THE MECHANICAL UNIVERSE

High School Adaptation

QUAD IV

FROM KEPLER TO EINSTEIN

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THE MECHANICAL UNIVERSE

High School Adaptation

A co-production of the
California Institute of Technology
University of Dallas
and
Southern California Consortium

QUAD IV

FROM KEPLER TO EINSTEIN

Kepler's Laws
Introduction to Waves
Temperature and the Gas Law
Curved Space and Black Holes

An Annenberg/CPB Project

National Science Foundation

Materials Development Council

Donald J. Barron, Wheaton High School, Wheaton, Maryland
Debra G. Cannon, Grand Prairie High School, Grand Prairie, Texas
Judith B. Healey, Plano Senior High School, Plano, Texas
Dr. Charles R. Lang, Omaha Westside High School, Omaha, Nebraska
William J. Layton, Palisades High School, Pacific Palisades, California
William H. Leader, Loara High School, Anaheim, California
Franceline C. Leary, Troy High School, Troy, New York
Donald A. Martin, Cistercian Preparatory School, Irving, Texas
Katherine E. Mays, Visiting Fellow, AAPT
Wilfred H. Oswald, Napa High School, Napa, California
Donald M. Sparks, North Hollywood High School, North Hollywood, California
Dr. George O. Taylor, Jr., The Baylor School, Chattanooga, Tennessee
Courtney W. Willis, University High School, Greeley, Colorado

Developer and Project Director of The High School Adaptation

Dr. Richard P. Olenick
Associate Professor of Physics, University of Dallas

Curriculum Consultant and Editor

Dr. Kathleen A. Martin, Associate Professor of Education
Texas Christian University

Evaluators

Geraldine R. Grant
J. Richard Harsh

Project Manager

Don Delson

Project Secretaries

Nancy Renwick
Judy Post

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Developer and Project Director of The Mechanical Universe
Dr. David L. Goodstein, Professor of Physics and Applied Physics
California Institute of Technology

Executive Producer

Sally V. Beaty, President, Southern California Consortium

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FOREWORD

Today, scientific and educational leaders are seriously concerned about the quality of science and mathematics education in the United States. It is as though the problems have been rediscovered, 25 years after Sputnik! In addition to those problems which have repeated themselves, today many qualified science and mathematics teachers at the pre-college, college, and university levels are being lured from the classroom by higher-paying jobs in business and industry. Many classrooms, therefore, have become the responsibility of instructors with limited preparation in the subject matter they are called upon to teach. And yet, more than ever the nation's current economic, social, and political needs call for a technologically literate population.

The Mechanical Universe, which served as the basis for the high school materials, addresses one critical need in science education by providing video and print materials that can serve as the basis of a solid, introductory college-level physics course. The video offers an exciting array of audiovisual resources for classroom instruction: close-ups of complicated experiments; extensive computer animation sequences that make abstract concepts and mathematical processes understandable; historical reenact-ments that provide a philosophical fabric for the development of ideas of physics.

The Mechanical Universe, part of the Annenberg/CPB collection, has as its primary purpose the provision of a quality learning experience for those whose lives cannot fit into the traditional campus schedule. This 52-program introduction to physics also offers a partial answer to some of the current problems of science education, for it can be used to upgrade skills of secondary science teachers and to provide supplementary support in the college and university classes.

Through the sponsorship of the National Science Foundation, selected programs of **The Mechanical Universe** have been adapted for use in high school. These materials represent the same quality and innovation as the college series, but they are presented in shorter and less mathematically oriented tapes that can be used in a wide variety of high school curricula. Teachers who find themselves teaching high school physics in spite of limited preparation will discover that, by enrolling in **The Mechanical Universe** course and using the adaptations in their classes, they will enjoy the confident feeling that they are presenting their students with quality instruction.

INTRODUCING

THE MECHANICAL UNIVERSE

High School Adaptation

The adaptations of **The Mechanical Universe** were created by twelve outstanding high school physics teachers (the Materials Development Council) through the generous support of the National Science Foundation. The clear purpose of the Council and the entire staff was to produce quality materials that would be used to improve instruction in physics. No one was satisfied with the goal of producing materials that would simply motivate or fascinate students, or would provide a change of pace. From the start, the challenge was to create materials which could make wise use of the power of television in developing a sound and solid understanding of physics.

Herewith the fruit of these labors: sixteen modules each consisting of a video adaptation from **The Mechanical Universe** with written support materials. Each module stresses conceptual understanding of underlying physical principles. The written materials support the video dimension of the modules. These support materials provide the teacher with additional background information and mathematical derivations, pre-video and post-video questions, applications, demonstrations, and evaluation questions.

The Mechanical Universe was originally developed for lower-division college courses in physics. The materials from **The Mechanical Universe** that have been adapted for use in high schools were field tested in 1984–86 by over 100 high school physics teachers located in schools widely scattered across the county in both urban and rural communities that serve various socio-economic populations. As a result of the assessment of the field testing, the videos were re-edited and the written materials were focused more directly on the videos to provide the best support possible for teachers.

PREFACE

These materials are intended for all teachers of high school physics. Teachers new to the arena of physics will discover rigorous, conceptual video presentations of traditional and not-so-traditional topics in classical physics. We hope that each word of the written materials will be savored. They are your resources and we hope that you tap them to capture the excitement of **The Mechanical Universe**. Experienced teachers will find a different slant to classical physics in the space age: a humanizing, compelling, integrated approach to the greatest revolution in the history of Western civilization. These teachers, too, we hope, will find the written materials continually refreshing resources.

Although **The Mechanical Universe** is a calculus-based course, the excerpts for high school use were selected to focus on concepts. That is not to say that the videos for high school use are not rigorous; they present sound logic at every stage in the development. Mathematics is occasionally used in the high school materials as a language to relate ideas concisely. In many cases the original mathematical derivations have been modified to be appropriate to the high school level. Nonetheless, mathematical derivations go by quickly in the video and we hope that teachers will replay these sections for their students. The mathematical background sections of the modules, we expect, will be read by all teachers even though they may not necessarily present to their classes the same level of mathematics provided in the print materials. We hope that teachers as well as students will gain a better appreciation of the vital role of mathematics in physics.

No laboratory component is currently suggested. The reason is not because we judge a physics laboratory component to be unimportant or uninteresting. On the contrary, we believe that demonstrations and laboratories lie at the heart of a sound education in high school physics. Instead we concentrated on what we could offer best: instruction through television. There are dozens of laboratory manuals which can be appended easily to these materials and we expect that each teacher will decide how best to handle the laboratories. On the other hand, since many demonstrations and applications to everyday life are presented in the video, we identified simple, short, and effective demonstrations that tie into concepts in the video. We hope that all physics teachers will enjoy performing them.

Not all the topics covered in the modules are conventional to high school physics curricula. *Angular Momentum* and *Harmonic Motion*, effectively covered in the videos, are two topics which are not necessarily a part of every curriculum. *Navigating in Space*, on the other hand, represents an exciting application of Kepler's ellipses and Newton's gravity that is not covered in typical curriculum. Other topics, such as *The Fundamental Forces* and *Curved Space and Black Holes*, provide tantalizing looks at twentieth century physics from the perspective of classical physics.

The Mechanical Universe is the story of the Copernican revolution, why it was necessary, and how it unfolded in the work of Galileo, Kepler, and Newton. It is the story of the eventual wedding of the heavens with the earth through the synthesis of mechanics and astronomy. History is presented in the series, not for the sake of historical detail, but for a fuller sense of how scientific thought proceeded through the intellectual searches and triumphs of men who reshaped the society of their times. We hope the infectious spirit of **The Mechanical Universe** will inspire teachers and students and will contribute to a lifelong scientific interest in the workings of the universe.

ACKNOWLEDGEMENTS

The adaptations of these instructional materials for high school use would not have been possible without the assistance of a long list of people who aided through the dedicated use of their diverse and specialized skills.

Heading the list is Professor David L. Goodstein, of Caltech, whose inspiration and guiding force in the creation of **The Mechanical Universe** led to the development of these materials.

Program Direction

Dr. Richard P. Olenick
Nancy Renwick, Secretary

Curriculum Development

Dr. Kathleen A. Martin

Production of Videotapes

Sally V. Beaty, Executive Producer
Dr. James F. Blinn, Computer Animator
Peter F. Buffa, Producer
Jack G. Arnold, Story Editor
Glenn Kammen and Peter Robinson, Videotape Editors

Program Management

Don Delson, Project Manager
Judy Post, Project Secretary

Program Evaluation and Field Testing

Geraldine R. Grant
J. Richard Harsh
Diana D. Price, Secretary

Videotape Distribution

Judy Sullivan, Product Distribution
Jean Coltrin, Director of Marketing

Special Mention

Dr. Howard Hubbard, (Supervisor of Mathematics and Science, Long Beach Unified School District, CA., retired), for review of materials
Lynn Strech, Artist
Judy Sullivan, Southern California Consortium, Design and typesetting

Finally, we offer special thanks to Mary Kohlerman of the National Science Foundation.

Materials Development Council
Irving, Texas
July 1987

STRUCTURE OF THE MATERIALS

The written materials are designed to support and extend the VIDEO presentation of each module. The format and content of the materials are designed to help the user (1) to integrate the concept(s) presented in the VIDEO with traditional high school materials, (2) to supplement and promote conceptual understanding of the phenomena presented in the VIDEO, and (3) to infuse the students with a new spirit of inquiry concerning the mechanics of physics.

Each module is composed of components of written materials. Each component is intended as a resource to promote active engagement of the learner in developing conceptual understanding of the physical phenomena. The five components of the print materials are:

TEACHER'S GUIDE

Pre-Video Activities*

Content and Use of the Video – describes what the VIDEO does and does not cover.

Terms Essential for Understanding the Video – includes the definitions of terms listed in the STUDENT'S GUIDE, discussion of critical elements or relationships.

What to Emphasize and How to do It – includes the objectives of the module, references to demonstrations, possible applications, and suggestions for correcting common misconceptions.

Points to Look for in the Video – includes common misconceptions when relevant; characteristics and questions concerning critical elements presented in the VIDEO. Answers to questions in the STUDENT'S GUIDE are included.

Post Video Activities*

Everyday Connections and Other Things to Discuss – suggests additional questions to promote student participation and discussion. An essential purpose of the questions is to engage students in review and clarification of the concepts.

Summary – reviews the key concepts that have been presented.

STUDENT'S GUIDE

(designed for duplication and distribution)

Introduction – a brief statement about the content or purpose of the VIDEO.

Terms Essential to Understanding the Video – includes terms or critical elements of the VIDEO, with definitions and explanations provided in the TEACHER'S GUIDE.

Points to Look for in the Video – includes common misconceptions when relevant; characteristics and questions concerning critical elements presented in the VIDEO along with figures representative of key points in the VIDEO.

TEACHER RESOURCES

Supportive Background Information – summarizes additional historical, physical, and mathematical information that relate to the topics and content presented in the VIDEO.

Additional Resources – includes demonstrations and applications the teacher may use to extend and enrich the treatment of the topic.

Evaluation Questions – provides ten multiple-choice questions dealing with the objectives of the module and two essay questions that require student's explanations of certain concepts related to the topics.

*The repeated showing of the video (in full and part) is essential to student understanding. The division of activities into prevideo and postvideo activities, therefore, is somewhat artificial. It is likely that most, if not all, prevideo activities will precede the initial showing of the video. Sections of the video will undoubtedly be sprinkled throughout the postvideo activities, with a full showing being used for closure where time permits.

QUAD IV

KEPLER'S LAWS

HOW CAN THE MOTIONS OF THE PLANETS BE DESCRIBED? Although Copernicus had proposed that the earth is not the center of the universe, an accurate description of the motions of the planets was not developed until the pioneering work of Johannes Kepler. No seventeenth century astronomer searched more for order in the heavens and had a life fined more with turmoil than he. This video depicts the momentous effort Kepler made to find a mathematical description for the motion of the planets, in particular Mars. The three laws Kepler subsequently formulated provided the essential groundwork for Isaac Newton's later development of the universal law of gravitation.
Running time: 14:52

INTRODUCTION TO WAVES

WHAT MAKES A WAVE? Water waves, shock waves, sound waves, and waves on a spring are all examples of mechanical waves. In this video, the generation and characteristics of mechanical waves are introduced and explored. The oscillatory motion of balls linked by springs-coupled oscillators – provides a model of the way such waves travel. This model is extended to describe sound and water waves.
Running Time: 10:20

TEMPERATURE AND THE GAS LAWS

WHY DO GASES EXPAND WHEN HEATED? The connection between temperature and energy emerged after a series of investigations into the behavior of gases undertaken by Boyle, Charles, and other scientists. In this video, the observable properties of gas, such as temperature and pressure, are tied to microscopic events, such as gas molecules striking container wells. Through the use of computer animated sequences, the difference between heat and temperature is formulated and the ideal gas law is developed.
Running time: 16:42

CURVED SPACE AND BLACK HOLES

WHY DO ALL BODIES FALL WITH THE SAME CONSTANT ACCELERATION? Newton realized that the law of falling bodies was a consequence of the equality between inertial mass and gravitational mass of an object. Einstein realized that that equivalence stemmed from a more fundamental feature of nature – the inability to distinguish solely by experiments done within a single reference frame whether that frame is undergoing uniform acceleration or is in free fall. In this video, the principle of equivalence and its consequences – curved space and black holes – are introduced and explored.
Running time: 12:36

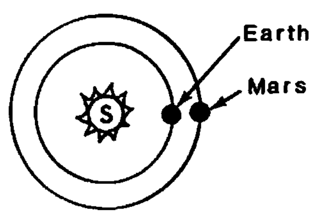
TEACHER'S GUIDE TO KEPLER'S LAWS

CONTENT AND USE OF THE VIDEO - Kepler's laws are fundamental to the study of motion of the planets. Although most high school physics teachers examine simple orbital motion in relation to centripetal motion, few examine in detail the real variable motion of the planets. The video introduces Kepler's laws, not from the perspective of a mathematical analysis, but from the viewpoint of the historical development of the ideas. As such, this video could be used successfully at various stages in the study of physics.

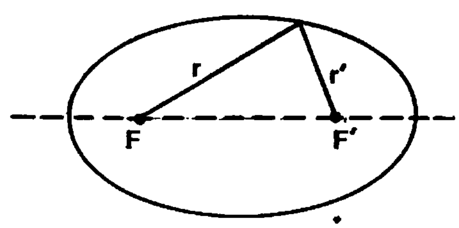
Used as an introduction, Kepler's laws demonstrate one dimension of the scientific process--how methodical work and persistence eventually lead to the development of a new idea. The video could also be used to tie together a number of ideas presented in other modules. For example, it could be shown after the combined study of Newton's laws, centripetal force, and energy. Another appropriate time to show the video would be after a study of periodic motion. Also, *Angular Momentum* together with *Kepler's Laws* is a proper introduction to *Navigating in Space*.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - The video introduces terms which the student might have heard but whose technical use is unclear. Therefore, it would be helpful to list these terms and brief definitions on the board and discuss them with the students.

Mars in opposition--the position of Mars is the same, as viewed either from the earth or the sun.



ellipse--the set of all points in a plane the sum of whose distances r and r′ from two fixed points F and F′ (the foci) is a constant.

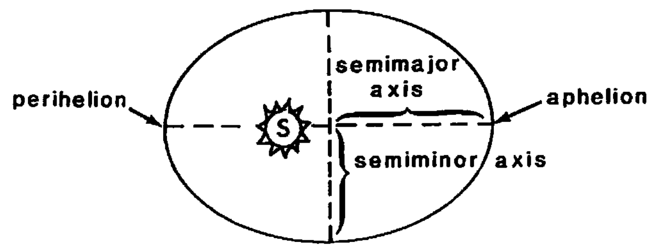


perihelion--the point of a planet's orbit nearest to the sun.

aphelion--the point of a planet's orbit farthest from the sun.

semimajor axis--half the distance of the long axis of the ellipse.

semiminor axis--half the distance of the short axis of the ellipse.



eccentricity--the degree of departure from circularity; the more eccentric the orbit, the flatter its appearance.

WHAT TO EMPHASIZE AND HOW TO DO IT

- This video presents a historical review of how a mathematical genius took extensive observational data and extracted some common relationships of the movements of planets. Although the *results* of Kepler's work are valuable and essential to us, the video emphasizes the *nature* of scientific research. The presentation illustrates the circuitous routes that scientists often take in analyzing data, developing hunches, experiencing failures, stating hypotheses, seeking replications of findings, and then gradually developing laws of relationship. The video emphasizes that Kepler's descriptions of planetary motion stand as one of the classic investigations in the history of physics.

Objective 1: Describe why and how Kepler went about determining the orbit of Mars.

Frequently students don't recognize the importance of Kepler's study of Mars' orbit. Discuss with your students why it was significant that Kepler chose to study the orbit of Mars rather than any of the other planets which were then observable. Be sure they understand that Tycho's measurements of the slightly eccentric orbit of Mars provided the only data which would have enabled Kepler to conclude that the planets travel in elliptical rather than circular paths around the sun. Replay the section of the video describing Kepler's determination of Mars' orbit.

If your students have little background regarding the nature of the ellipse, you might replay the latter part of the video to examine its characteristics. DEMONSTRATION #5, a construction of the ellipse, would be helpful here. If the materials are available, you might also work through DEMONSTRATION #4, the triangulation of Mars' orbit.

Objective 2: Recognize Kepler's laws:

(a) Each planet orbits the sun in an elliptical path, with the sun at one focus;
(b) A line from a planet to the sun sweeps out equal areas in equal times; and
(c) The square of the period of a planet's elliptical orbit is proportional to the cube of the length of its semimajor axis.

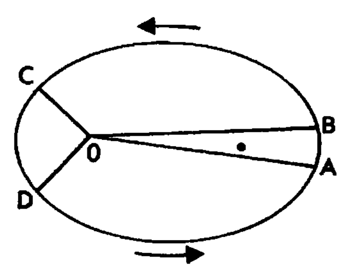
(a) Although students may be aware that planets travel in elliptical orbits around the sun, they may not be familiar with the nature of that elliptical motion. You might want to perform DEMONSTRATION #1, a swimsuit material model of Kepler's first law, and then discuss the various paths and velocities of the steel balls.

(b) Ask your students what determines the orbital speed of the planets. Be sure they understand that the law of universal gravitation describes the force between a planet and the sun (similar to the force of the "gravity" well on the rolling balls in DEMONSTRATION #1). As a planet moves in its elliptical path away from the sun, gravitational force diminishes and so does the planet's speed. Conversely, the closer a planet is to the sun, the greater the gravitational force and the greater the planet's speed.

(c) You might ask your students why Halley's comet takes so long to reappear compared with Encke's comet. Halley's comet has a very eccentric orbit; the aphelion is between the orbits of Neptune and Pluto. Therefore, according to Kepler's third law, the period is very long--76 years. On the other hand, Encke's comet has a period of only 3.3 years due to its considerably less eccentric orbit and smaller semimajor axis.

DEMONSTRATIONS #3, Kepler's third law of harmony, and #4, the triangulation of Mars' orbit, will be helpful in discussing Kepler's second and third laws. Since the concepts presented in these demonstrations are ones emphasized in the video, it should not be difficult to generate discussion.

Your students will probably be interested in the applications of Kepler's laws. You might draw the following figure on the board and then ask them how Kepler's law of equal areas can be used to show why a comet's orbital speed varies as it orbits the sun, which is located at O in the figure.



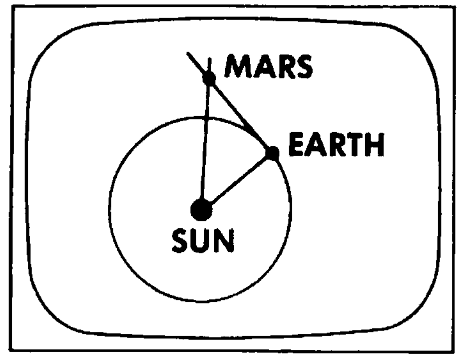
In the figure above, the time the comet takes to sweep out area AOB and area COD is the same. According to Kepler's second law, the two areas are also equal. Then the comet must move from A to B in the same time as from C to D. Since the length of arc CD is greater than AB, the comet must be moving faster as it nears the sun.

Another interesting application of Kepler's laws relates to the orbital path of satellites. Ask your students if they know the difference between the orbit paths of American and Russian satellites.

American satellites are usually placed in nearly circular equatorial orbits. Communication is maintained by way of several communication centers along a wide belt around the earth. Russian satellites are placed in highly eccentric polar orbits. From the northern latitudes of their communication centers, the satellite is nearly always in communication "sight." During the time when the satellite is out of sight below the Russian horizon, the speed is so great that the time of blackout is minimal.

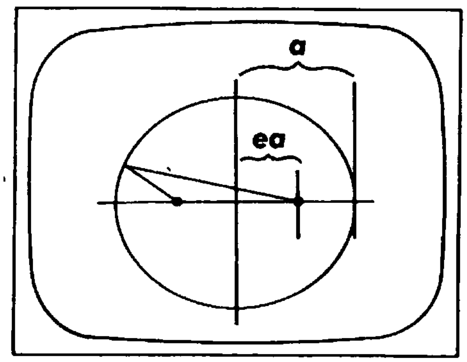
POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and frames from the video.

What method did Kepler use to determine the orbit of Mars? Would this method have worked if he had chosen to "wage war" on a planet other than Mars?



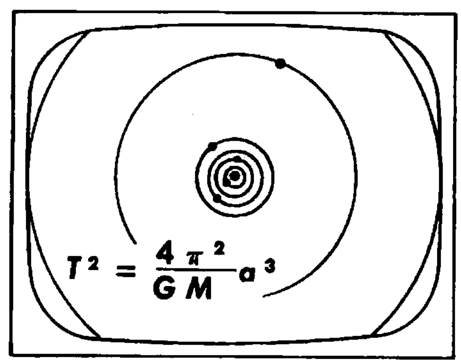
Kepler used the method of triangulation. He knew the orbit and timetable of earth and the length of the Martian year. In a Martian year, Mars returns to the same place in the sky, but the earth would not be in its same place. By using the position of the earth at the same stage of successive Martian years, Kepler could triangulate to find points on the orbit of Mars. His method would not have worked with other known planets since Mars has the only planetary orbit eccentric enough--about 9%--to be noticeably noncircular using data available at that time.

What are the characteristics of an ellipse?



1. *An ellipse has two foci.*
2. *The sum of the distances from the foci to a point on the ellipse is constant.*
3. *The degree of "off-centeredness" of an ellipse is termed its eccentricity.*
4. *An ellipse describes a continuous and consistent path.*

In your own words, explain the meaning of Kepler's third law. How does this law explain the physical relationship between the planets as we actually see them in the night sky?



Kepler's third law states that the square of the period, T, of a planet's orbit is proportional to the cube of the semimajor axis, a, of the orbit. Mathematically we have $T^2 \propto a^3$. The third law tells us that the farther a planet is from the sun, the longer the planet takes to orbit. Since the planets have different periods, their positions as viewed from the earth appear somewhat erratic. Nonetheless, the third law expresses the underlying physics.

EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in this video, you might pose the following questions to your students.

1. **Why is it not necessarily true that Saturn and its moons can be viewed on any given night?**

   Saturn's average radius of orbit is much larger than that of earth. Therefore, its period is considerably longer. As both planets orbit the sun, there are times when our nighttime viewing shows us a section of the skies that does not include Saturn.

2. **How do Kepler's laws apply to our interplanetary probes?**

   To send a probe directly to another planet would require more expenditure of energy than our spacecraft can supply. Therefore, only enough energy is used to get the probe out of the earth's orbit. The sun's gravitational field does the rest, bending the motion of the probe into an elliptical path. Care must be taken, however, that as the probe reaches its aphelion, the planet is there to greet it. The launch date must be carefully timed to take into account both the period of the probe and the period of the planet to ensure a happy meeting.

SUMMARY - The important theme of this module is that the motions of planets can be described precisely by Kepler's three laws. Contrary to Plato's dictum of the perfection of circular motion, Kepler's first law states that the planets' orbits are ellipses with the sun at one focus. What has become known as Kepler's second law states that an imaginary line from the sun to a planet sweeps out equal areas in equal times. Kepler's third law states that the square of the period of a planet's orbit is proportional to the cube of the semimajor axis of its orbit. Together, Kepler's laws provide a kinematics of heavenly motion, the essential groundwork for Newton's later development of the universal law of gravitation.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO KEPLER'S LAWS

INTRODUCTION - This video presents a historical review of how a mathematical genius took extensive observational data and extracted some common relationships of the movements of planets. Although the *results* of Kepler's work are valuable and essential to us, the video emphasizes the *nature* of scientific research. The presentation illustrates the circuitous routes that scientists often take in analyzing data, developing hunches, experiencing failures, stating hypotheses, seeking replications of findings, and then gradually developing laws of relationship.

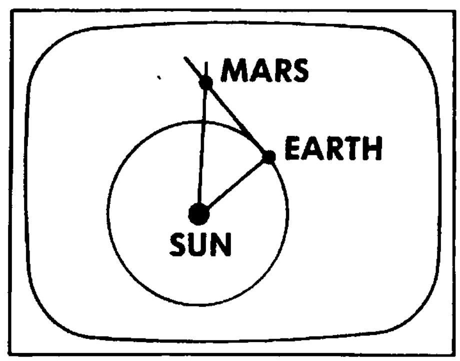
Terms Essential for Understanding the Video

ellipse
Mars in opposition
perihelion
aphelion
semimajor axis
semiminor axis
eccentricity

**** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ****

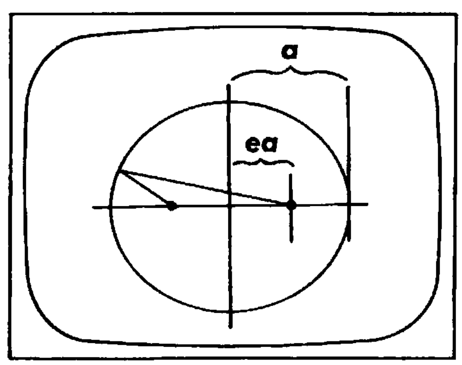
Points to Look for in the Video

What method did Kepler use to determine the orbit of Mars? Would this method have worked if he had chosen to "wage war" on a planet other than Mars?

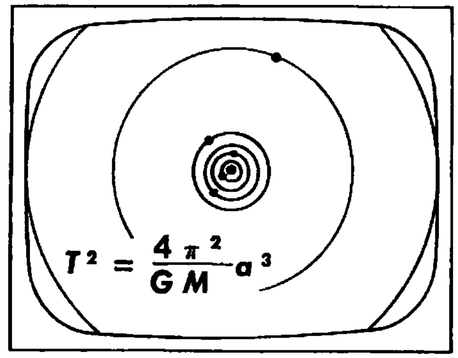


In watching the video, note how Kepler, using a series of observations, could triangulate precise points on the orbits of Mars. If you do not understand how Kepler used the method of triangulation, ask your teacher to replay the related section of the video.

What are the characteristics of an ellipse?



In your own words, explain the meaning of Kepler's third law. How does this law explain the physical relationships between the planets as we actually see them in the night sky?

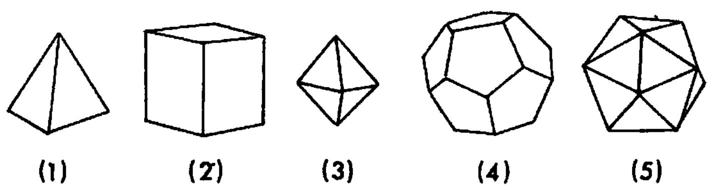


TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - At one time Isaac Newton indicated that his work rested on the shoulders of giants. Two of these, Tycho Brahe and Johannes Kepler, laid the foundation for Newton's analysis of the motion of planets.

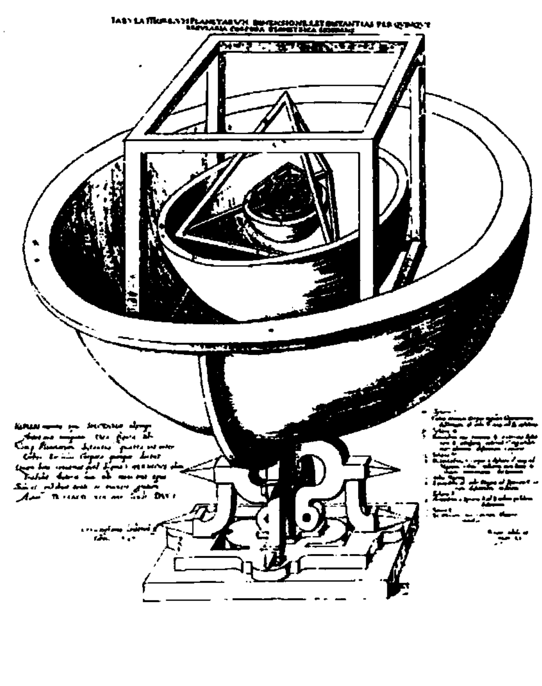
Tycho was an ill-tempered Danish lord who tongue-lashed kings, tormented peasants, and sported a metal nose (his own being lost in a youthful duel over mathematics). At the same time he was a measuring maniac, a fussy, precise man who opened a new era of observation in science. He had realized the need for more accurate astronomical data than what were available at that time. Princes of Europe vied with one another for his services as court astronomer and astrologer. As a result, he was able to demand and build the finest observatory of his day. In this observatory on an island off the coast of Denmark, Tycho took up his lifelong task of determining the relative positions of the sun, moon, and stars. All of his observations were done prior to the invention of the telescope. So accurate were these observations that he modified the error in the knowledge of the planetary positions from about ten minutes to two minutes of arc.

At the beginning of the year 1600, Tycho formed a strange alliance with an impoverished mathematical genius named Johannes Kepler. Kepler was a small, nearsighted man who in early life suffered the affliction of illness and the torments of classmates. He gradually learned the resources of his own mind and was able to transform his inner agony into creative achievements. While teaching a class, he stumbled onto the idea that certain symmetrical solids could be the key to the universe. Only five figures, called Platonic solids, have faces which have equal sides and angles.



The five Platonic solids (1) tetrahedron, (2) cube, (3) octahedron, (4) dodecahedron, (5) icosahedron.

Kepler imagined a model in which these five solids could be nested, one inside the other. Between the solids could be fitted spheres just touching the corners of the solids. By trial and error, varying the order of one solid within another, he found a way to arrange them so that the spheres fit to about 5% of the actual planetary distances. Kepler concluded that there were only six planets since he thought it impossible that the number of these regular solids and the intervals between the known-planets could be equal by chance.



Later, Kepler turned to a more promising problem, perhaps realizing that his model could not make any predictions on the motions of planets. He began a search for a mathematical relation between a planet's distance from the sun and the time needed to complete one orbit. And to accomplish this he needed to acquire the best astronomical data in Europe--those of Tycho Brahe.

Tycho and Kepler found they needed one another. Kepler needed Tycho's accurate astronomical data. Tycho, to get his data organized in useful order, needed Kepler's mathematical genius. This tenuous relationship lasted 18 months. At a banquet, Tycho overdrank and then held back his water beyond the demands of courtesy. After 11 days of an acute urinary infection, he died. In order to keep the invaluable astronomical data from the hands of the probate court, Kepler promptly stole Tycho's papers.

Armed with these data, he spent the next six years and some seventy failures trying to fit the orbit of Mars to a uniform circular path. He finally realized it was hopeless. Rather than the circle, the orbit of Mars fit a different elegant curve--the ellipse. Plato's perfect circles, which had captivated the minds of all observers for twenty centuries, finally had to be abandoned. Kepler formulated his first law: **EACH PLANET ORBITS THE SUN IN AN ELLIPTICAL PATH WITH THE SUN AT ONE FOCUS.**

Noticing that the speed of the planet was greater as it drew closer to the sun and slower as it receded, he determined his second law: **THE LINE FROM THE SUN TO THE MOVING PLANET SWEEPS OUT EQUAL AREAS IN EQUAL INTERVALS OF TIME.**

It took Kepler ten more years before he formulated what has become known as the third law, called the law of harmony: **THE SQUARE OF THE PERIOD OF A PLANET IS PROPORTIONAL TO THE CUBE OF THE LENGTH OF THE SEMIMAJOR AXIS OF ITS ORBIT.**

Kepler's laws provided a more drastic departure from ancient theory than did the Copernican system. His three laws of heavenly motion became an important foundation for Newton's analysis of the heavens. Before Newton could establish a mechanics which would explain both celestial and terrestrial motion, the celestial motions themselves had to be described adequately. In other words, a kinematics of the heavens was needed. Kepler provided just that.

Although Kepler is remembered for his description of planetary motion, he also attempted to build a theory of celestial dynamics. In an effort to explain the phenomenon of planetary motion, he theorized that there must be a force emanating from the sun which sweeps the planets in their orbits around the sun. Kepler's proposal had revolutionary significance, not because it was correct, but because it was the first attempt to assign a physical cause to celestial motion. His theory constituted a fundamental break with Aristotelian doctrine, thereby bringing the physics of the heavens closer to that of the earth.

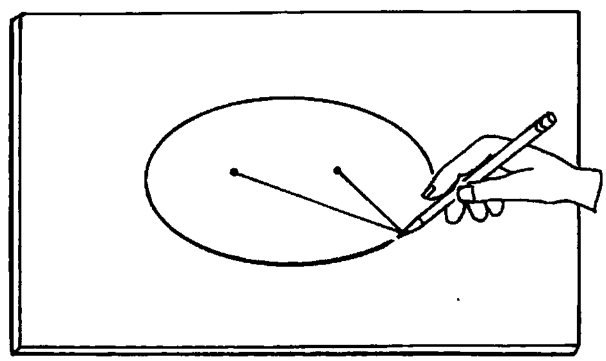
ADDITIONAL RESOURCES

Demonstration #1: Construction of the Ellipse

Purpose: To demonstrate the construction of an ellipse and the relationship of the foci to the elliptical path.

Materials: Bulletin board, tacks, string, and pencil.

Procedure and Notes: Back a bulletin board with white paper. Cut a piece of string between 12 and 16 inches long. Tack the two ends to the board approximately 8 inches apart on an imaginary horizontal axis of the paper. Insert a pencil in the loop of the string and pull it taut to form a triangle. Move the pencil around the paper keeping the string taut. In the resulting ellipse, the tacks represent the foci.



Ask students what will happen if:

(1) the foci are moved farther apart.
(2) the foci are moved closer together.
(3) the foci are located at the same point.

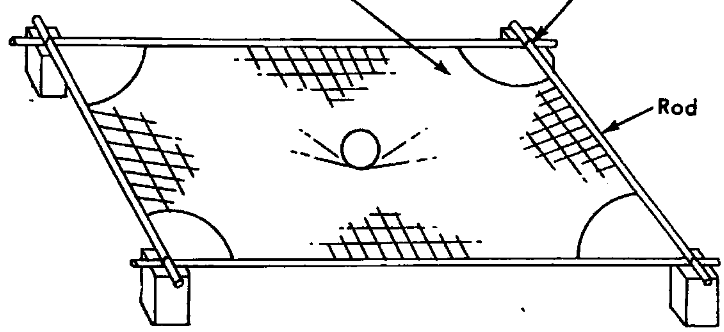
Explanation: Moving the foci farther apart increases the eccentricity of the ellipse, whereas moving them closer together decreases the eccentricity. If the foci are located at the same point, the consequent path is circular.

Demonstration #2: A Swimsuit Model of Kepler's First Law

Purpose: To illustrate varying planet velocities as a planet moves along its elliptical path.

Materials: Four aluminum rods (about 1 m long), Lycra (swimsuit material), one heavy steel ball, several small steel balls.

Procedure and Notes: Set up the materials as shown below. Place the heavy ball in the center. By rolling the smaller balls at different angles and velocities, paths of various shapes can be produced.



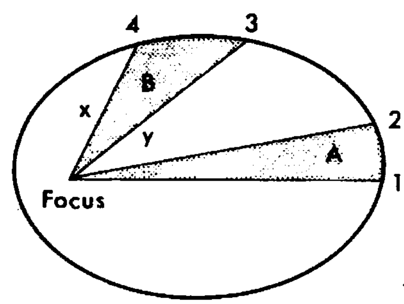
Explanation: The depression caused by the heavy ball will simulate a "gravity" well or a centrally directed force on the rolling balls. The initial speeds and angles will result in various shaped orbits. As the balls "fall" more deeply into the "gravity" well, the gravitational force increases their velocities. Conversely, they lose velocity as they move farther away.

Demonstration #3: The Law of Equal Areas

Purpose: To examine the relationship between areas swept out by a line from the focus to the elliptical path and the corresponding length of the path.

Materials: Overhead projector transparency of an ellipse laid over graph paper, and a clear plastic ruler.

Procedure and Notes: Draw any small area A that represents the area swept out by a planet in, say, a month. Count the number of squares in that area. Prepare a second area B by first drawing Line x. Then move Line y until the number of squares in B equals the squares in A. Note that for equal areas, the length of the path from Point 1 to 2 is less than the path from 3 to 4. Since the areas are equal, Kepler's second law requires that the times also be equal.



Explanation: This models the actual motion of planets around the sun. Kepler determined that the area swept out by the line from planet to sun was equal for equal intervals of time.

Demonstration #4: Thought Experiment--Kepler's Third Law of Harmony

Purpose: To illustrate the relationship of the period of a planet to its average distance from the sun.

Materials: Blackboard.

Procedure and Notes: The circle is a special ellipse with eccentricity of zero. Therefore, a circularly orbiting satellite should follow Kepler's third law. Find the required orbital velocity for a satellite at any particular distance from the earth's center.

Then find the circumference of that orbital path. Knowing the orbital velocity and the length of its path, find the period of one orbit. This should be identical to the period found by the third law.

Formal Proof

Let M = mass of the earth, m = mass of the satellite, r = radius of the orbit from the earth's center and set $F_{grav} = F_{centripetal}$. Then we have

$$\frac{GmM}{r^2} = \frac{mv^2}{r} ,$$

$$v = \sqrt{\frac{GM}{r}} .$$

Knowing that the circumference of a circle is equal to $2\pi r$, we have

$$d = vt ,$$

$$t = \frac{d}{v} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = 2\pi r\sqrt{\frac{r}{GM}} ,$$

$$t^2 = 4\pi^2 r^2 \frac{r}{GM} ,$$

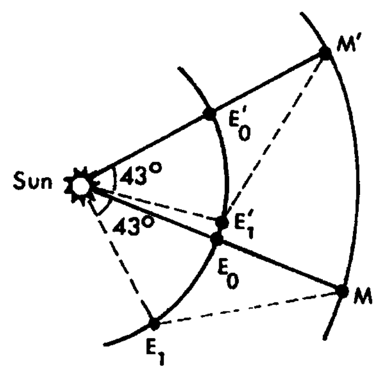
$$t^2 = \frac{4\pi^2 r^3}{GM} .$$

Demonstration #5: Triangulation of Mars' Orbit

Purpose: To demonstrate Kepler's procedure in determining the orbit of Mars.

Materials: The method of triangulation shown in the video is available from most major catalogue suppliers, e.g., Sargent Welch, Cenco, Fisher, etc.

Procedure and Notes: This procedure consists of examining eight pairs of star photographs of Mars. Each pair is taken exactly one Martian year apart. Students use a set of coordinate overlays to gather data used to triangulate the orbit of Mars.



Kepler used observations of the position of Mars separated by one Martian year. By using the position of the earth at the same stage of successive Martian years, Kepler could triangulate to find a point on the orbit of Mars, as depicted in the figure, where point M was fixed by sighting along the lines E_0M and E_1M. Then he could choose a second point, for example M′, the position of Mars and the next time the earth--now at E'_0--was on a straight line between the sun and Mars. When Mars returned to M′ after one further Martian year, the earth would be at E'_1 as shown in the figure, allowing a second triangulation to fix M′. Tycho's data amassed over a quarter of a century allowed Kepler to fix 12 points on the orbit of Mars in this manner. He could not force the points to fit a circular orbit, but rather he found the orbit of Mars to be an ellipse. The disagreement between the data and the best circular path was about 8′ of arc. Here the improvement from the 10′ uncertainty in the data available to Copernicus to the 2′ uncertainty in Tycho's data proved crucial. Had Copernicus not tried to use epicycles but attempted instead to fit the orbit of Mars to a circle in this way, he would have succeeded in matching the best observations available to him. But Kepler, working with Tycho's improved data, could not fit the orbit of Mars to a circle. Faced with the choice of giving up the Platonic ideas of circular motion or violating Tycho's magnificent observations, he chose to believe the observations.

EVALUATION QUESTIONS

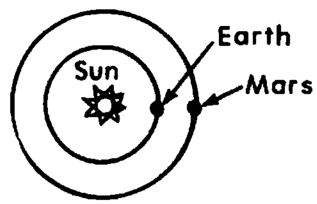
1. **To confirm his ideas about elliptical paths, it is fortunate that Kepler chose to study the orbit of Mars rather than the orbit of another visible planet because**

   A. only Mars was close enough to be seen with the instruments on hand at that time.
   B. Brahe's notes on the other planets were skimpy.
   C. with the data available, only Mars' orbit was eccentric enough.
   D. the periods of earth and Mars are close enough to allow continual viewing.

2. **When Kepler used the method of triangulation to determine the orbit of Mars,**

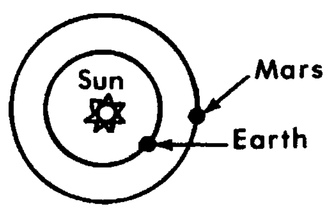
   A. he observed the position of Mars on successive earth years.
   B. he used the position of Mars on successive Martian years.
   C. he relied on observations of when it was on the opposite side of the sun from the earth.
   D. he used data giving the position of Mars every four months.

3.

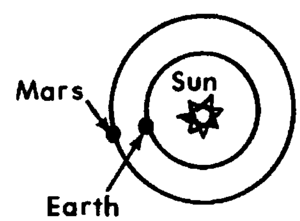


Examine the diagram above of Mars and Earth in opposition. Which of the drawings below could depict a situation one Mars year later?

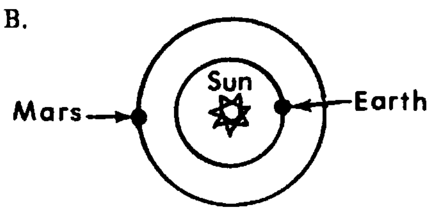
A.



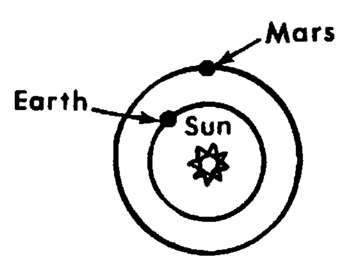
C.



B.



D.



4. Two of Jupiter's moons are being observed. One is seen passing the other. It can be concluded that the moon which overtakes and passes the other is

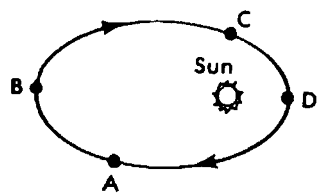
A. closer to the planet.
B. farther from the planet.
C. more massive than the other moon.
D. less massive than the other moon.

5. Kepler and Tycho complemented each others genius. Kepler was known mostly for genius in the area of

A. mathematics.
B. astronomical observations.
C. astronomical measurements.
D. determining star positions.

6. If the figure indicates the path of Halley's comet, at what point would the speed of the comet be the greatest?

A. Point A
B. Point B
C. Point C
D. Point D



7. If the eccentricity of Mars' orbit about the sun were to approach zero, what would the new appearance of the orbit be?

A. An ellipse
B. A circle
C. A parabola
D. A hyperbola

8. Jupiter has a larger orbit about the sun than Mars does. According to Kepler's third law,

A. Jupiter takes greater time to complete one orbit about the sun.
B. Mars takes greater time to complete one orbit about the sun.
C. the periods of all planets are the same, so Jupiter and Mars take the same time to complete one orbit about the sun.
D. none of the above is true.

9. One of the things indicated by Kepler's second law is that

A. all the planets move fastest in January.
B. each planet orbits the sun with constant speed.
C. a planet moves faster when it is closer to the sun.
D. a planet moves slower when it is closer to the sun.

10. Two planets orbit a star in circular orbits. One planet has a period of one year. The other planet is twice as far from the star and has a period that is

    A. greater than two years.
    B. equal to two years.
    C. less than two years.
    D. equal to one year.

ESSAY QUESTIONS

11. In your own words, describe the relationship of the size of an orbit to its period. Illustrate this by discussing the difference between the orbit of Halley's comet and the orbit of Mars.

12. Draw two ellipses using the same length of string. Make the foci in Ellipse A twice as far apart as in Ellipse B. Explain why there are greater variations in the speed of a planet with an orbit like Ellipse A than there are for a planet with an Ellipse B orbit.

KEY

1. C
2. B
3. A
4. A
5. A
6. D
7. B
8. A
9. C
10. A

SUGGESTED ESSAY RESPONSES

11. According to Kepler's third law, the square of the period of a planet is proportional to the cube of the semimajor axis of the orbit. This means that planets or comets that are farther from the sun take longer to complete an orbit about the sun. Halley's comet, for example, has an orbit that takes it out past Neptune; its semimajor axis is much larger than that of Mars. In accordance with Kepler's third law, Halley's comet has a much longer period (76 years) than Mars does (1.9 years).

12. Ellipse A is much flatter than Ellipse B since the foci are twice as far apart. This indicates that Ellipse A also has a greater eccentricity. If one focus of each ellipse is imagined as the sun, then a planet orbiting along Ellipse A can be found at a greater distance from the sun and also a corresponding closer distance than for Ellipse B. According to Kepler's second law, the closer (farther) a planet is from the sun in its orbit, the faster (slower) it is moving. Therefore, the variations in speed are greater for the more eccentric Ellipse A.

TEACHER'S GUIDE TO INTRODUCTION TO WAVES

CONTENT AND USE OF THE VIDEO - The video is intended as an introduction to the generation and characteristics of waves and consequently should be used prior to a detailed treatment of waves and wave phenomena. Coupled oscillators are presented as a mechanical model for waves; therefore, the basic ideas of harmonic motion should have been taught prior to showing this video.

The video illustrates the oscillatory motion of balls linked by springs and then relates that motion to other mechanical waves such as sound waves and water waves. The generation of the two basic types of waves, longitudinal and transverse, is introduced and the basic characteristics of waves such as wavelength, frequency, and wave velocity are discussed. This video does not explore the wave phenomena of reflection, refraction, interference, or diffraction.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - The following terms describing wave characteristics would be helpful for students to know prior to viewing:

medium--the general name given to a substance, which may be a gas, a liquid, or a solid, that is composed of interconnected particles.

mechanical wave--a disturbance that travels through a medium with no net motion of the medium.

crest--the highest part of a wave.

trough--the lowest part of a wave.

amplitude--the size of a wave disturbance that is measured from crest to rest position.

period--the time for a complete cycle of a wave; the time for two successive crests or two successive troughs to pass by a given point.

wavelength--distance between successive crests or between successive troughs.

frequency--the number of crests to pass a given point per second.

compression--the repeated concentration of molecules of a medium created by a mechanical wave.

longitudinal wave--a wave in which the particles of the medium move parallel to the direction in which the wave is traveling.

transverse wave--a wave in which the particles of the medium move perpendicularly to the direction in which the wave is traveling.

WHAT TO EMPHASIZE AND HOW TO DO IT - In our universe, waves are common natural phenomena. Shock waves, water waves, and pressure waves, as well as sound waves are just a few familiar examples. Waves that are propagated through a medium are called mechanical waves. Not all waves observed in nature are mechanical waves. Light is a type of wave in which no medium at all is required for propagation. Instead, electric and magnetic fields oscillate. The video presents a model of mechanical waves. It develops the concept that the transfer of energy through a medium of interconnected particles constitutes a mechanical wave.

Objective 1: Recognize that a wave passes through a medium without any net motion of the medium.

When students first consider waves, they often think that when a wave travels in a medium such as water, the entire medium moves. It is important to correct this misconception. You might first ask students to give examples of waves. Their responses may include sound waves, water waves, and radio waves. You might mention (á la *America, The Beautiful*) "amber waves of grain." Ask them if the stalks of grain move over the ground as a wave of grain travels. Point out that in wave motion there is no overall motion of the medium.

DEMONSTRATION #1 or #5 (see ADDITIONAL RESOURCES) could be used at this time to support the discussion. Again emphasize that there is no net motion of the medium, although something seems to be moving with the wave.

Objective 2: Describe the motion of particles in transverse and longitudinal waves.

Once students realize that a wave is a disturbance, the question arises as to how that disturbance is propagated through a medium. The video explicitly uses animated scenes of coupled oscillators to illustrate wave generation and motion. Stop the video when it shows longitudinal waves generated in a series of coupled oscillators. Ask students what type of motion the masses connected by springs undergo. Point out that because the masses are linked together by springs, which provide restoring forces on the masses, the motion of one influences the motion of those near it. In the mechanical model of a medium developed in the video, the springs are replaced by electric forces that link together molecules of a medium.

DEMONSTRATION #6 could be used to help clarify this point. DEMONSTRATION #4 could be used to point out the analogy between the spring coupling of the pendulum bobs and the electric force coupling of the atoms in the table top.

A discussion of water waves should explain how surface tension couples the motion of the particles on the surface of the water. A discussion of sound waves should stress that the coupling is the contact force between molecules in the air.

Objective 3: Define frequency, period, amplitude, and wavelength, and understand how they are related.

The video establishes the relationships among the wave characteristics of speed, wavelength, and frequency. It may help to stop the video at this point and write these relationships on the board. Discussion should emphasize the *relationships* among terms and not simply the definitions of the terms.

The period, T, is the amount of time required for a particle in the wave to make a complete cycle. The frequency, f, is the number of complete cycles a particle makes per unit of time. Period and frequency are reciprocals of one another, or $T = 1/f$.

The amplitude of a wave, A, is the maximum distance a particle in the wave is displaced from its rest position. The wavelength of a wave, l, is the distance in the direction of the wave's velocity between adjacent regions of identical motion, e.g., from compression to compression in a longitudinal wave or from crest to crest in a transverse wave.

Using DEMONSTRATIONS #2 and #3, again stress the definitions and relationships among wavelength, frequency, and amplitude. For example, as the frequency of sound is increased, how does this influence the wavelength and velocity of the sound? After the students understand the qualitative aspects of velocity, frequency, and wavelength, the ideas can be reinforced by putting the formula on the board and working a few numerical examples. The SUPPORTIVE BACKGROUND INFORMATION provides illustrations of basic wave properties within the context of both longitudinal and transverse waves.

Objective 4: Recognize that any medium in which displaced particles experience a restoring force is capable of supporting mechanical waves.

The video explicitly points out that all mechanical waves follow the same basic principles. When this line is narrated, stop the video and ask the students what principles are being referred to. The basic principles of wave motion are extended to water waves.

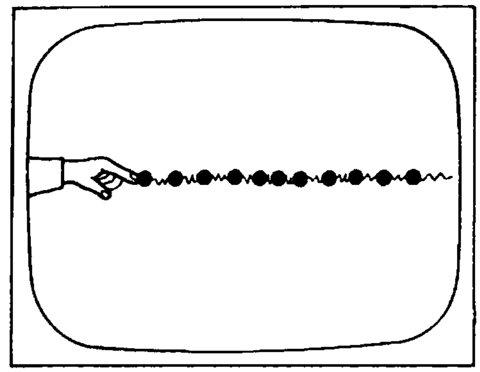
The section in which water particles are shown moving in circles is another place where stopping and replaying the video will enhance student understanding. Point out that the circular motion of the water particles leads to the wave phenomenon. You might ask students whether waves are just an illusion of some apparent motion or whether they have reality. The *actual* and the *apparent* movements need to be separated clearly in students' minds. To foster responses, ask if water waves can cause damage. Discuss the fact that water waves can produce physical effects and, therefore, are not mere illusions. Be sure the students understand that a wave transmits energy and momentum.

After viewing the video, it would be well to re-emphasize that a wave passes through a substance without any net motion of the substance. DEMONSTRATIONS #4 and #5 are excellent for this purpose and also illustrate the difference between longitudinal and transverse waves. If DEMONSTRATION #3 is used, it would be well to stress that sound is a longitudinal wave even though it is often represented as a transverse wave. This transverse representation is probably used because it is so much easier to draw and because the oscilloscope trace is transverse.

POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and related frames from the video.

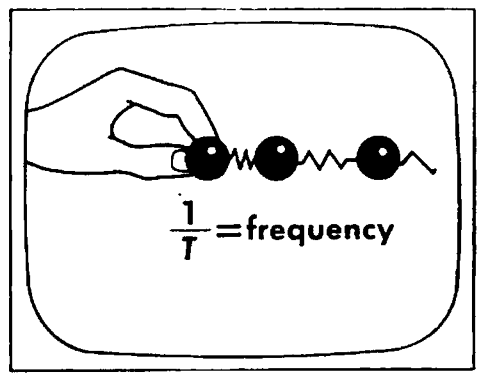
Why don't incoming ocean waves bring more and more water on to the shore until the beach is submerged?

This is a difficult question. It involves a common misconception about waves. Students cannot be expected to answer it fully prior to viewing the video. However, it is precisely this kind of question that probably prompted early scientists to investigate wave motion. When an impulse moves through a system, no single oscillator goes very far, but the disturbance moves right along.



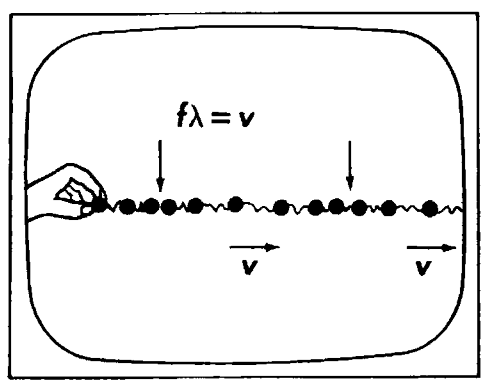
What is the relationship between the frequency of a wave and its period?

The inverse of the period is the frequency.



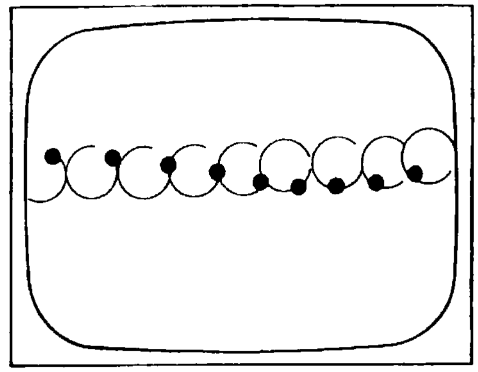
Describe the relationship between frequency, wavelength, and velocity.

The wavelength equals the period times the speed of the wave, or, in other words, frequency times wavelength equals velocity. (The wavelength is the distance between the vertical arrows.)



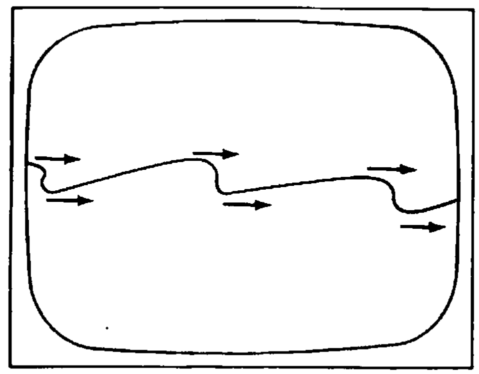
What are the oscillators in a water wave? How do they differ from the oscillating masses connected by springs?

The oscillators in a water wave are the tiny particles of water. The oscillating motion of these particles is a simple circular path. The oscillating masses connected by springs move back and forth.



What makes an ocean wave "break"? How is the motion of each bit of water in a breaking water wave different from the motion of each bit of water in a non-breaking wave?

The wave "breaks" when the top of the wave moves faster than the bottom because the bottom water is slowed by contact with the seabed. In the non-breaking wave, the motion of the particles is simply circular. In the breaking wave, the circular motion of the particles is distorted.



EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the ideas contained in the video, you might pose the following questions to your students.

1. **When lightning strikes some distance away, the thunder may not be heard until several seconds after the flash is seen. Why?**

   Light from the lightning travels much faster than the sound wave.

2. **You are some distance from a band and observe that all the instruments from tuba to flute are playing in time. What does this tell you about the speed of sound?**

   Since you observe that all of the instruments are playing in time and that their sounds reach you simultaneously, you can assume that all frequencies in a given medium travel at the same speed.

3. **If one coil of a slinky were to be painted a bright color and a pulse perpendicular to the stretched slinky were initiated, what would be the motion of the painted coil? What would happen if a longitudinal pulse were generated?**

   In the first case the coil would move only in a transverse direction and not along the slinky. In the second case the coil would be seen to move and return in the direction of wave motion.

4. **An astronaut using an air hammer to break up a boulder on the moon need not wear earplugs. Why?**

   Sound waves need a medium such as air or water through which to travel. On the airless surface of the moon, no sounds can be heard.

SUMMARY - Bits of matter vibrating collectively, not independently, give rise to waves. A wave is a disturbance that travels through a medium. A good model for understanding wave phenomena is that of coupled oscillators. According to this model, when particles of the medium undergo oscillations, the net effect of their coupled motion is the wave. There is no net displacement of the medium when a wave travels.

Waves are classified according to the direction of the motion of the particles in the medium. In longitudinal waves the particles oscillate along the direction of propagation of the wave. Transverse waves are disturbances in which the particles oscillate perpendicularly to the direction of propagation.

All waves have common characteristics: amplitude, frequency, wavelength, and wave speed. The wave speed v, frequency f, and wavelength λ, for any wave are related by $v = \lambda f$. The speed of the wave through a medium depends on the nature of the medium and the way in which the bits of matter in it are coupled.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO INTRODUCTION TO WAVES

INTRODUCTION - This video develops the concept of mechanical waves, describes the characteristics of longitudinal and transverse waves, and develops the relationships among such characteristics of waves as velocity, frequency, amplitude, and wavelength.

Terms Essential for Understanding the Video

medium
mechanical wave
crest
trough
amplitude
period
wavelength
frequency
compression
longitudinal wave
transverse wave

**** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions you should ask your teacher to replay these sections. ****

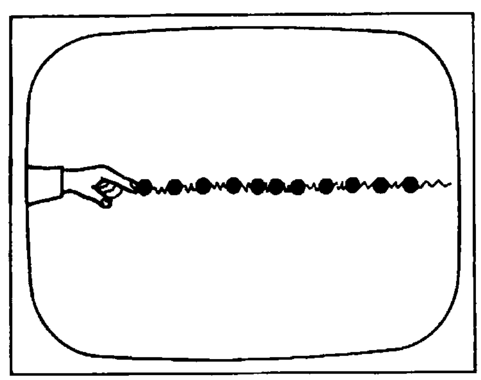
Points to Look for in the Video

Why don't incoming ocean waves bring more and more water on to the shore until the beach is submerged?

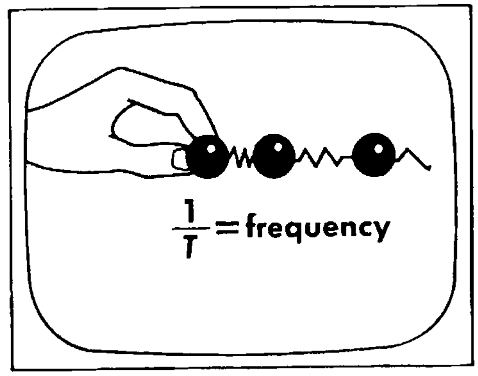
This is a difficult question because it involves a common misconception about waves. You cannot be expected to answer it fully prior to viewing the video. However, it is precisely this kind of question that probably prompted early scientists to investigate wave motion. Perhaps the following excerpt from the video will help you to answer the question:

When an impulse moves through a system, no single oscillator goes very far, but the disturbance moves right along.

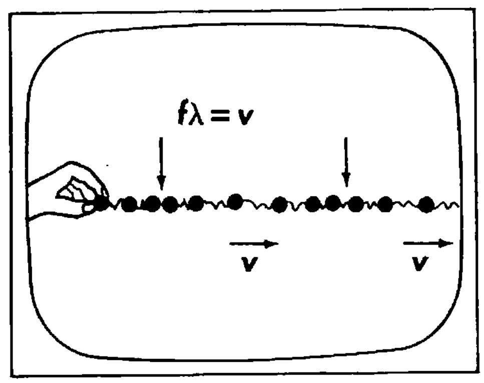
Be sure that you understand this point when it comes up in the video. If you don't, ask your teacher to stop the tape and replay that part.



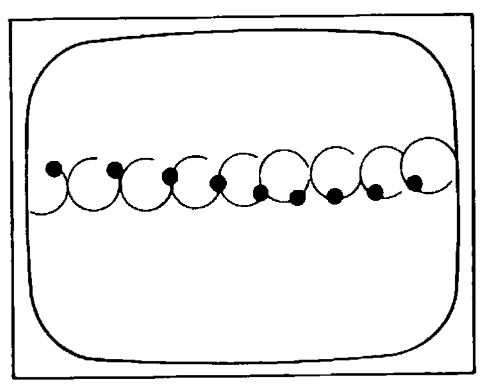
The relationship among wave characteristics is important for understanding the behavior of waves. Note in the video how one relationship is substituted for another.



What is the relationship between the frequency of a wave and its period?



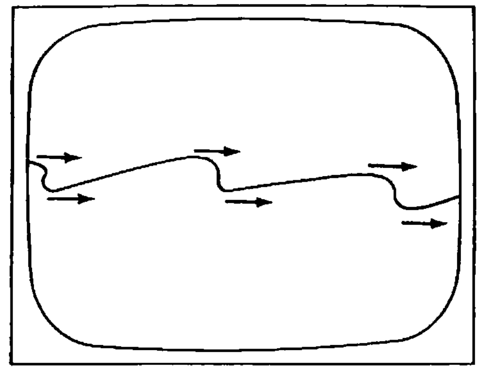
Describe the relationships among frequency, wavelength, and velocity.



What are the oscillators in a water wave? How do they differ from the oscillating masses connected by springs?

In watching the video, look for the computer animation of a water wave. Note that in a water wave each bit of matter on the surface moves around in a little circle, each circle is slightly offset from the next, all together giving the familiar undulation of the watery surface.

What makes an ocean wave "break"? How is the motion of each bit of water in a breaking water wave different from the motion of each bit of water in a non-breaking wave?



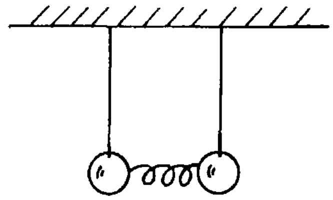
Perhaps this section of the video will help answer these questions. Your teacher may need to replay it several times.

In viewing the video, note what happens as the deep part of the wave moves faster than the shallow part.

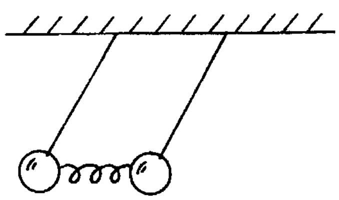
TEACHER RESOURCES

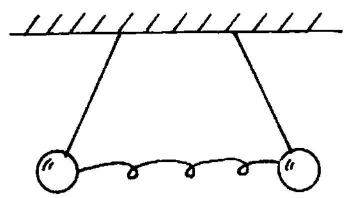
SUPPORTIVE BACKGROUND INFORMATION - Sound waves are just one example of wave phenomena. They are created by something vibrating in the air or other liquid or solid medium. The vibrating object sets the medium into motion, compressing and expanding the medium's density with each vibration.

Coupled oscillators provide another model of mechanical waves. Bits of matter vibrating in unison create mechanical waves. Two pendulums connected by a weak spring are said to be coupled oscillators.



Depending on how the system of pendulums is disturbed initially, different types of oscillation may occur.





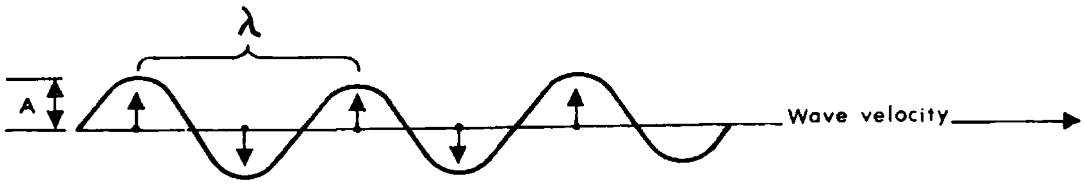
In the first case the system behaves as a single pendulum. In the second case the energy is exchanged back and forth between the pendulums.

This description serves to model the way waves travel through air, water, and solids. In the latter case, the "pendulums" are the atoms in the crystal and the "springs" are the coulomb electric forces between atoms in the crystal lattice.

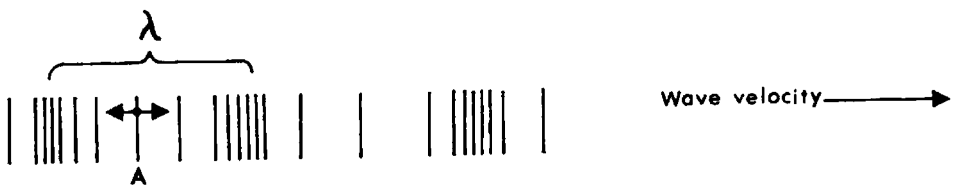
If there are many coupled oscillators and one is put into motion, the disturbance that travels transfers energy through the interactions of the springs and masses.

Waves are classified according to the direction of the motion of the particles in the medium. In **longitudinal waves** the particles oscillate along the direction of propagation of the wave. **Transverse waves** are disturbances in which the particles oscillate perpendicularly to the direction of propagation.

All periodic wave phenomena can be characterized by various properties. The terms crest and trough refer to the highest and lowest parts of the wave. **Amplitude** A is the maximum displacement of the particle (oscillator) in the medium, as shown below for a transverse wave.



Similarly, in a longitudinal wave, **Amplitude** A represents the maximum displacement of an individual particle's motion, as shown below. (This is particularly well illustrated in the video.)



Frequency f is the measure of the number of wave crests which pass by an observer each second. It is the reciprocal of the **period** T of the wave which is the time for one complete wave cycle to occur.

$$T = 1/f.$$

Wavelength λ, is the distance from one crest to another, from one compression to the next, or the distance between any two adjacent points on the wave. **Wavespeed** v is the product of the wavelength and the frequency.

$$v = f\lambda$$

The speed of the disturbance, or wave, through a medium depends on the nature of the medium and the connection between one bit of matter and the next (e.g., the stiffness of the springs in the coupled pendulums). For a given mass, a weak linkage results in a slow wave, a strong linkage produces a faster wave.

ADDITIONAL RESOURCES

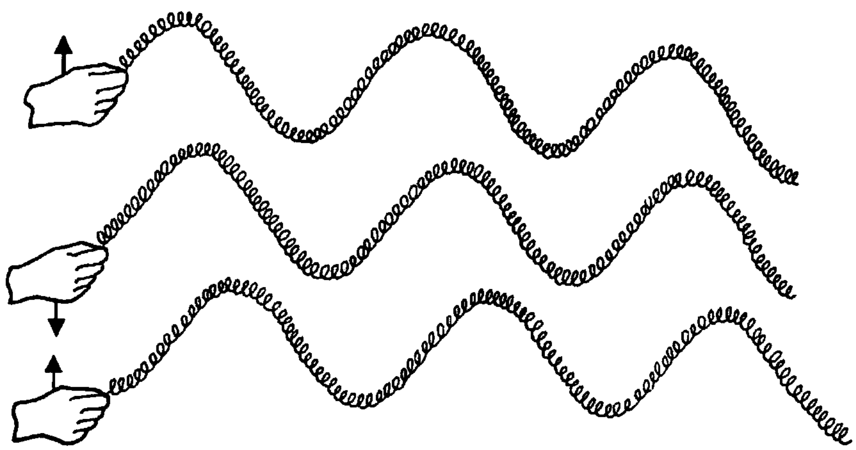
Demonstration #1: The Slinky or "Snake" Spring

Purpose: To show the motion of waves and differentiate between transverse and longitudinal waves.

Materials: Slinky or a long metal coiled spring.

Procedure and Notes: Attach one end of the device to a wall and stretch the device out along the classroom floor.

1. Give the device a sharp snap at one end perpendicular to the medium and parallel to the floor to create a transverse wave.
2. Give a series of pulses in succession to create a train of waves.



3. Vary the frequency of the pulses and note the wavelength changes.
4. Give a sharp push in the direction of the medium to create a longitudinal wave.

Explanation:

1. Note that the individual coils move perpendicular to the device while the pulse, and hence the energy, moves in the direction of the device.
2. Look for repetition in the pulses to observe one wavelength.
3. As the frequency is greater, the wavelength is shorter.
4. In the longitudinal pulse the particles move back and forth in the same direction as the motion of the wave.

Demonstration #2: Tuning Forks

Purpose: To show that a disturbance produces waves, an increase in frequency produces higher pitch, and a greater amplitude produces a louder sound.

Materials: Various tuning forks

Procedure and Notes:

1. Strike a tuning fork and note the sound. Restrike the fork and place the bottom end on a rigid surface such as a chalk board and note the increase in loudness.
2. Strike various forks, one at a time, and note the changes in pitch. Point out the relationship between the physical size of the fork and frequency and pitch produced.
3. Strike a tuning fork and dip the vibrating tines in a beaker of water.

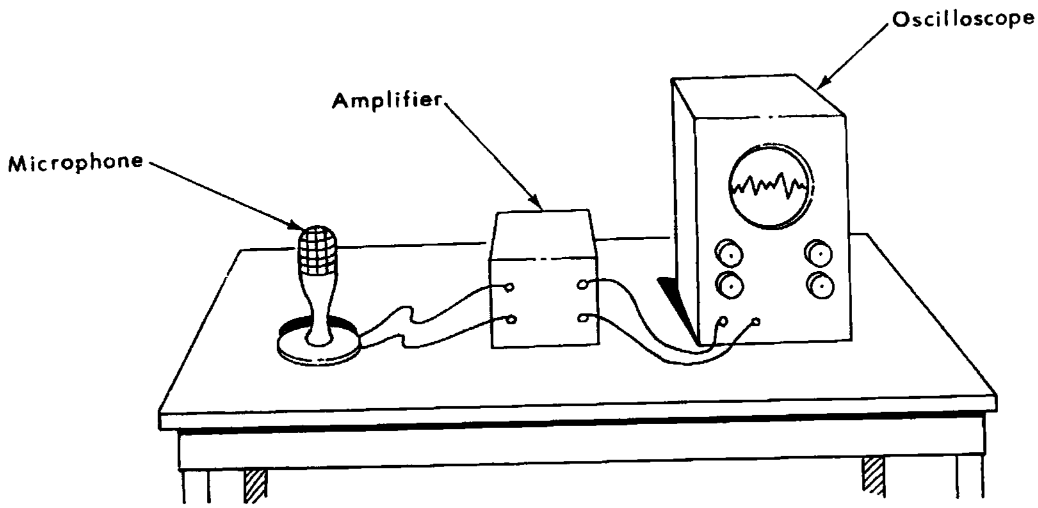
Explanation:

1. The vibration of the tines produces a series of compressions and rarefactions which result in a sound wave. When the bottom end of the fork is placed on the chalkboard, the entire area of the board is set into vibration. Since this increased area causes more air to be set into vibration, a louder sound is produced.
2. The fork will vibrate with a frequency determined by the mass and stiffness of the tines. When the frequency of the vibrating air is perceived, it is called pitch.
3. The motion of the vibrating tines is made more visible by the splashing water.

Demonstration #3: Oscilloscope Waveform Display

Purpose: To show the relationship between frequency and pitch, amplitude and loudness, and electrical signal and sound produced.

Materials: Oscillator or signal generator, amplifier, speaker, oscilloscope, microphone and tuning fork.



Procedure and Notes:

1. First wire the oscillator (not shown) and amplifier directly to the speaker (not shown) and place the oscilloscope across the speaker terminals. Observe the scope and listen as the gain is increased.
2. With an appropriate gain setting, alter the frequency and note the changes in pitch as well as the changes in the oscilloscope trace.
3. Now use a microphone (and amplifier if necessary) connected to the oscilloscope and show the trace produced by a tuning fork, your voice, etc. Notice how the amplitude of the trace decreases as the sound of the tuning fork dies.

Explanation:

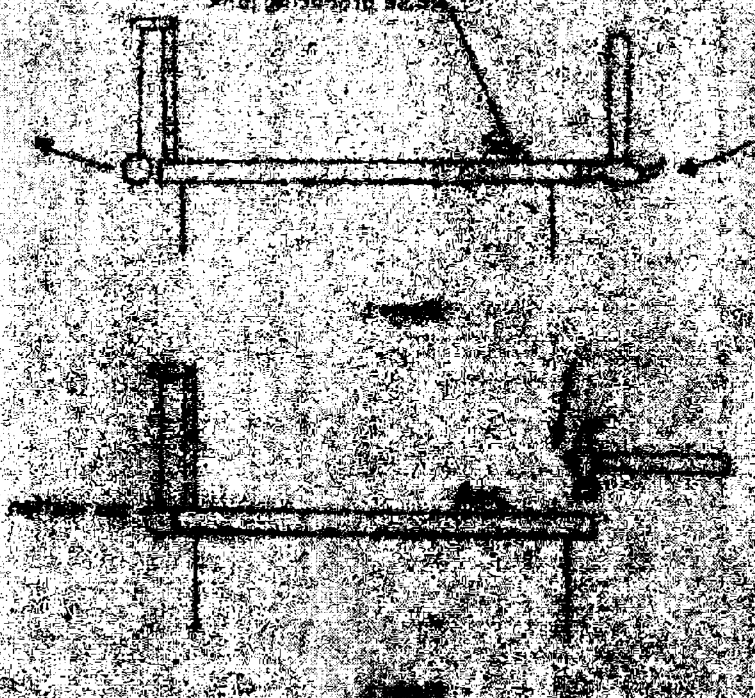
1. When the amplitude is increased in the amplifier, more electrical energy is transferred to the speaker enabling it to move more air. Thus, the sound wave is more intense.
2. As the frequency is increased, the pitch is higher. Also, as the frequency is increased, the wavelength, as illustrated by the oscilloscope, decreases.
3. The sound of the tuning fork is converted to electrical vibrations by the microphone and is displayed on the oscilloscope. Pitch and amplitude relationships are depicted by the oscilloscope trace.

Demonstration #4: Shock Waves in a Table Top

Purpose To show that wave motion can occur without any net motion of the medium. Also to illustrate transverse and longitudinal shock waves.

Materials Pendulum bob, demonstration table which is attached to the floor, hammer, tuning forks or other materials which will vibrate and make noise.

Procedure and Notes 1. Suspend a pendulum of about 200 grams mass so that it is in contact with the top edge of the demonstration table. Place tuning forks or jangly material on the table near where the hammer blow will be struck.



[illegible]

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Demonstration #4: Shock Waves in a Table Top

Purpose: To show that wave motion can result without any net motion of the medium. Also to illustrate transverse and longitudinal shock waves.

Materials: Pendulum bob, demonstration table which is attached to the floor, hammer, tuning forks or other materials which will vibrate and make noise.

Procedure and Notes:

1. Suspend a pendulum of about 200 grams mass so that it is in contact with the top edge of the demonstration table. Place tuning forks or jangly material on the table near where the hammer blow will be struck.

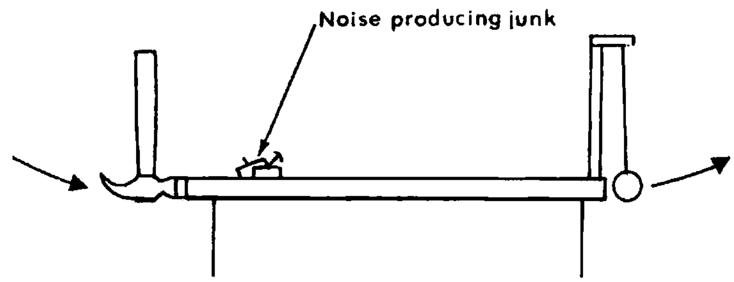


Figure 1.

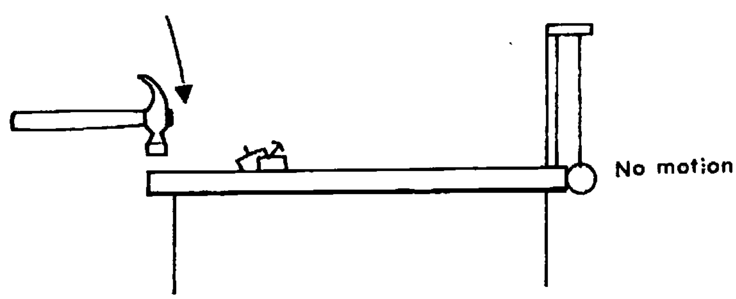


Figure 2.

2. Give the other end of the table a sharp blow with the hammer as illustrated in Figure 1 above. The bob will fly off the other end of the table. You will probably note only a small amount of noise from the material you placed on the table top.
3. Repeat the experiment giving the table top a "transverse" blow as illustrated in Figure 2 above. In this case the bob's motion will be considerably less and the material on the table will make a greater noise.

Explanation: The longitudinal blow produces a longitudinal pulse which displaces the pendulum bob but not the material on the table top. The transverse blow has the opposite effect. In both cases the energy moves through the table top without any net motion of the table itself.

Demonstration #5: Motion and Waves

Purpose: To give several examples of how a wave is propagated without any net motion of the medium.

Materials: Wave machine, momentum collision ball apparatus, a large drill bit, a row of students.

Procedure and Notes:

1. Use the wave machine and concentrate on the motion of a single particle.
2. Discuss the net motion of the balls in the momentum apparatus. Point out that something moves through the apparatus, yet the apparatus stays in the same place.
3. Slowly rotate a large drill bit as the students view it from the side. Note the apparent motion in one direction, yet there is no net motion. (The old-fashioned barber's pole is a similar phenomenon.)
4. Line up a row of students side by side and push the first one sideways for a more animated demonstration. Prepare for the consequences!

Explanation: In each case the transfer of energy by the wave is accomplished without any net motion of the substance through which the wave moves.

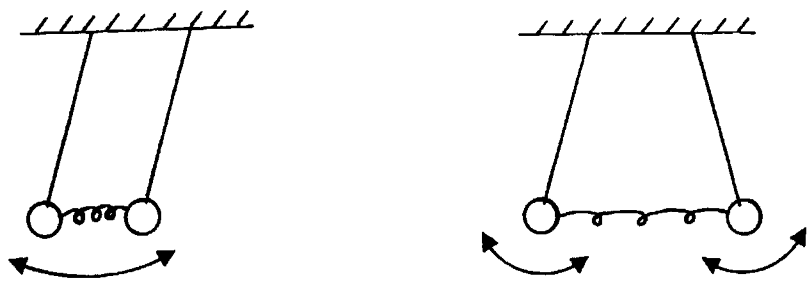
Demonstration #6: Coupled Oscillators

Purpose: To show various modes of oscillation for two or more coupled pendulums.

Materials: Coupled pendulum demonstration. (This can be commercially obtained or constructed.)

Procedure and Notes:

1. Disturb two coupled oscillators by vibrating both in unison. Observe the mode of oscillation.



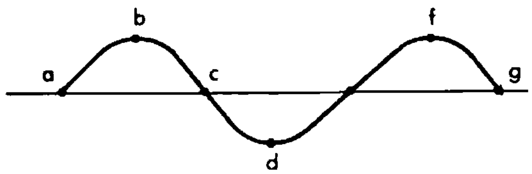
2. Disturb two coupled oscillators by vibrating one only and observe the mode of oscillation. Note the complete transfer of energy from one to the other.
3. Repeat with three or more coupled oscillators and observe the motion.

Explanation: In the first case the system behaves as a single pendulum or two opposite ones. In the second and third cases the energy is exchanged back and forth between the balls as with the particles in a wave.

EVALUATION QUESTIONS

1. Indicate the interval that represents one full wavelength.

   A. ac
   B. bd
   C. ag
   D. cg



2. As the pulse moves past Point A, which one of the following best illustrates the motion of Point A?

   A. 

   C. 

   B. 

   D. 

3. When a sound is made louder, which of the following features is increased?

   A. Wavelength
   B. Frequency
   C. Wave velocity
   D. Amplitude

4. Two waves on identical strings have frequencies in a ratio of 2 to 1. If their wave speeds are the same, how do their wavelengths compare?

   A. 2:1
   B. 1:2
   C. 4:1
   D. 1:4

5. In order for a medium to be able to support a mechanical wave, the particles in the wave must be

   A. frictionless.
   B. isolated from one another.
   C. able to interact.
   D. very light.

6. A wave has an amplitude of 2 cm and a frequency of 12 Hz. What is its period?

   A. 1/12 s
   B. 1/24 s
   C. 12 s
   D. 24 s

7. **Which of the following is *not* a characteristic of all mechanical waves?**

   A. They consist of disturbances in or oscillations of a medium.
   B. They transmit energy.
   C. They travel in a direction that is at right angles to the direction of the vibrating particles of the medium.
   D. They are created by a vibrating source.

8. **A musical note which is higher than another has which wave property?**

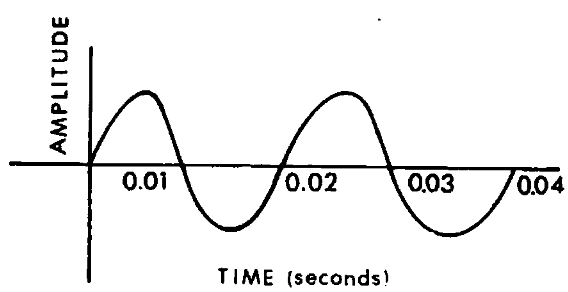
   A. Lower frequency
   B. Shorter wavelength
   C. Greater amplitude
   D. Longer period

9. **If you strike a horizontal metal rod vertically from above, what can be said about the waves created in the rod?**

   A. The particles oscillate horizontally along the direction of the rod.
   B. The particles oscillate vertically, perpendicular to the direction of the rod.
   C. The particles oscillate in circles, perpendicular to the rod.
   D. The particles travel along the rod.

10. **The following graph describes a wave. What is the frequency of this wave?**

   A. 100 Hz
   B. 50 Hz
   C. 0.02 Hz
   D. 0.01 Hz



ESSAY QUESTIONS

11. **In a short essay, discuss the similarities and differences between longitudinal and transverse waves.**

12. **Write a short essay describing in your own words the mechanical model of how waves travel in a medium.**

KEY

1. D
2. B
3. D
4. B
5. C
6. A
7. C
8. B
9. B
10. B

SUGGESTED ESSAY RESPONSES

11. Both longitudinal and transverse waves represent disturbances in a medium and can be described by wavelength, frequency, and amplitude. The speed, wavelength, and frequency are related by $\nu = v/\lambda$. For longitudinal waves, the particles of the medium move parallel to the direction in which the wave travels. For transverse waves, the particles of the medium move perpendicularly to the direction in which the wave travels.

12. In a mechanical model of wave propagation, the atoms of the medium are considered to form a collection of harmonic oscillators that are connected to one another. Masses connected to each other by springs is an analogous situation. When one oscillator is displaced from its equilibrium position and released, it begins to oscillate. Because it is connected to adjacent oscillators by springs, the adjacent oscillators also begin to oscillate. The collective oscillations form the disturbance that comprises the wave.

TEACHER'S GUIDE TO TEMPERATURE AND THE GAS LAWS

CONTENT AND USE OF THE VIDEO - Although the video is intended as an introduction to a unit on heat and the behavior of gases, it can also serve as an excellent review of these concepts. If used as a starting point in the study of thermodynamics, the video can be more effective after students have examined Newton's laws, energy, elastic collisions, and momentum. One of the strengths of the video is the presentation of a number of computer simulations which visually tie observable properties, such as pressure and temperature, to microscopic events, such as gas molecules striking container walls. Through these animated sequences the video explores the difference between heat and temperature. In addition it describes the commonly used temperature scales and develops three important gas laws, namely Boyle's law, Charles' law, and the ideal gas law. Because of its emphasis on the gas laws, the video relates to chemistry as well as to physics.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - Since the following terms are introduced in the video, it might be helpful to discuss these briefly with students prior to viewing.

temperature--a number assigned on a definite scale to compare degrees of "hotness" and "coldness"; technically it is a measure of the average kinetic energy, $\frac{1}{2}mv^2$, of atoms and molecules.

heat--that form of energy which is transferred from one body to another solely by virtue of the difference in their temperatures; the term heat is often used generically to encompass the energy of random motions of atoms and molecules.

thermal energy--energy in the form of random motion of atoms and molecules.

pressure--force divided by area (a scalar quantity).

Boyle's law--the product of the pressure and the volume of a gas equals a constant (PV = constant when temperature doesn't change).

Charles' law--all gases expand or contract by the same fractional amount with a given change in temperature, i.e., 1/273 of its volume at 0°C per celsius degree rise when pressure is held constant.

ideal gas law--the temperature is related to the product of pressure and volume by the expression $PV = NkT$.

Temperature Scales

	Boiling Water	Freezing Water	Absolute Zero
Fahrenheit	212°F	32°F	-459°F
Celsius	100°C	0°C	-273°C
Kelvin	373K	273K	0K

absolute zero--the temperature at which a gas has a minimum kinetic energy and volume; the lowest temperature definable.

WHAT TO EMPHASIZE AND HOW TO DO IT - This video shows how the heat in a gas is related to the pressure, temperature, and volume of the gas. The gas laws are formidable to students because they involve quantities that depend on the amount of material and quantities that do not. In addition, a common misconception is to confuse temperature with heat and to think of the two as one and interchangeable.

The video lesson attempts to distinguish between the two measures of heat and temperature. Heat is presented as a form of energy that is exhibited as the random motions of atoms and molecules. Temperature is depicted as the measure of the kinetic energy of atoms and molecules. Notice that the amount of heat in a body depends on its size, but the temperature does not. Newton's laws of motion can also be applied to the quantities of material and the measures of heat and temperature.

Objective 1: Compare the three temperature scales.

Prior to viewing the video, it could be useful to explain to your students the three major scales--Celsius, Fahrenheit, and Kelvin--that have been developed to measure temperature. Working a few problems may reinforce the ideas that demonstrate the relationship between these scales. For example, if there is a decrease of 10 degrees on the Fahrenheit scale, what are comparable decreases in the Celsius and Kelvin scales? Familiarity with the temperature scales and their relationships will facilitate viewing of the video.

Objective 2: Recognize the difference between heat and temperature.

Students should be aware of the distinction between heat and temperature before seeing the video. Be sure they understand that heat is the random motion of atoms and molecules and thus depends on the number of molecules, i.e., the size of the body. The average kinetic energy of moving atoms and molecules is proportional to the temperature. Temperature is independent of the size of an object.

A typical misconception in this area is that the temperatures of two bodies combine to give a cumulative total. You might ask students what happens when a cup of water at 70°F is mixed with a cup of water at 100°F. A typical response is that the resultant mixture has a temperature of 170°F. You might respond with something like, "Do you mean that mixing two cups of warm coffee will give you hot coffee?" The students will know from experience that mixing hot and cold water results in warm water.

To emphasize further the distinction between temperature and heat, you might next ask, "Will two hot water bottles give you more warmth than one?" Point out that the amount of heat contained in the bottles is additive, as their common experience will attest.

Objective 3: Describe the relationship of pressure to volume of a gas.

You might ask your students the same question as is posed in the video: How does gas blow up a balloon? The ensuing discussion will provide an opportunity to establish firmly the relationship between pressure and volume as stated in Boyle's law, DEMONSTRATIONS #2 and #3 on the cartesian diver and growing marshmallow, respectively, reinforce the concept that pressure is inversely proportional to volume.

In the derivation of pressure, the videotape shows a single molecule bouncing off of a wall. You should stop the tape and point out that these are *microscopic* collisions (transfer of momentum by individual molecules) that lead to a *macroscopic* effect (pressure). Since these scenes go by quickly you might replay them.

Objective 4: Explain the relationship of volume and pressure to temperature of gas.

Students particularly enjoy the section of the video focusing on hot air balloons. Consequently, you might expand a discussion about balloons by asking what keeps a hot air balloon flying? The relationship of volume and pressure to temperature involves some mathematical derivations that are sometimes confusing to students. The SUPPORTIVE BACKGROUND INFORMATION explains these relationships in detail as well as deriving the proportionality constant.

The video contains a computer animation sequence that visually demonstrates the resulting higher temperature in a container at higher pressure. Since the video moves too quickly at this point for the viewer to understand fully the connection, it is important to pause and/or replay these video segments and to discuss the relationship between pressure and temperature.

DEMONSTRATION #1 on the expanding balloon and #4 on the heated balloon help in developing a conceptual understanding of the relationship between volume and temperature. Encouraging your students to relate what happens to the balloons in the demonstrations to what happens to those in the video can reinforce the concepts.

Objective 5: Apply the ideal gas law to the explanation of gas phenomena.

You might explore the relationship between quantities in the ideal gas law by asking what happens if the temperature of a gas is increased and the volume is held constant, and so on. To connect the discussion with the video, you might remind the students how the size of letters is varied in the equations to symbolize the relationships.

POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and related frames from the video.

What is the difference between heat and temperature?

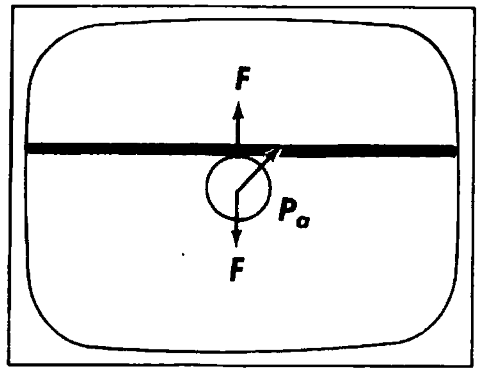
*The **absolute temperature** (Kelvin scale) is a measure of the energy of random motion of molecules and is equal to two-thirds the average kinetic energy of a **single** molecule of gas. **Heat** is the energy transferred from one body to another through a difference in temperature and is also generically referred to as the thermal energy of a body--the total kinetic energy of **all** the molecules of the gas.*

Which quantity, heat or temperature, depends on the amount of material present? Which quantity does not depend on the amount of material present?

Heat depends on the amount of material present. Temperature does not depend on the amount of material present.

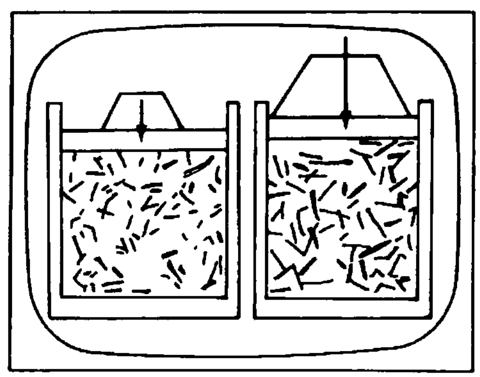
What is happening in this frame of the video?

Each time a gas molecule hits a wall, it gives the surface a tiny push. By bouncing off the wall, the molecule changes its momentum. The change in momentum means a force has acted on the molecule. Therefore, a reverse force has acted on the wall. As more and more molecules strike the wall, the individual pushes begin to merge. The force per unit area exerted by the molecules is called the pressure of the gas.



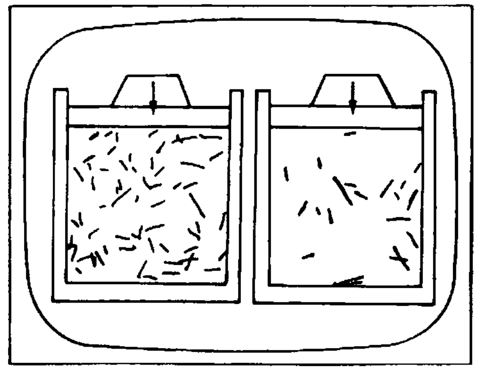
What relationship between pressure and temperature is depicted in this frame of the video?

When a gas is hotter, each collision with the wall exerts greater pressure. Heating a gas increases its pressure. Point out to the students that a greater weight (exerting more pressure) is needed to contain the molecules when heated. (You will need to observe the motion of molecules in the video. The increased speeds of the molecules cannot easily be shown in the drawings; longer lines represent more rapid movement of the molecules.)



What relationship between temperature and the number of molecules is depicted in this frame of the video?

The hotter a gas, the fewer molecules it takes to maintain the same pressure. Point out to the students that the container with fewer but more rapidly moving molecules exerts the same pressure as the container with more but slower moving molecules. The longer lines in the drawings represent more rapid motion.



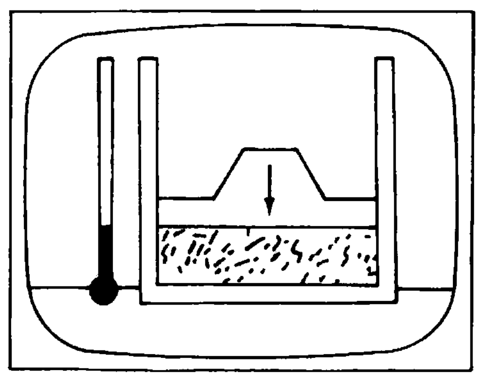
What relationships are conveyed through this equation?

The average kinetic energy of a molecule, whether it increases or decreases, is directly proportional to both the volume and the pressure of a gas. That is, if P is reduced, V must increase so that their product PV remains constant.

$$PV = \frac{2}{3} N\bar{K}$$

What does this frame tell you about the relationship of temperature to pressure and volume?

As the temperature rises, both pressure and volume increase. The video shows that a container with less volume and pressure has a lower temperature.



What is the meaning of the ideal gas law?

The ideal gas law refers to a gas whose behavior is described exactly by Boyle's law and the law of Charles and Gay-Lussac. Boyle's law states the product of the pressure times the volume of a gas is a constant at a given temperature. The quantity PV is also proportional to the number of gas molecules N and otherwise depends on the average kinetic energy K of a molecule, which presumably depends only on temperature. This suggests that PV/N is a function of temperature only. Gases expand when heated, so as temperature increases so does the quantity PV. Thus we anticipate that PV should increase with temperature. The exact dependence on temperature was established by Charles--for each Celsius degree rise in temperature, the volume of a gas expands by 1/273 of its volume at 0°C. The ideal gas law, PV = NkT, summarizes the interrelationship among the quantities of pressure, volume, and temperature of a gas.

$$PV = \frac{2}{3} N\bar{K}$$

$$k = 1.38 \times 10^{-23} \frac{J}{K}$$

EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in the video, you might pose the following questions to your students.

1. **Weather balloons are partially filled when released from earth. Why is this so?**

   As the balloon rises in the atmosphere, the atmospheric pressure on the outside decreases and so the balloon expands. Filling them only partially allows for this expansion.

2. **How can an inflated balloon maintain its shape?**

   Molecules of the gas hitting the inside surface of the balloon exert a pressure that is equal to the pressure caused by the elasticity of the balloon and the air molecules outside.

3. **Why does a bicycle tire pump warm up as you use it?**

   As the piston of the pump is pushed down the air is compressed. Work is being done on the gas. Work, a form of energy, turns into increased kinetic energy of air molecules, i.e., higher temperature. Heat flows from the gas to warm the pump.

4. **Why does a diesel engine need no spark plugs?**

   The piston in the diesel engine compresses the air in the cylinder to twenty or more times atmospheric pressure. Very high temperatures result. When the fuel is injected into the cylinder, it explodes without the need for a spark plug.

5. **Why should tire pressure be measured before a trip and not during it?**

   Manufacturers assume that the readings will be taken before the trip while the tires are cool. The friction of the tires against the road during a trip causes the temperature of the tires to rise. As the temperature rises the pressure also rises. The pressure reading would therefore not correctly indicate whether the tire is properly inflated.

6. **Why should empty aerosol cans not be placed in a fire?**

   The volume of gas in an aerosol can is physically limited and an "empty" can still has some gas in it. If the temperature were to be substantially increased, the pressure inside would increase until the can could explode, causing harm to anyone nearby.

SUMMARY - This module focuses on the ideas of temperature and pressure by examining the microscopic origins of each quantity. Temperature, which is the measure of the hotness or coldness of something, refers to a number on an assigned scale. On the absolute scale, temperature is directly proportional to the average kinetic energy of atoms and molecules and is independent of the amount of material. Through the idea of temperature the microscopic motions of gas atoms are related to the macroscopic quantities of pressure and volume as summarized by the ideal gas law.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO TEMPERATURE AND THE GAS LAWS

INTRODUCTION - This video draws a distinction between temperature and heat by focusing on the microscopic origins of each quantity. It also presents the ideal gas law which summarizes the interrelations among pressure, temperature, and volume of a gas.

Terms Essential for Understanding the Video

temperature
heat
thermal energy
pressure
Boyle's Law
Charles' law
ideal gas law
temperature scales
absolute zero

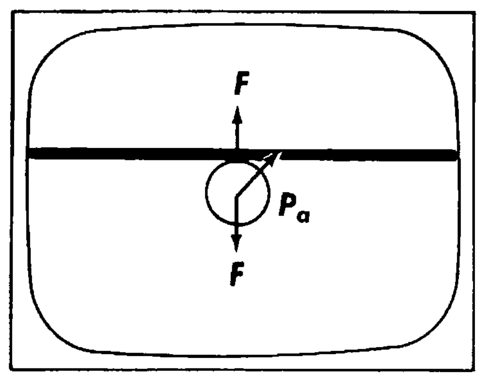
**** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ****

Points to Look for in the Video

What is the difference between heat and temperature? Which quantity, heat or temperature, depends on the amount of material present? Which quantity does not depend on the amount of material present?

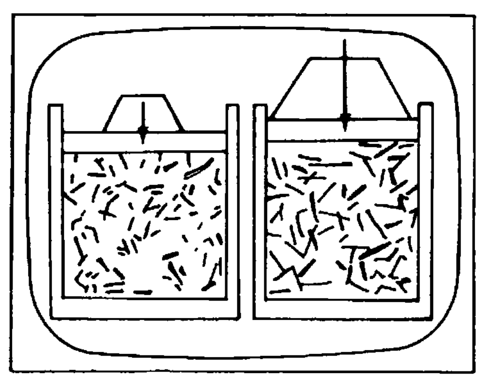
What is happening in this frame of the video?

Your teacher may stop the video at this point to discuss the interaction of the gas molecule with the container wall.

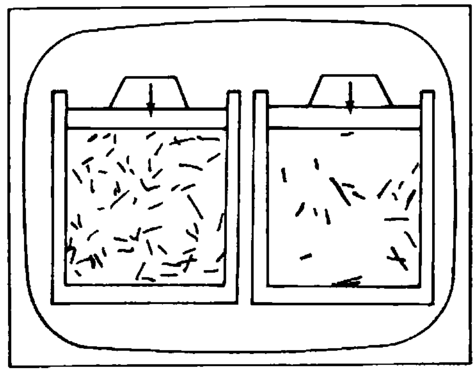


What relationship between pressure and temperature is depicted in this frame of the video?

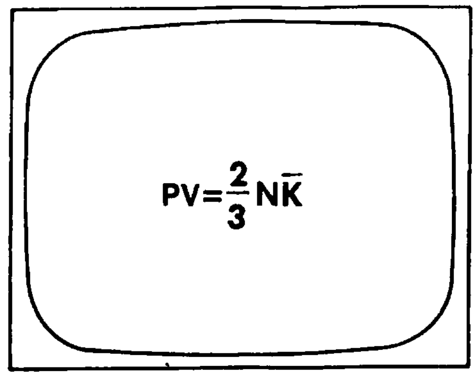
While watching the video, note that as the motion of molecules in the container becomes more agitated, a greater weight (exerting more pressure) is needed to contain the gas.



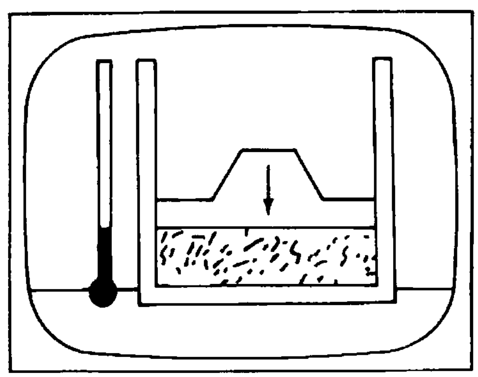
What relationship between temperature and number of molecules is depicted in this frame of the video?



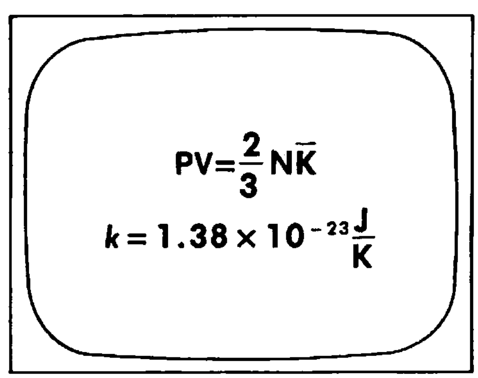
What relationships are conveyed through this equation?



What does this frame tell you about the relationship of temperature to pressure and volume?



What is the meaning of the ideal gas law?



TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - A blend of scientific theories is particularly apparent when pursuing the topic of *Temperature and the Gas Laws*. Galileo developed an air thermometer. Newton had a theory, although inaccurate, about the pressure and volume of a gas. In the eighteenth century, the work of Joule, Maxwell, and Boltzmann led to the kinetic theory of matter. They found that the pressure in a gas is proportional to the density (i.e., proportional to the number of molecules divided by the volume) and also proportional to the average kinetic energy of the molecule. Although Newton's theory of gases was erroneous, his laws of motion contributed to the discovery of the constant of proportionality. This relationship is developed in the section entitled MATHEMATICAL BACKGROUND INFORMATION:

$$P = \frac{2}{3}\left[\frac{NK}{V}\right], \tag{1}$$

where

P = pressure in the gas molecules,
N = number of gas molecules,
V = volume of gas,
K = average kinetic energy of a molecule.

The kinetic theory of matter states that all matter is made of particles called molecules. In gases, the molecules are moving rapidly and independently of each other. If the gas is confined in a vessel, the molecules must collide with the walls, exerting a pressure at the moment of collision. The total pressure of the gas will be the sum of the pressures of the individual molecules. The above relationship also makes use of Avogadro's theory that equal volumes of gases at the same pressure and temperature will have equal numbers of molecules.

In the late 1600's, Robert Boyle, a chemist and a physicist, found that, as a gas is compressed without changing its temperature, its pressure increases directly with its density. This has become known as Boyle's law; at a fixed temperature, the product of the pressure and the volume is constant, or

$$PV = \text{constant}. \tag{2}$$

If its temperature remains unchanged, the pressure is inversely proportional to the volume. Rewriting Eq.(1) gives

$$PV = \frac{2}{3}NK. \tag{3}$$

The constant in Boyle's law is equal to 2/3 of the number of molecules times the average kinetic energy of all of the molecules. Kinetic energy involves motions of molecules; heat is a manifestation of that motion. Heating a gas will increase the kinetic energy, thereby causing the pressure to rise, the volume to expand, the density to decrease, or any combination of these effects.

The relationship between the temperature and the volume of a gas was stated first by Jacques Alexandre Ceasare Charles, a French scientist and a hot-air balloon enthusiast. For each Celsius degree rise in temperature, the volume of any gas expands by 1/273 of its volume at 0°C, i.e., all gases have the same coefficient of expansion. Similarly, if a gas cools, its volume contracts by 1/273 for every decrease in Celsius degree. Providing the pressure remains constant, this relationship may be stated:

$$V/T = \text{constant}, \tag{4}$$

where V = gas volume,

and T = temperature of a gas in kelvins.

Thus the change in volume of a gas is directly proportional to the change in temperature, provided the pressure of the gas remains unchanged.

The ideal gas law combines the above equations into a more usable general form.

$$(PV)/T = Nk, \tag{5}$$

or

$$PV = NkT. \tag{6}$$

The constant k is the Boltzmann's constant and has the value $k = 1.38 \times 10^{-23}$ joule/kelvin.

The ideal gas law does not apply to gases at very low temperatures and very high pressures. At low temperatures and high pressures, attractive forces become important. The size of the particles are also no longer negligible.

MATHEMATICAL BACKGROUND INFORMATION - The following information is provided to assist the teacher in the derivation of the equation, $P = [(2/3)NK]/V$. In the video it is logically deduced that pressure is proportional to the density of a gas [$P \propto N/V$, (Boyle's Law)] and also to the average kinetic energy of a molecule. Using Newton's laws of motion, it can be shown that the constant of proportionality is 2/3.

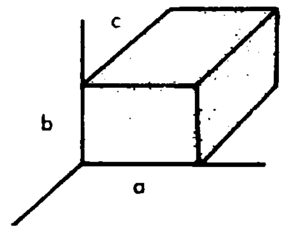


Figure 1.

Consider N molecules in a closed box of volume V, and each having a mass, m, as shown in Figure 1, and occupying a volume, V. To keep the discussion simple, let each molecule have the same velocity, v, and let all move in the same direction. In the diagram, Figure 2, this is the -x direction. Newton's second law, in the form, net $F = \Delta(mv)/\Delta t$, gives the force on the face, bc, of the box due to the molecules striking and rebounding. Each molecule has a momentum, $-mv$, before striking the face bc and momentum, $+mv$, after striking it. Therefore, the change in momentum is given by

$$\Delta(mv) = (mv_{\text{final}} - mv_{\text{initial}}) ,$$

$$\Delta(mv) = [mv - (-mv)] = 2mv .$$

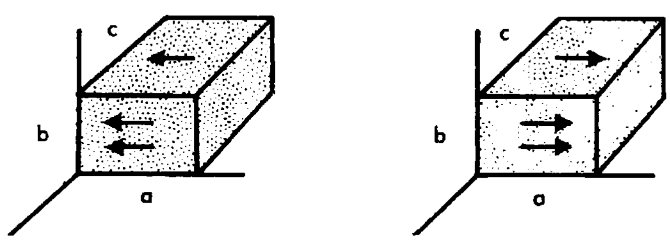


Figure 2.

Each time a molecule strikes the face, bc, the change in its momentum is $2mv$.

In a time, Δt, a molecule moves a distance equal to $v\Delta t$ meters. Between successive collisions with the face bc the molecule must travel twice the width of the box $2a$. The number of collisions for each molecule in time, Δt, is therefore:

$$\frac{\text{distance traveled in time } \Delta t}{\text{distance traveled between collisions}} = \frac{v\Delta t \text{ collision}}{2a} .$$

Applying Newton's second law yields,

$$\text{net Force} = \frac{(\text{Momentum Change per Collision})}{\Delta t}\ (\text{total collisions})\ .$$

$$\left[\frac{2mv}{\Delta t}\right]\left[\frac{v\Delta t}{2a}\right] = \frac{mv^2}{a}\ .$$

Hence the force exerted by each molecule is mv^2/a. Since there are N molecules, the total force is $N(mv^2)/a$. Pressure is force/area. The pressure on face bc is $[N(mv^2)]/[a(bc)]$ or $N(mv^2) = V$, where V is the volume of the box (abc).

Because of the random nature of molecular motion, only one-third of the molecules in the box would be expected to strike the face, bc. Others would strike face ab or ac with the same number of collisions in a time t. Thus the pressure on the face, ac, or any other face is one-third the value computed or

$$P = 1/3\ Nmv^2/V.$$

Since the average kinetic energy can be expressed as, $K = 1/2\ mv^2$ or $2K = mv^2$, the pressure can be expressed as $P = 2/3\ NK/V$. Pressure is proportional to the density of a gas N/V and the average kinetic energy, K, with a constant of proportionality of 2/3.

ADDITIONAL RESOURCES

Demonstration #1: The Expanding Balloon

Purpose: To demonstrate the relationship between heat and volume of a gas.

Materials: Flask; burner; and round balloon.

Procedure and Notes:

1. Attach a round balloon over the mouth of a small flask (about 250 ml).
2. Pass the flask slowly back and forth through a burner flame.
3. The balloon will expand.

Explanation: Heating the air in the flask causes the air molecules to move more rapidly, thereby increasing the pressure inside the flask and balloon. Since the balloon is elastic, the volume of gas in the balloon increases correspondingly.

Demonstration #2: Cartesian Diver

Purpose: To illustrate the relationship between pressure and volume of a gas.

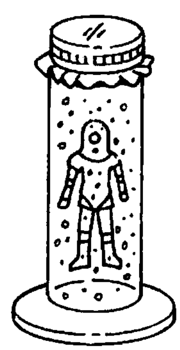
Materials: Clear plastic shampoo or dishwashing bottle; glass eyedropper or commercial Cartesian Diver; and water.

Procedure and Notes:

1. Fill the bottle nearly to the top with water.
2. Fill the eyedropper with water until it is just slightly lighter than an equal volume of water and floats upright in the bottle.
3. Seal the bottle tightly and squeeze. The eyedropper will sink to the bottom of the bottle. When the pressure is released, the eyedropper will float to the top again.

Explanation: When the bottle is squeezed, the pressure on the water increases. This, in turn, increases the pressure on the small amount of air in the eyedropper and thereby decreases its volume. With less water displaced by the air, the eyedropper sinks.

Note: Many additional forms of the cartesian diver may be built. A fruit jar with an inverted pill bottle as the diver works well when a hand is pressed down on the open end.



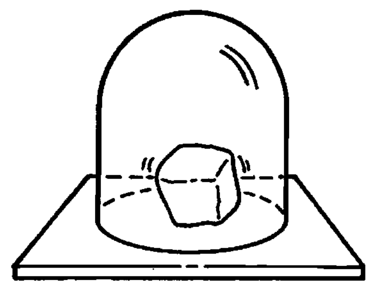
Demonstration #3: The Growing Marshmallow

Purpose: To illustrate the relationship between pressure and volume.

Materials: Marshmallow, vacuum pump, and bell jar

Procedure and Notes: Place the marshmallow on a platform; cover it with the bell jar; and then evacuate the jar.

Explanation: The marshmallow will increase in volume as the air pressure decreases because of air bubbles trapped inside. This is predicted by the ideal gas law. The demonstration may also be performed with shaving cream in place of the marshmallow.



Demonstration #4: Heated Balloons

Purpose: To demonstrate the relationship between temperature and volume.

Materials: Large beaker; smaller nesting beaker; small round balloon; boiling water; and ice water.

Procedure and Notes:

1. Inflate the balloon until it will fit in the bottom of the large beaker with room to spare on all sides, then knot the balloon. Using the smaller beaker, hold the balloon in the larger beaker as you pour enough boiling water into the larger beaker to cover the balloon. Note the space occupied by the balloon. *Be careful not to spill boiling water on yourself.*
2. Pour out the hot water and replace it with ice water. The volume of the balloon will be noticeably smaller.

Explanation: Cooling the confined air within the balloon decreases the velocity of the randomly moving air molecules. With lower average velocity, they exert less pressure against the inside of the balloon, and atmospheric pressure will decrease the volume of air.

EVALUATION QUESTIONS

1. **The boiling point of water occurs at approximately 373° on which of the scales below?**

   A. Fahrenheit
   B. Celsius
   C. Kelvin
   D. The boiling point of water occurs at the same temperature on all three of the above scales.

2. **Two identically shaped rigid containers, A and B, hold gas. The temperature in both containers is raised by the same amount and it is observed that the pressure in container A increases more than the pressure in B. You can conclude that**

   A. the temperature of the gas is higher in A than in B.
   B. the number of gas molecules is higher in A than in B.
   C. the temperature of the gas is lower in A than in B.
   D. the number of gas molecules is smaller in A than in B.

3. **If a balloon filled with helium is placed in a freezer, its volume must**

   A. increase.
   B. decrease.
   C. remain the same.
   D. first decrease then increase as the temperature of the helium drops.

4. **If the volume occupied by a gas is decreasing, its temperature**

   A. will decrease in all cases.
   B. will decrease if the number of molecules and the pressure remain the same.
   C. will remain the same if the number of molecules and the pressure remain the same.
   D. will increase if the number of molecules and the pressure remain the same.

5. **If only the temperature of two objects is known, it is always possible to determine the**

   A. total internal kinetic energy.
   B. total internal potential energy.
   C. direction of heat flow between them.
   D. phase or state of the objects.

6. **The air pressure in a tire increases after it has been driven for a while due to**

   A. the rubber shrinking at higher speeds.
   B. an increase in the number of air molecules inside the tire.
   C. the air molecules in a stationary tire gathering at the top of the tire.
   D. the friction of the road increasing the temperature of the tire.

7. **If a gas is heated and allowed to expand, its density will**

   A. increase.
   B. decrease.
   C. remain the same.
   D. first increase, then decrease.

8. As the gas in an insulated container is compressed, the temperature of the gas will

   A. increase.
   B. decrease.
   C. remain the same.
   D. first decrease, then increase.

9. If the temperature remains the same as a helium filled balloon rises in the atmosphere, its volume should

   A. increase.
   B. decrease.
   C. remain the same.
   D. first increase, then decrease.

10. If you double the pressure of an ideal gas while halving the temperature, the volume of the gas will

   A. quarter.
   B. double.
   C. quadruple.
   D. remain the same.

ESSAY QUESTIONS

11. Write a short essay describing in your own words why a hot air balloon is lighter than air.

12. Write a short essay describing the origin of pressure in a gas and its connection to temperature.

KEY

1. C
2. B
3. B
4. B
5. C
6. D
7. B
8. A
9. A
10. A

SUGGESTED ESSAY RESPONSES

11. All hot air balloons have heaters that warm the air inside the balloon. Heating the air increases the kinetic energy of all the molecules, and when a gas is hotter, each collision exerts greater pressure on the inner walls of the balloon. The balloon expands until the pressure on the inner and outer walls is equal. Because of the heater, fewer air molecules are needed to provide the balance of pressure from inside the balloon. Fewer air molecules means that the air inside the balloon weighs less than the equivalent volume of air outside the balloon. Consequently, the balloon is lighter than air.

12. On the microscopic level, the pressure of a gas is due to collisions of moving molecules of the gas with the walls of the container. Raising the temperature of a gas increases the kinetic energy of all the molecules. Consequently, the collisions are more violent--a greater change in momentum occurs when hotter molecules hit the walls. This greater change in momentum means a greater force is exerted on the wall in each collision. The added effect of the continually colliding molecules produces the pressure. Therefore, increasing the temperature of a gas increases the pressure, if the volume and number of molecules is held constant.

TEACHER'S GUIDE TO CURVED SPACE AND BLACK HOLES

CONTENT AND USE OF THE VIDEO - Although the video introduces concepts rarely mentioned in high school physics textbooks, it treats a topic of high interest and one which has been widely discussed in the press, in TV specials, and in feature films. The video and supporting materials can be effectively integrated at several different times during a physics course. It can be used as (1) a concluding unit of mechanics, (2) a follow-up to the discussion of gravity, (3) a component in the study of modern physics, especially where relativity is discussed, or (4) as a wrap-up for the entire year.

The video assumes that students are familiar with the concept of gravitational force as presented in *The Apple and the Moon*. If your students have seen that video, you might want to review with them the reason for the moon not falling to the earth.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - The video introduces several terms with which students may not be familiar. Therefore, it might be helpful to list these terms and discuss them briefly.

inertial mass--the property of an object which gives it resistance to acceleration, i.e., the m in $F = ma$.

gravitational mass--the property of an object which gives it the ability to attract other masses, i.e., the m in $F_g = GMm/R^2$.

principle of equivalence--the principle that states that there is no way solely within a reference frame to tell whether the frame is undergoing a constant acceleration or whether it is in free fall in a gravitational field.

spacetime--the four-dimensional "fabric of the universe" in which matter exists and moves.

curved space--space which is not flat like a plane but curved, such as the surface of a sphere.

geodesic--the shortest path between two points in any kind of curved geometry.

event horizon--the spherical surface around a black hole; nothing can escape from inside this surface.

WHAT TO EMPHASIZE AND HOW TO DO IT - The video develops the concept that gravity can be viewed as the curvature of spacetime which affects both matter and light.

Objective 1: Distinguish between inertial mass and gravitational mass.

Most students never recognize that the mass in Newton's second law, that is, inertial mass, does not have to be equal to the mass appearing in the universal law of gravitation. You should point out that there is no a priori reason why the two masses should be equal. DEMONSTRATION #1, which compares the lift versus kick box, provides an excellent opportunity to discuss with students the difference between gravitational mass and inertial mass. Regarding the two boxes used in the demonstration, ask your students the following:

(a) Which box required a larger force to lift it?

The box filled with weights has a greater gravitational mass and, therefore, requires more force to lift it. Gravitational mass determines how strongly one body attracts another.

(b) Which box must be kicked with a greater force to give equal acceleration?

The box filled with the weights has a greater inertial mass and, therefore, requires more force to move it. Inertial mass determines how strongly a body resists changing its motion.

Be sure your students understand that mass m plays an essentially different role on both sides of the following equation:

$$ma = -G\frac{mM_E}{R_E^2} .$$

You might ask your students what proof we have that gravitational mass and inertial mass are equivalent. You may need to remind them that all objects in a uniform gravitational field move with the same acceleration. For this to be true, the force of gravity on an object must be directly proportional to its inertia.

Objective 2: Give an example of Einstein's Principle of Equivalence.

Although we know that inertial mass and gravitational mass are the same for every body, emphasize to your students that it is not obvious *why* they are equivalent. The SUPPORTIVE BACKGROUND INFORMATION describes Newton's investigation of the equivalence. It also offers several illustrations of Einstein's *principle of equivalence*. In conjunction with these illustrations and those in the video, you may want your students to discuss the questions about the elevator presented at the end of this TEACHER'S GUIDE.

Objective 3: Realize that the classical view contained in Newton's universal law of gravitation does *not* explain the bending of light past a massive object.

Most students are familiar with Euclidean geometry and have difficulty visualizing the geometry of curved space. DEMONSTRATION #3, which shows straight lines on a curved surface, helps to introduce an important distinction, i.e., in curved space, parallel lines intersect if extended. You will want to relate the demonstration to the video discussion of a journey on the surface of a sphere.

DEMONSTRATION #2, which is on laser light and projectile motion provides students with a rubber ball analogy that will help them in the analysis of the light beam presented in the video. Make sure your students understand that Newton's universal law of gravitation did not predict that light beams would bend in a gravitational field. Einstein's analysis did predict the bending of light past a massive object, not as a function of gravitational force, but rather by describing its path along a geodesic. The SUPPORTIVE BACKGROUND INFORMATION parallels the video in its description of Einstein's theory.

It is important that students realize that only very massive bodies, such as stars, curve spacetime sufficiently to affect the path of light noticeably. Within our solar system, Einstein's Universe is almost indistinguishable from Newton's Universe. Only the most precise instruments of science stretched to the limits of their abilities can detect the tiny difference.

Objective 4: Trace the connection between the General Theory of Relativity and our present day understanding of black holes.

Students are intrigued by the notion of black holes. The black hole constitutes one of those times and places in the universe where only Einstein's general theory of relativity can describe and connect the phenomena. The video describes the explosion and consequent collapse of a star to a black hole. The explanation is complex. Besides replaying that section of the video several times, you may need to supplement the discussion using the SUPPORTIVE BACKGROUND INFORMATION. Several discussion questions relating to black holes are presented at the end of this TEACHER'S GUIDE.

POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and related frames from the video.

What is the difference between the gravitational mass and the inertial mass of an apple?

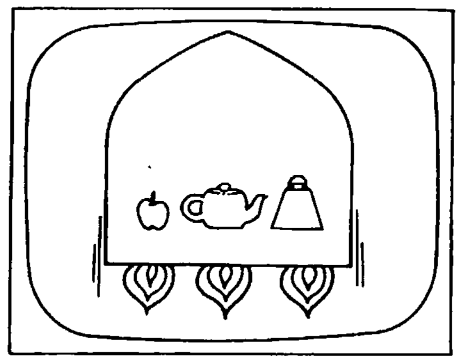
The gravitational mass of an apple reacts to the gravitational force created by the (gravitational) mass of the earth. The gravitational masses of both an apple and the earth appear in the force acting on the apple which is given by the universal law of gravitation. The inertial mass of an apple resists changes in motion; it is the mass that appears in Newton's second law.

$$m_a a = -G \frac{m_a M_E}{R_E^2} \hat{r}$$

$$a = -G \frac{M_E}{R_E^2} \hat{r}$$

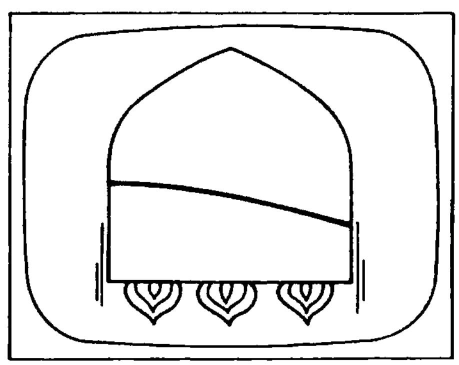
How does Einstein's principle of equivalence explain the equality of gravitational mass and inertial mass?

Einstein's principle of equivalence posits that uniformly accelerated motion and free fall in a uniform gravitational field are indistinguishable. One consequence is that all bodies fall with the same acceleration, which implies equality of gravitational mass and inertial mass. The principle of equivalence asserts no experiment can be performed locally to distinguish a uniformly accelerated form from one in free fall.

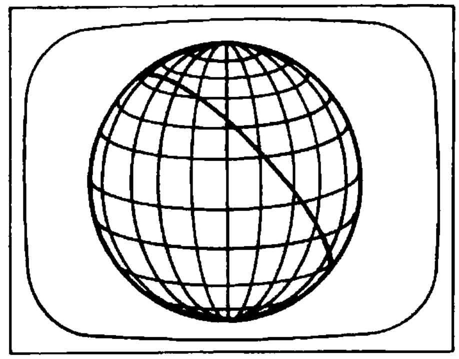


What did Einstein conclude as a consequence of imagining the path of a light beam in an upwardly accelerating frame and in a stationary frame in a gravitational field?

Einstein concluded that light doesn't travel in straight lines. Instead it bends ever so slightly because of the gravitational force of the earth. Since space itself is curved, it makes no sense to talk of straight lines but only of geodesics--paths light takes.

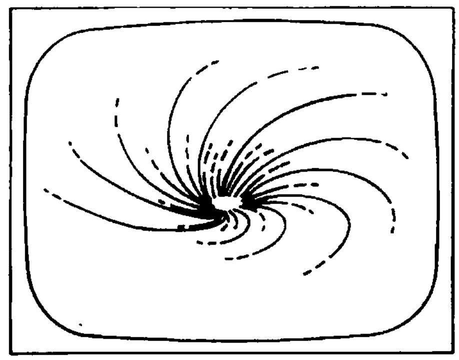


Einstein's theory changed the Newtonian notion of bodies moving in a straight line. How is this illustrated on the globe?



In watching the video, note that on a flat map the shortest path between two points is the familiar straight line that joins them. But not on the surface of a globe. Here the shortest path between two points is not a straight line, but a great circle.

Where is one place in the universe where conditions are so extreme that Einstein's theory of curved space-time is needed to describe it?



A black hole can be explained only by Einstein's theory of curved spacetime. While a gravitational field seems to bend light beams, only massive bodies, such as stars, curve spacetime sufficiently to affect noticeably the path of light. If the curvature is extreme, a black hole will result. In a black hole things can fall in but nothing can ever get out. Not even light can escape. That is why it is called a black hole.

EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the ideas contained in the video, you might pose the following questions to your students.

1. **What are some everyday experiences of being in accelerated reference frames?**

   Everyday experiences of an accelerated reference frame include a car accelerating and going around a corner and a plane taking off.

2. **Apply the principle of equivalence to the example of an apple falling from a tree (a) on earth, and (b) in a laboratory in space accelerating upward at g.**

   (a) *Near the surface of the earth, gravity applies a force to the apple. The apple falls with the acceleration of gravity.*
   (b) *In the accelerating lab in intergalactic space the apple is released from the tree and the tree accelerates upward with $a = g$. As a result, it appears the apple is falling with the acceleration of gravity; thus, any body in the laboratory must fall with the acceleration of gravity regardless of its mass.*

3. **Newton felt that the moon fell toward the earth because of the gravitational force. How would Einstein explain it?**

   Einstein would not say that the moon is subject to any force. Instead he would explain that the moon naturally follows a geodesic in the curved spacetime surrounding the earth.

4. **If you get into a closed elevator and push the "UP" button, can you determine if you are accelerating upwards or if the earth has suddenly become more massive?**

   According to Galileo's law of inertia, there is no experiment that can be done inside a closed system to determine whether that system is at rest or moving with a constant velocity. Einstein's principle of equivalence extends that concept to acceleration and gravity. It states there is no experiment that could determine if a system is in a uniform gravitational field or undergoing constant acceleration.

5. **If you get into a closed elevator at the top of a tall building and the supporting cable suddenly breaks, you would feel apparent weightlessness, at least for a time. During this time could you determine whether the elevator car is actually falling or whether the earth has suddenly disappeared?**

   The answer to this question is the same as the answer to the preceding question while the elevator car is in free fall. The collision of the car with the ground could also be explained by the sudden appearance of a very large mass.

6. **How do they "weigh" an astronaut in outer space?**

   In outer space, the weight of an astronaut cannot be determined. However, his or her inertial mass m can be determined by applying a known force and measuring the resulting acceleration, i.e., $m = F/a$.

7. **How do we detect black holes?**

   Since black holes cause and are caused by enormous gravitational fields, they should exhibit gravitational effects on other nearby objects. There are three methods we might use to detect black holes if they do exist:

   (a) *The motion of nearby stars would be perturbed.*
   (b) *Light that passes near a black hole would be bent.*
   (c) *As objects fall toward a black hole they would lose energy which would be given off in the form of x-rays.*

   In practice, methods (a) and (c) are used to identify black holes.

8. **Where are black holes?**

   Black holes might exist anywhere in the universe. Present theory suggests that there are rather large black holes at the center of some galaxies such as the Milky Way. Some research is presently aimed at detecting such black holes. Other research searches for binary systems in which a black hole might have a visible companion from which matter is falling into the black hole and giving off x-rays.

9. **What evidence is there for black holes?**

   At present, research indicates that black holes probably do exist. Cygnus X-1, an x-ray source in the constellation Cygnus, seems to fit the theory of black holes. This star has a massive, unseen companion and is a strong emitter of x-rays, which fits the theoretical properties of black holes.

SUMMARY - This module focuses on one of the deepest mysteries of physics: why all bodies fall with the same constant acceleration (in vacuum). Newton realized that the law of falling bodies was a consequence of the equality between inertial mass and gravitational mass. Inertial mass measures the tendency of an object to resist changes in its motion and appears in $F = ma$. Gravitational mass is responsible for the gravitational attraction of bodies and appears in $F = -GMm/r^2\,\hat{r}$. Einstein realized that this equivalence stems from a more fundamental feature of nature--the inability to distinguish solely by experiments done within a single reference frame, whether that frame is undergoing uniform acceleration or is in free fall. The principle of equivalence became the cornerstone of his theory of gravity, the General Theory of Relativity. According to Einstein's theory, matter (say, the sun) warps, or curves, the surrounding spacetime. That curvature, in turn, "tells" other matter (such as planets) how to move. The natural motion of a body near another very massive one is along a geodesic. Even light follows the curves of spacetime. Einstein's theory led to the prediction of extremely massive, compact objects around which the spacetime is so curved that not even light can escape. These objects--black holes--may be remnants of exploding stars. The observational evidence supporting their existence continues to mount.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed *prior* to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as part of the activities that *follow* the viewing(s) of the VIDEO and give closure to the lesson.

STUDENT'S GUIDE TO CURVED SPACE AND BLACK HOLES

INTRODUCTION - This video draws a distinction between gravitational mass and inertial mass, provides background for Einstein's development of the principle of equivalence, and describes some consequences of his theory of relativity including the path of a planet around the sun, the path of a ray of light, and the phenomenon of black holes.

Terms Essential for Understanding the Video

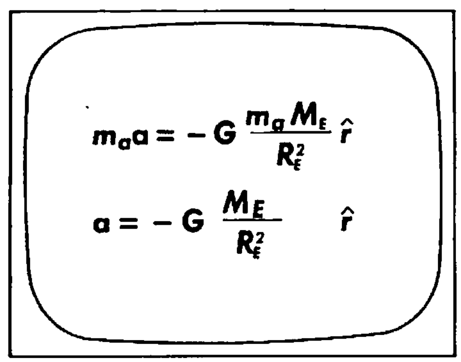
inertial mass
gravitational mass
principle of equivalence
spacetime
curved space
geodesic
event horizon

**** NOTE: Parts of the video, especially mathematical equations may go by quickly on the screen. If you have questions you should ask your teacher to replay these sections. ****

Points to Look for in the Video

What is the difference between the gravitational mass and the inertial mass?

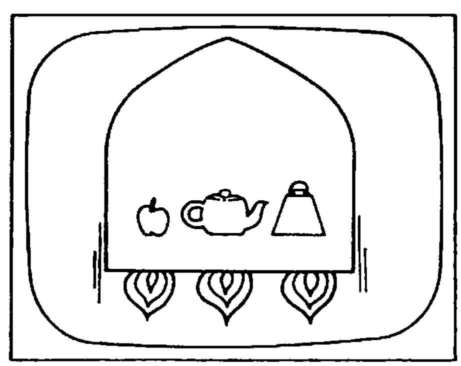
In viewing the video note that gravitational mass appears in Newton's law of universal gravitation, $F_g = Gm_1m_2/r^2$, and that inertial mass appears in Newton's 2nd Law, $F = ma$. When the forces representing these two laws are equated, the gravitational and inertial mass of the object cancel, demonstrating that the acceleration of gravity does not depend on the mass of the object at all.



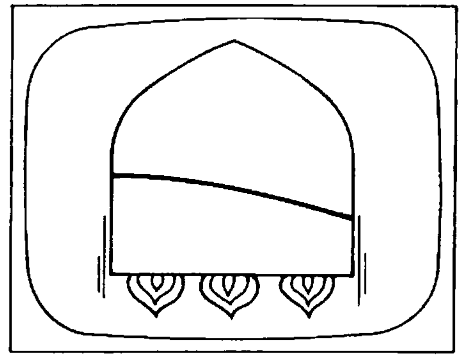
How does Einstein's principle of equivalence explain the equality of gravitational mass and inertial mass?

The related section of the video helps answer this question.

In viewing this video, note that no experiment of any kind done entirely inside the laboratory could determine whether the objects fall because of the pull of gravity or because the laboratory is accelerating upwards in outer space.

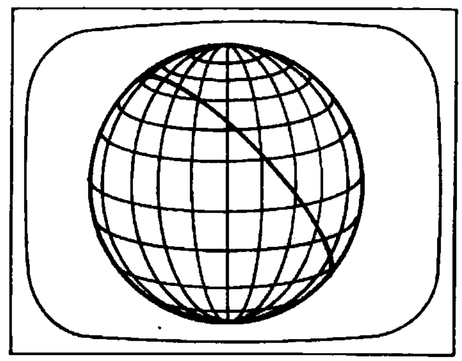


What did Einstein conclude as a consequence of imagining the path of a light beam in an upwardly accelerating frame and in a frame undergoing free fall in a gravitational field?

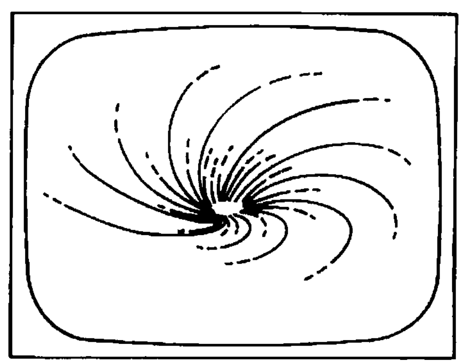


The section of the video indicated by the drawing to the right helps answer this question. In watching the video, note that if the rocket accelerates upward while the beam goes straight across, it must seem to bend downward just a tiny bit.

Einstein's theory changed the Newtonian notion of bodies moving in a straight line. How is this illustrated on the globe?



What is one place in the universe where conditions are so extreme that Einstein's theory of curved space-time is needed to describe it?



Hint: Consider how the curvature of space changes for a more massive object.

TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION[1]

The Principle of Equivalence

When the force of gravity is used in Newton's second law,

$$ma_i = \frac{M_E m_g}{R^2},$$

and the inertial mass, m_i, and gravitational mass, m_g, are set equal, the mass of the falling object cancels out of the equation. The result is that all bodies fall with the same constant acceleration.

Mass plays an essentially different role on the two sides of the equation. On the left-hand side of the equation, the mass stems from F = ma and is a measure of the resistance of a body to be accelerated. In this role it is called the *inertial* mass. The mass on the right-hand side of the equation comes from the universal law of gravitation, $F = GmM_E/R^2$ and measures the magnitude of the *gravitational* mass. It is not obvious why these two kinds of mass should be exactly the same for every body. This question has troubled generations of physicists from Newton to Einstein.

There have been numerous experiments to determine to what extent inertial mass and gravitational mass are equal. Newton investigated the equivalence by studying the period of a pendulum with interchangeable masses. His experiment consisted of looking for a variation in the period of the pendulum, using bobs of different composition. He found no such change, and from an estimate of the sensitivity of his method, he concluded that the inertial mass and gravitational mass cannot differ by more than one part in a thousand. Recent experiments which are capable of detecting a variation of one part in 10^{11} have indicated no variation within the precision of the experiment.

The answer to the problem of equivalence between inertial mass and gravitational mass forms the basis of Albert Einstein's general theory of relativity, his theory of gravity. Einstein did not believe in coincidence. Rather he thought that there must be some deep principle at work, a principle which would have the equivalence of gravitational mass and inertial mass as a simple, inevitable consequence. The principle he adopted is called the *principle of equivalence.*

The principle of equivalence states that there is no way to tell solely within one frame of reference whether that frame is undergoing constant acceleration or is in free fall. Imagine a laboratory as a closed box with a scientist inside who makes measurements but who cannot see outside. In one situation the box is on earth, as in Figure 1(a). In another situation, the box is somewhere in intergalactic space--very far from any body that could exert a gravitational force. However, this box is accelerated with a uniform acceleration in the upward direction equal in magnitude to g, as indicated in Figure 1(b). In Figure 1(a), the ball falls downward on account of the gravitational force of the planet. The ball in Figure 1(b) would appear to go "down" because the floor would come "up" to meet it.

[1]This section is excerpted from the college level textbook, *The Mechanical Universe*, by R.P. Olenick, T.A. Apostol, and D.G. Goodstein, Cambridge University Press, New York (1985).

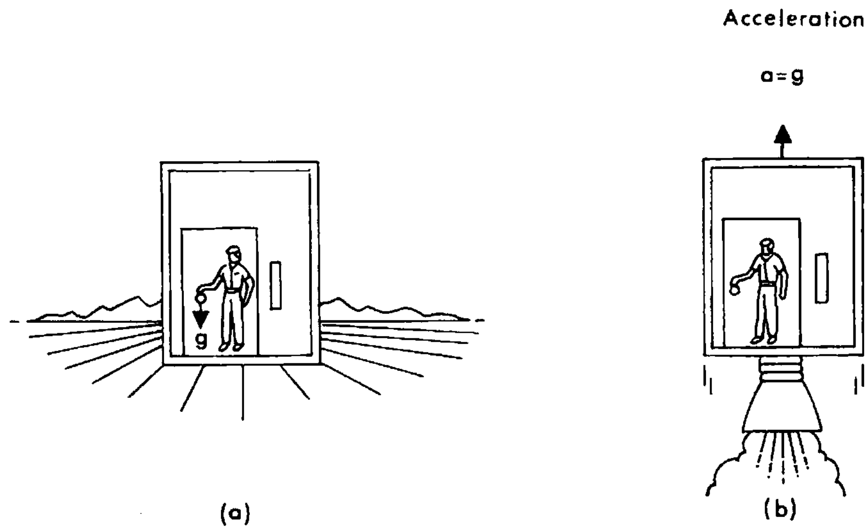


Figure 1. Free fall on the earth (a) and in an accelerating frame (b) illustrates the principle of equivalence.

Now take Newton's apple falling out of a tree. On earth, gravity applies a force to the apple and it falls with acceleration g as in Figure 1(a). Still using the laboratory box in intergalactic space as in Figure 1(b), the apple is released from the tree, but the tree continues to accelerate upward (as does the observer inside the box) with acceleration g. As soon as the apple is released it becomes inertial, which means it no longer is accelerated. The tree however, is accelerated upward, with an acceleration equal to g, and so it appears that the apple falls with acceleration g. Thus any body must fall with exactly the acceleration g regardless of mass.

The law of falling bodies and the apparent equivalence of inertial and gravitational mass are simple consequences of the principle of equivalence. This is fine for falling bodies, but what does the principle of equivalence imply for the propagation of light?

Imagine sending a light beam horizontally inside a box. The observer sees the light beam travel across the box. While the beam is traveling across the box in space, the box is accelerated upward with $a = g$. As the beam crosses the box with speed c, the box moves up a distance $gt^2/2 = g(L/c)^2/2$, where L is the width of the box and t is the time it takes for light to travel across the box. As the light moves across, the box moves upward and the beam hits a lower point than it started from by just that distance, as depicted in Figure 2. The principle of equivalence tells us that the observer cannot do any experiment that will determine whether the box is accelerated or in a gravitational field. This means that the same results must be observed in a closed box on earth: light curves downward as it crosses the box. In other words, a gravitational field bends light beams. This was the first prediction Einstein produced from his analysis. In 1919 an experiment was performed during a total solar eclipse to observe the bending of starlight. The success of that experiment made Einstein into a world-famous folk hero.

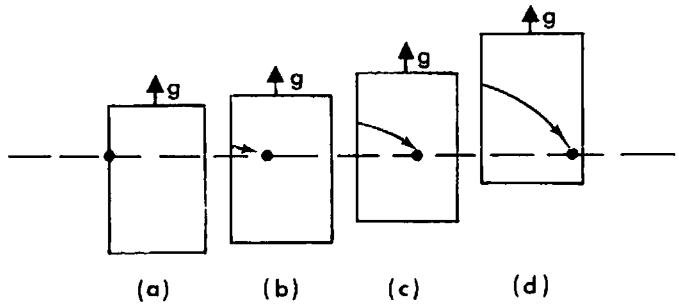


Figure 2. Successive stages in the path of a light beam traveling horizontally in a box which is accelerated upward.

Einstein's Theory of Relativity

If light travels in curved paths, then what do we mean by a straight line? Einstein said that it is meaningless to speak of straight lines--it is space which is curved. Not only is space curved, but the four-dimensional fabric of the universe--spacetime--is curved. This means that both rulers and clocks change their properties as they move through gravitational fields.

The idea of a straight line has meaning in the physics we know--in the law of inertia. According to the law of inertia, a body moving along at some speed continues to move in a straight line unless something interferes with it. If a straight line no longer has meaning in physics, then the law of inertia must be reformulated.

It was Einstein who made the reformulation. He realized that the idea of a straight line can be generalized for curved spacetime or curved space, or anything, by the idea of the optimal distance between two points. On a flat plane surface, the shortest distance between two points is along a straight line. On the surface of a sphere, the shortest distance between two points is not along a straight line but along a great circle, as illustrated in Figure 3. In any kind of curved geometry the shortest path between two points is called a geodesic; it is the most economical way to get from one point to another.

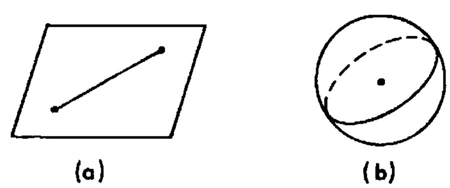


Figure 3. The shortest path between two points is (a) in a plane, a straight line; and (b) on a sphere, a great circle.

Einstein described the bending of starlight by the sun, not as a gravitational force from the sun which makes the light alter its straight-line path, but rather by saying that light travels at a constant speed along a geodesic in the spacetime curved in the vicinity of the sun. According to Einstein, the orbit of the earth does not have to be thought of, as it must in Newtonian physics, as a compromise between the earth's inertia, which wants to make it fly off in a straight line, and the force of the sun, which wants to keep it bound to the sun. Instead, the earth can be described as moving inertially, without any forces, along the geodesic in the local spacetime created by the presence of the sun. The earth follows its characteristic orbit because it is traveling along a geodesic. In the presence of an extremely massive object, such as the sun, the geometry of spacetime is locally disturbed, so geodesics which were once straight lines in flat spacetime become curved lines. According to Einstein, what happens near a massive object may be interpreted, not in terms of a gravitational field, but in terms of curvature of spacetime.

In Einstein's theory of gravity, forces may be done away with and replaced by the curvature of spacetime. There is an historical irony in this reformulation. Galileo thought that bodies, if unimpeded, would keep on moving, not along a straight line, but in a perfect circle parallel to the surface of the earth. It was only later that Newton and Descartes discovered that inertia tends to move objects along straight lines. Einstein's reformulation is much closer to Galileo's picture. In Einstein's picture, the nearly circular orbit of the earth is itself inertial motion.

Black Holes

How does mass cause local spacetime to curve? Einstein developed a consistent set of equations--Einstein's field equations. They are among the most mathematically difficult equations in all of physics. Nonetheless, there are intriguing predictions found in these equations.

The mass of an object causes spacetime in its vicinity to curve. According to Einstein, an object in a curved spacetime has more energy than it would in flat space. Because it has more energy, it has more mass, since $E = mc^2$. And because this object has more mass it causes more curvature in spacetime. There is a kind of unstable feedback effect built into Einstein's theory of gravity. Under ordinary circumstances the effect is extremely small--so small that only the most exquisitely delicate observations and experiments can detect the difference between Einstein's theory and Newton's theory. However, there are other circumstances in which the difference in theories is extremely large and the feedback effect becomes important. When the feedback effect causes an increase in mass, which causes a corresponding increase in curvature, which creates more mass and more curvature ad infinitum, the object collapses into what is known as a black hole.

It takes a great deal of mass in a very small space to create a black hole. In our present understanding of the universe, there are only two circumstances in which black holes might be created. One of them, the primeval black hole, arises from the extraordinary events that attended the birth of the universe. The other circumstance occurs when a very large mass, such as a very large star, much larger than our sun, runs out of nuclear fuel. A star is made up of gases that attract each other gravitationally, but are kept from collapsing by the fact that they are burning fuel inside, and radiation is pushing outward, preventing collapse. When the star runs out of nuclear fuel, it will collapse gravitationally, and if it's big enough and does not blow off too much matter while collapsing, it will enter the state in which the collapse is unchecked leading to a state identified as a black hole. It's not difficult to specify the conditions for having enough mass in a small enough space to have a black hole. From mechanics we know that for a body to escape from the earth, it must have enough kinetic energy to overcome its gravitational potential energy:

$$\frac{1}{2}mv^2 = G\,\frac{Mm}{R}\ .$$

There is an escape velocity for any body of any mass to escape from a given mass, like the earth. That's the solution of this equation. If we set that velocity equal to the speed of light, then $c^2 = 2GM/R$, which tells us the amount of mass within a given radius from which not even light has sufficient velocity to escape. Actually, when a body moves at nearly the speed of light, its kinetic energy is no longer equal to $mv^2/2$. Nevertheless, the idea is essentially right, and the result $R = 2GM/c^2$ is exactly correct. When there is that much mass within that small a radius, then a black hole is formed. The R that solves this equation is called the Schwarzchild radius. The spherical surface having this radius is called the event horizon. Nothing can pass out of the event horizon--things can fall in but nothing can ever get out. That is why it is called a black hole. Not even light can escape.

There are other consequences of the general theory of relativity, one of which is the current view of all cosmologists that the universe began with a cataclysmic event known as the Big Bang. Before the Big Bang there was nothing, and then the universe exploded out of nothing, and it's still expanding. In a certain sense the Big Bang is related to the idea of a black hole. There is a kind of opposite solution of the same equation, which we might call a white hole. A black hole is something that everything falls into, a white hole is something that everything flies out of, a sort of black hole with a minus sign in front. The Big Bang resembles a white hole.

Once the Big Bang occurs, the universe is expanding and there are two possible things that can happen according to Einstein's equations. One is that it can go on expanding forever. The other is that there is so much mass in the universe that it is really gravitationally stable, and so it won't go on expanding forever. It will eventually lose all its kinetic energy against its own gravitational potential and start contracting. If it does that, then at the end of the universe there will be an event that is the reverse of the Big Bang, where everything comes back together again. In cosmology there is a name for that too; it is known as the Big Crunch. The Big Crunch is a little like the black hole, with the whole universe falling into nothing, but it differs technically from a black hole because the black hole is a kind of singularity in the curved space, where space itself is due to all the rest of the mass in the universe. The Big Crunch is the entire universe falling into nothing because there is nothing left to create curved spacetime.

ADDITIONAL RESOURCES

Demonstration #1: Lift vs. Kick Box

Purpose: To illustrate the difference between inertial mass and gravitational mass.

Materials: Two identical boxes, one left empty while the other is filled with heavy weights.

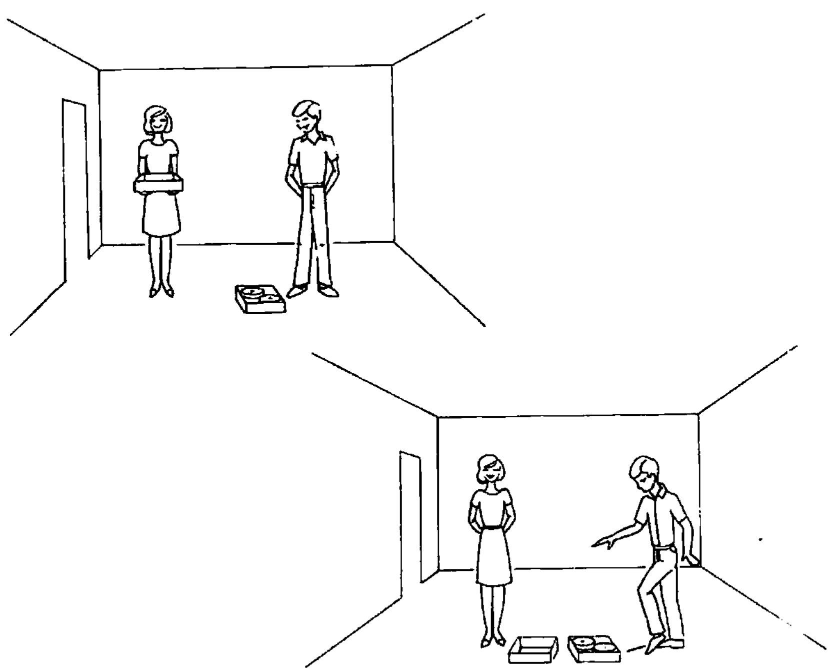
Procedure and Notes:

1. Set both boxes in the center of the room. Have two or three students come forward and lift the boxes.
2. Ask the students which box has the greater mass.
3. Next ask two or three students to kick the boxes gently.
4. Then ask the students which box seems to have the greater mass.
5. Discuss the two methods of determining the masses of the objects.

Explanation: When students lift the boxes, they are determining the relative weights of the boxes. Since weight depends on the pull of gravity on an object, the box with the greater weight also has the greater gravitational mass.

$$F = GM_eM_{box}/R^2$$

When the students kick the boxes, they are determining the inertial mass of the objects. Since the force required to accelerate an object is directly proportional to its inertial mass, the object that requires the greater force for a given acceleration has the greater mass. If students are still not convinced and feel that the heavier object is harder to kick because of more friction, put both boxes on laboratory carts and ask them to try again. The difference between the two will still be quite noticeable.



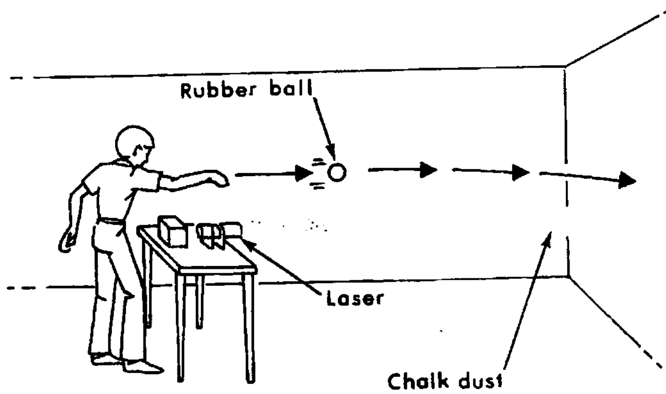
Demonstration #2: Laser Light and Projectile Motion

Purpose: To generate a discussion on the path of light and the path of a projectile in a gravitational field.

Materials: A small demonstration laser, some chalk dust, and a rubber ball.

Procedure and Notes:

1. Shine the laser across the room and sprinkle a bit of chalk dust in the beam so it can be seen clearly. Discuss the path of light.
2. Next throw the ball gently across the room. Ask how the motion differs from the light. Throw the ball harder and harder across the room. Ask how the path of the ball changes. Ask how hard you would have to throw the ball to have it go straight across the room. Could the light be bending a little?



Explanation: We say that light travels in straight lines. The ball, however, follows a parabolic path because of gravity. If the ball is thrown faster and faster, it curves down less and less. No matter how fast the ball is thrown, it will always curve a little. Although the ball always will fall a little, it could be propelled with a velocity high enough so that it appears to have followed a straight path. Encourage students to follow this same line of thought in their analysis of the beam of light.

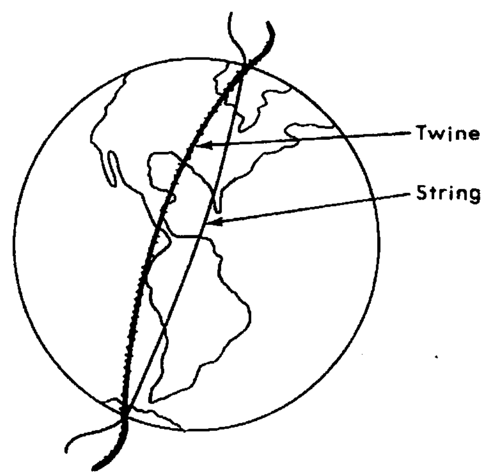
It takes light about 3×10^{-8} seconds to travel about 10 meters. Using $s = 1/2\ at^2$, students can calculate that in one second light would fall about 4×10^{-15} meters or about the diameter of a nucleus.

Demonstration #3: Straight Lines on a Curved Surface

Purpose: To demonstrate how straight lines behave on a two dimensional curved surface.

Materials: World globe, string, and twine.

Procedure and Notes: Cut equal lengths of string and twine that are long enough to go at least one-half way around the globe. Starting at the North Pole, place the string along a longitude line to the South Pole. Take the twine and place it along a different longitude line, beginning at the North Pole and ending at the South Pole. Throughout the demonstration, keep the strings taut.



Point out, that if you were to imagine starting at the North Pole and walking along either the string or the twine, you would end up at the South Pole. Along the way you would think that you were walking along a straight line, which on the spherical surface of the globe is a great circle.

Explanation: The geometry of curved surface is difficult to visualize and is different from plane Euclidean geometry.

Additional Video

The BBC television production of *Einstein's Universe*, that was presented on PBS by IBM, honors Einstein for his contributions to modern thought and presents his ideas to a wider public than ever before. As Einstein once wrote, "There is but one way to bring a scientist to the attention of the larger public. It is to discuss and explain, in language that can be generally understood, the problems and solutions that have characterized his life's work."

Peter Ustinov narrates the program and, as a layman himself, is constantly at pains to simplify difficult ideas and language. Many of the concepts are enacted or animated for easier comprehension. Written by science writer Niger Calder, the program is largely adapted from his book, *Einstein's Universe* (Viking Press, 1979). The program alternates between the simple presentation of core ideas and discussion by scientists of current applications of these ideas.

EVALUATION QUESTIONS

1. **Which of the following procedures could be used to distinguish any difference between an object's gravitational mass and its inertial mass?**

   A. A spring scale
   B. A triple beam balance
   C. An applied force
   D. None of the above

2. **Suzy is locked in a windowless laboratory. She notices that a beam of light is slightly bent towards the floor. From this she can conclude that**

   A. she is in an accelerating rocket.
   B. she is in a strong gravitational field.
   C. either A or B might be correct.
   D. neither A nor B could be correct.

3. **Einstein described the bending of light by the sun as due to which of the following?**

   A. The sun applying a force on the beam of light
   B. The light beam following a geodesic past the sun
   C. The light being blown by the solar wind
   D. None of the above

4. **After Newton formulated his law of universal gravitation, more than 250 years passed before it was replaced by Einstein's theory of curved spacetime. Which of the following reasons best accounts for this long delay?**

   A. Near the surface of the earth both theories predict almost identical results.
   B. The value of the universal gravitation constant was not measured during Newton's lifetime.
   C. Gravity bends light only near massive stars.
   D. None of the above

5. **You accidentally fall into a black hole. How could you communicate your predicament to your friends outside the black hole?**

   A. Send a radio message.
   B. Use a flare.
   C. Use a laser.
   D. None of the above.

6. **Light is not observed to bend as it travels past a basketball because**

   A. light travels too fast.
   B. the basketball has too little mass to curve spacetime detectably.
   C. light is not affected by gravity.
   D. the basketball has too small a diameter to affect light by its gravity.

7. If two light rays start out parallel to each other in spacetime that is curved like the surface of a sphere,

    A. their paths will always be parallel to each other.
    B. their paths will move farther way from each other.
    C. their paths will eventually cross.
    D. the paths will never cross.

8. Which one of the following phenomena is supportive of Einstein's general theory of relativity?

    A. Black holes
    B. Bending of starlight
    C. Curving of spacetime
    D. All of the above

9. If all bodies did *not* fall with the same constant acceleration near the surface of the earth (in a vacuum), you could conclude that

    A. the principle of equivalence does not hold.
    B. inertial mass and gravitational mass are not equal.
    C. you could tell whether or not a frame was undergoing uniform acceleration or was in free fall.
    D. all of the above statements are correct.

10. When light bends around the sun, it is

    A. traveling along a geodesic.
    B. not affected by the gravitational field of the sun.
    C. repelled by the gravitational field of the sun.
    D. moving more slowly than when it travels in empty space.

ESSAY QUESTIONS

11. Write a short essay describing the bending of starlight as it passes the sun.

12. Write a short essay describing the principle of equivalence and its implications for light.

KEY

1. D
2. C
3. B
4. A
5. D
6. B
7. C
8. D
9. D
10. A

SUGGESTED ESSAY RESPONSES

11. According to Einstein's general theory of relativity, the matter in the sun causes the surrounding spacetime to be curved. The curvature of spacetime dictates to other things in the vicinity of the sun, such as an orbiting planet or passing starlight, how to move, and replaces the classical idea of a gravitational force. Because of the curvature, starlight does not travel along a "straight line." Instead, it travels along a geodesic which represents the shortest distance between two points. Therefore, we say that starlight is "bent" as it passes the sun.

12. The principle of equivalence states that there is no way to tell solely within one frame of reference whether that frame is undergoing constant acceleration or free fall in a gravitational field. Regarding light, this means that an observer in an accelerated room (frame of reference) would see a beam of light bend as it passes through the room. The equivalence principle applied to this situation implies that the room could be accelerating or in a uniform gravitational field. Consequently, a gravitational field must bend light beams.