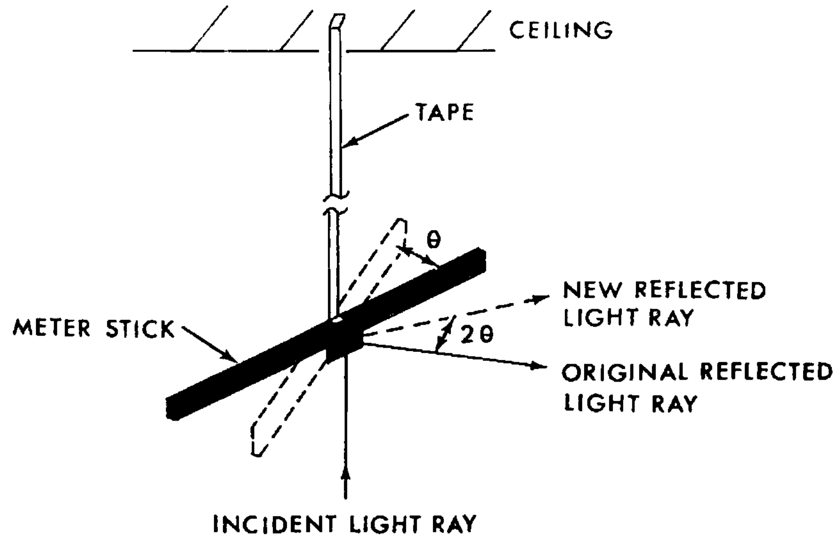


2. Tap one end of the meter stick lightly and note the motion.
3. Steady the meter stick and bring a charged rod close to the end of the meter stick. Here again note the motion of the balance.
4. Tape a small mirror to the meter stick and shine a beam of light onto the mirror, as shown in the figure below. Observe the motion of the reflected spot when the meter stick is disturbed as in Step 2.



Explanation:

Rotation of the meter stick can be easily produced by exerting a contact force, as in Step 2, or an electrical attraction, as in Step 3. The reflected beam of light in Step 4 actually rotates through twice the angle that the meter stick does. Cavendish used the gravitational force to move his torsion balance.



EVALUATION QUESTIONS

1. **Egg shells are held together by**

   A. the gravitational force.
   B. the electric force.
   C. the weak nuclear force.
   D. the strong nuclear force.

2. **Cavendish found the mass of the earth by using a torsion balance to measure the**

   A. electric force between the earth and the moon.
   B. strong force between two 1 kg masses.
   C. universal gravitational constant, G.
   D. pull of the earth on a known mass.

3. **In some models of the atom, electrons orbit the nucleus in much the same way that planets orbit the sun. Why can this analogy be made?**

   A. Both forces depend inversely on the square of the distance.
   B. Both forces are proportional to the masses of the attracting bodies.
   C. The electrical force, like gravity, depends on mass.
   D. The electrical force is really gravity acting at small distances.

4. **Which one of the following is *not* a consequence of the gravitational force?**

   A. The formation of galaxies.
   B. The organization of the solar system.
   C. A stone falling.
   D. The structure of the nucleus.

5. **Many physicists in a quest for a simple description of nature would like to imply that**

   A. the weak and strong nuclear forces are really the same.
   B. all forces follow inverse square laws.
   C. all forces are simply different manifestations of just one force.
   D. gravity and electrical forces are the same force.

6. **Friction is caused by**

   A. gravitational pull only.
   B. electrical forces.
   C. nuclear repulsion.
   D. strong interactions.

7. **Radioactive decays are a manifestation of**

   A. the strong nuclear force.
   B. the weak nuclear force.
   C. the electric force.
   D. the gravitational force.

8. Scientists have concluded that the moon is held in its orbit about the earth by gravity and not by the electrical force. Which of the following statements gives the reason for that conclusion?

   A. Only gravity depends inversely on the square of the distance between two bodies.
   B. The electrical force is canceled out by the strong force between the earth and the moon.
   C. The earth and the moon are essentially neutral and, therefore, have no electrical force between them.
   D. The gravitational constant is larger than the electrical constant in the respective force laws.

9. In normal, *everyday* situations the forces that you encounter are the consequences of only

   A. the weak and the electric forces.
   B. the strong and the weak forces.
   C. the gravitational and the strong forces.
   D. the electric and the gravitational forces.

10. Which of the two fundamental forces have similar equations describing their nature?

    A. gravity-weak nuclear.
    B. weak nuclear-strong nuclear.
    C. electrical-weak nuclear.
    D. gravity-electrical.

ESSAY QUESTIONS

11. Many physicists strive to unite the four fundamental forces of nature. What evidence would cause them to feel this may be possible? Explain your answer.

12. How are the gravitational and electrical forces similar? How are they different? When is each most dominant?

KEY

1. B
2. C
3. A
4. D
5. C
6. B
7. B
8. C
9. D
10. D

SUGGESTED ESSAY RESPONSES

11. Physicists have been intrigued by similarities in the natural forces. For example, the mathematical descriptions of the electrical and gravitational forces are identical. Although the nuclear forces are not totally understood, they both govern within the nucleus. Science has been able to unite many explanations of common phenomena to single truths of nature. The possibility that the four forces derive from a single source is very appealing.

12. Both forces can be described by an identical mathematical relationship. The intensity of both forces depends on the inverse square of separation. Both are directly related to the product of the quality that is responsible for the force. The gravitational force is an attractive force; but the electrical force can be either attractive or repulsive. Gravitation dominates over large distances since most bodies are electrically neutral because of a charge balance. However, for short distances (less than a meter) imbalance of charge can cause the electrical force to be much stronger than the corresponding gravitational force.