Relativity on Rotated Graph Paper: Calculations with Causal Diamonds

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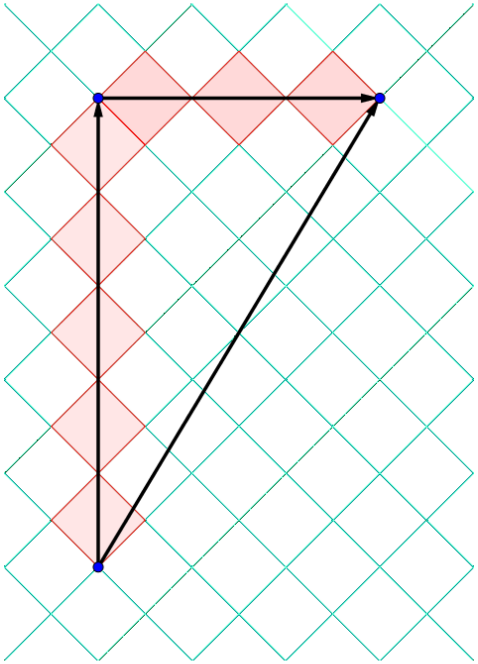

"Relativity on rotated graph paper"
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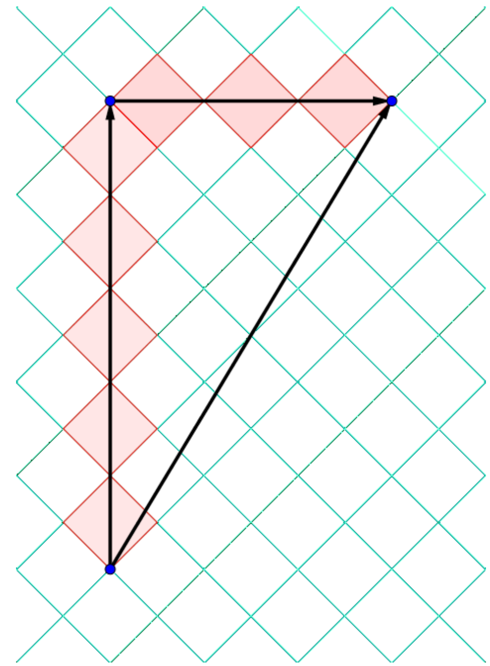
Relativity on Rotated Graph Paper

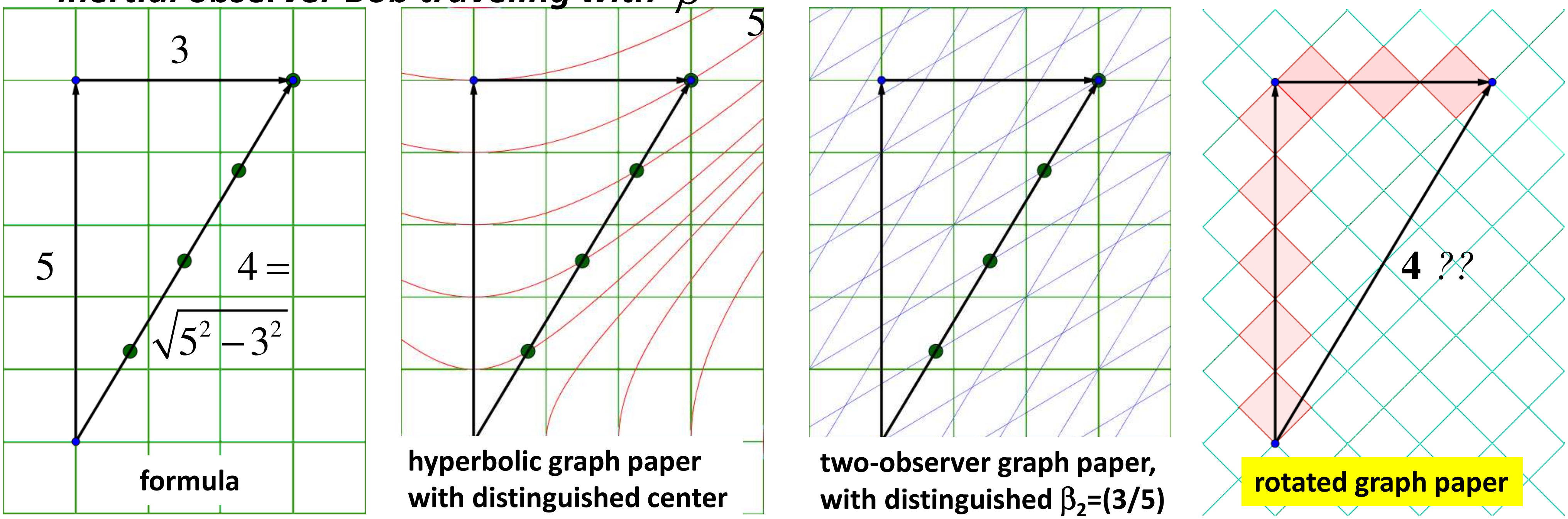
- an **ordinary Minkowski spacetime diagram**
  emphasizing light-signals
  allowing “ticks of a light-clock” to be visualized
- *physically motivated:*
  traced out by the **light-signals in a ticking light-clock**
- *method of calculation:*
  count boxes (“clock diamonds”) and do **simple algebra**
- the visualization **encodes many relativistic effects**
  and lends itself to numerous physical interpretations



- first developed for use in **algebra-based introductory** courses
- *new methods more appropriate for more advanced students*

Can you see the "4 ticks" on a spacetime diagram?

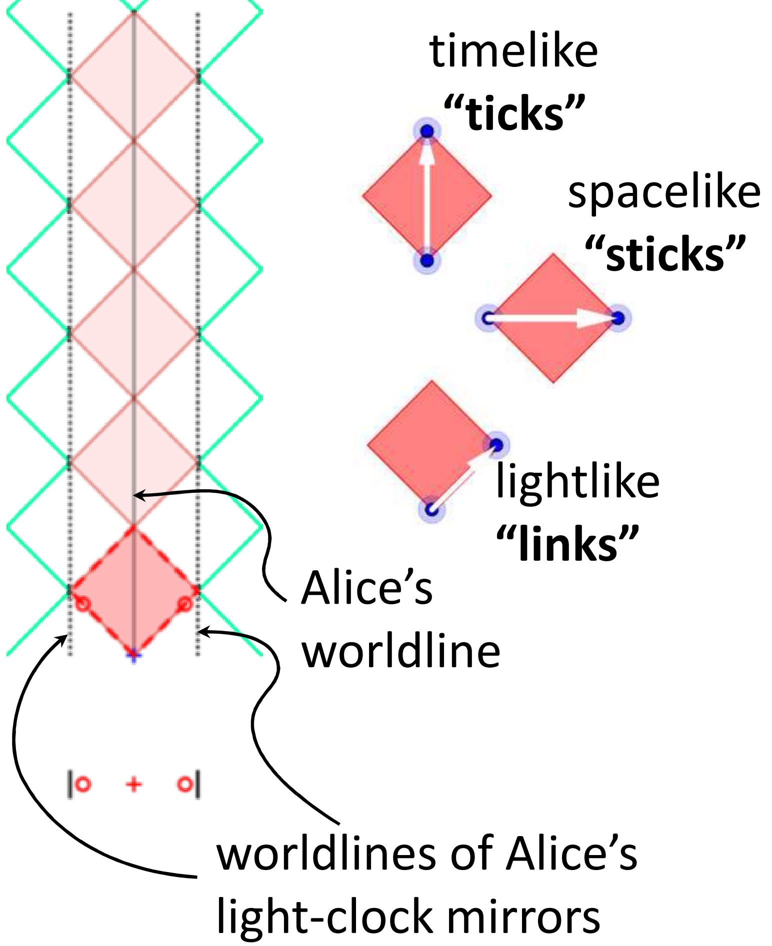
Inertial observer Alice at rest and
inertial observer Bob traveling with $\beta = \dfrac{3}{5}$



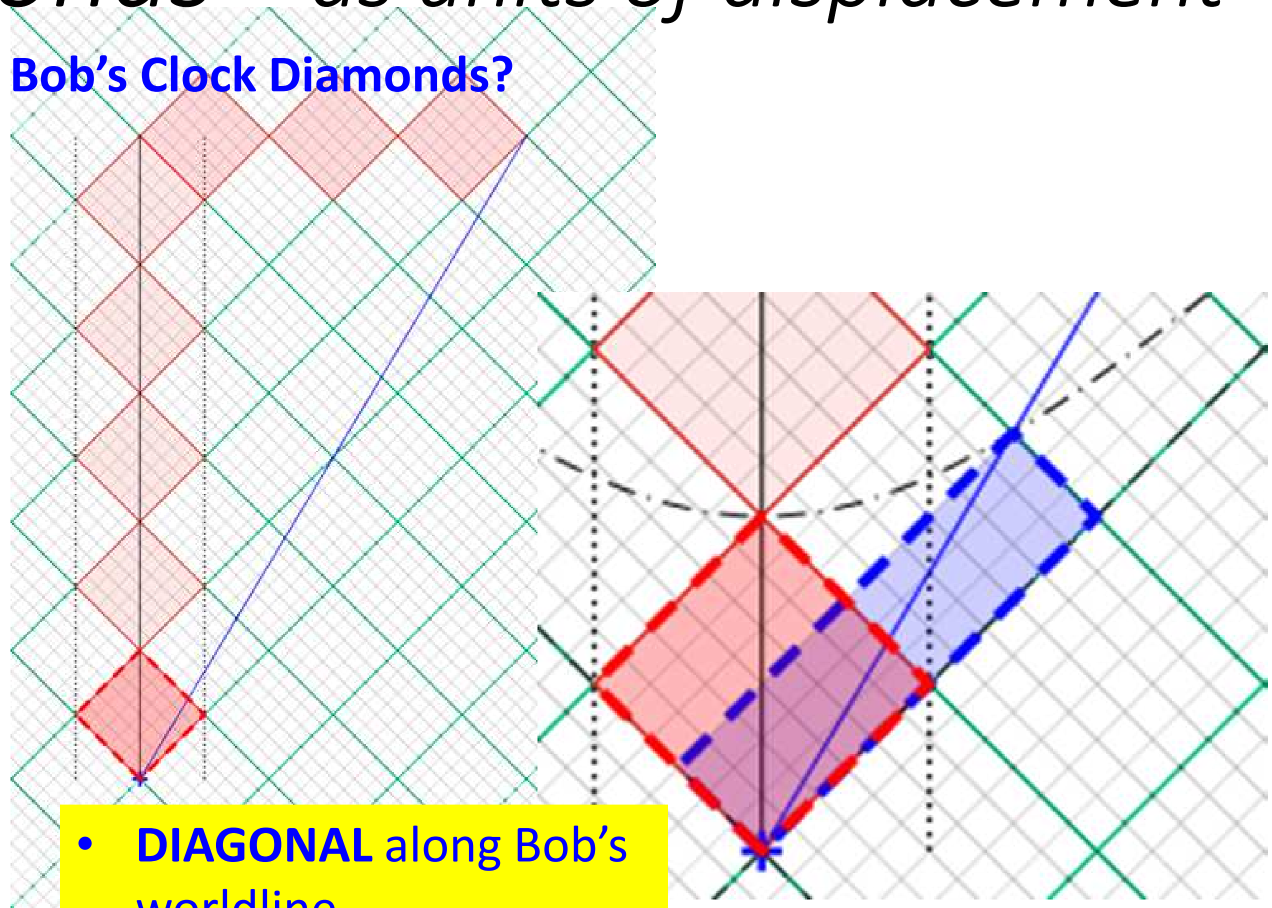
"Ticks" (a.k.a. "clock diamonds")
are constructed using the
light-signals in a longitudinal light-clock

Light-Clock Diamonds – as units of displacement

Alice's Clock Diamonds



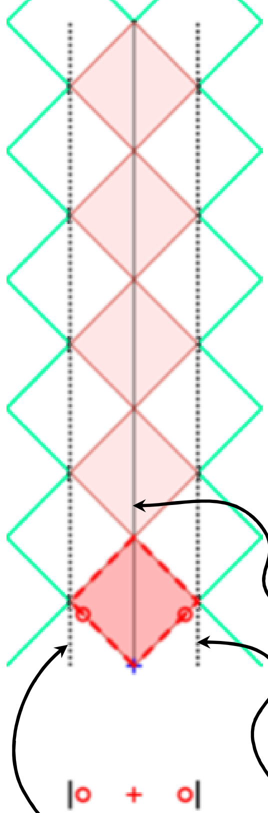
Bob's Clock Diamonds?

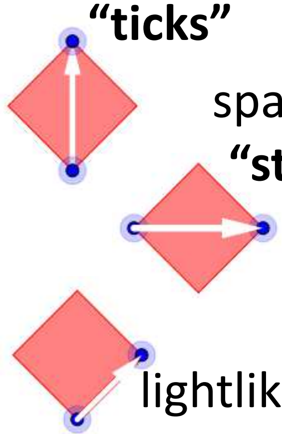


- **DIAGONAL** along Bob's worldline
- **SAME AREA** as Alice's Clock Diamond

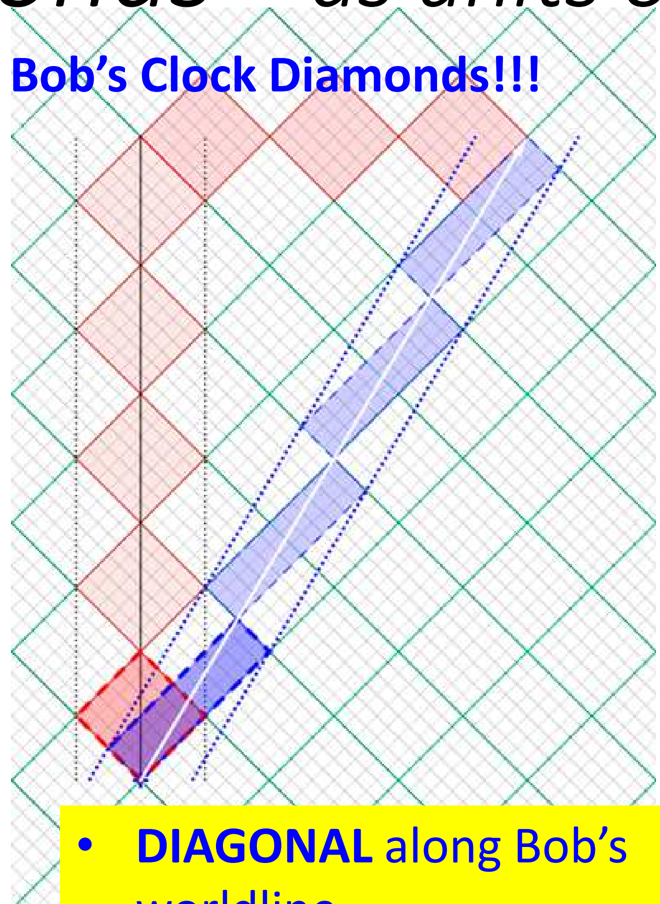
Light-Clock Diamonds – as units of displacement

Alice's Clock Diamonds

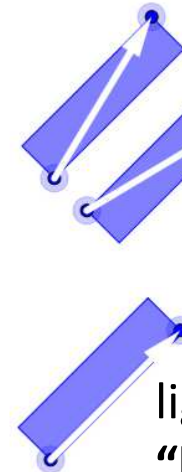


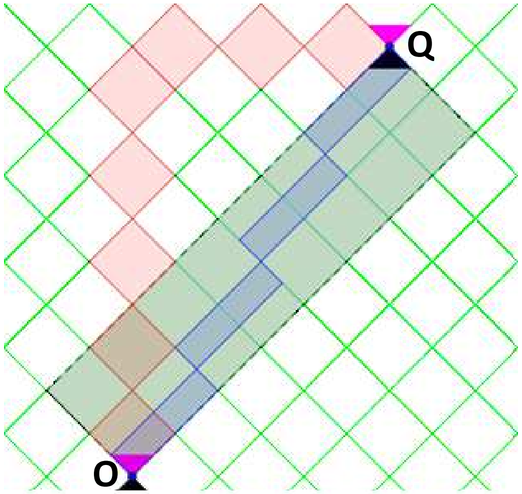


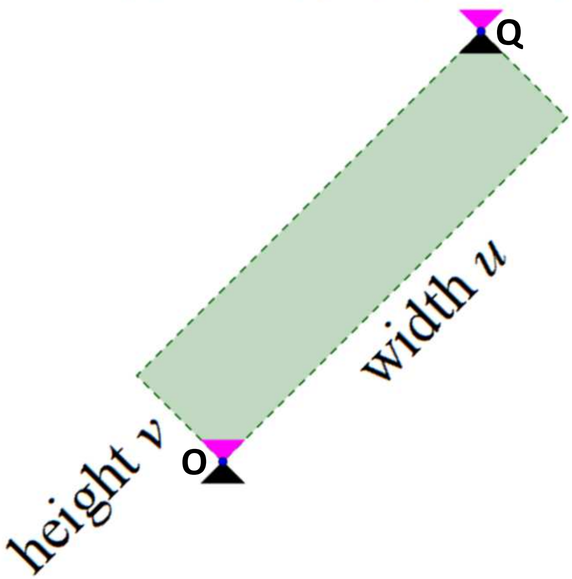
Bob's Clock Diamonds!!!



- **DIAGONAL** along Bob's worldline
- **SAME AREA** as Alice's Clock Diamond







Causal Diamonds

- **Intersection** of (the **future** light cone of event O) and (the **past** light cone of event Q).

  "events that can be influenced by O and can then influence Q"

- **area** of the diamond (in units of clock diamonds) = **squared-interval**

$$s^2 = (\text{width } u)(\text{height } v)$$

- **aspect ratio** of the diamond = **square of the Doppler Factor** (encodes velocity $\beta = (V/c)$)

$$k^2 = \frac{(\text{width } u)}{(\text{height } v)}$$

$$\text{Doppler } k = \sqrt{\frac{1+\beta}{1-\beta}}$$

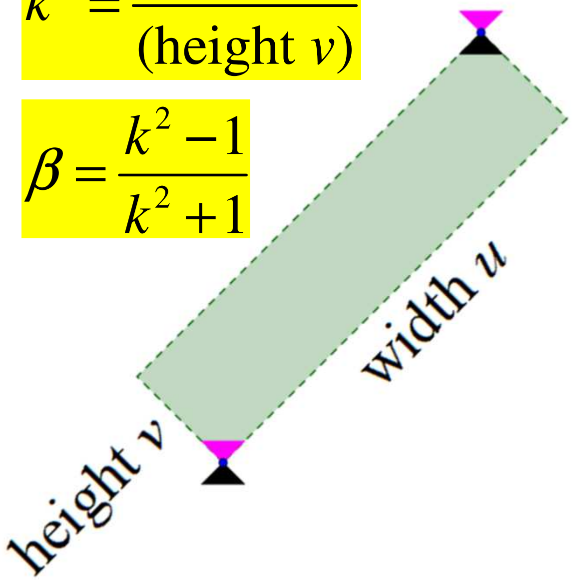
$$\beta = \frac{k^2 - 1}{k^2 + 1}$$

Causal Diamonds
with clock-diamond components

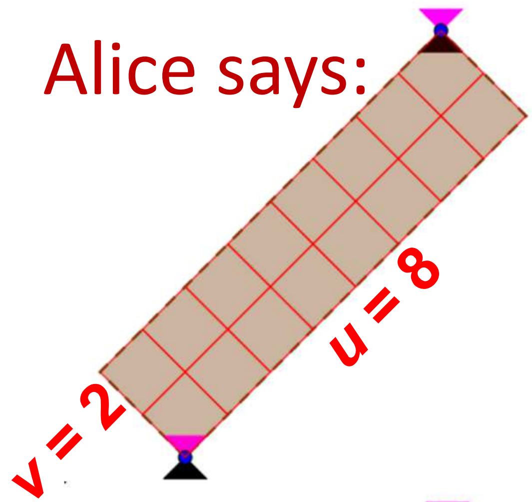
$$s^2 = (\text{width } u)(\text{height } v)$$

$$k^2 = \frac{(\text{width } u)}{(\text{height } v)}$$

$$\beta = \frac{k^2 - 1}{k^2 + 1}$$



Alice says:



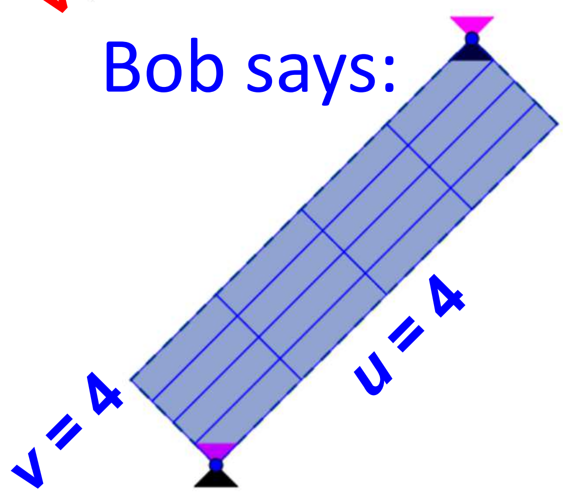
$$s^2 = (8)(2) = 16 = (\mathbf{4})^2$$

$$k^2 = \frac{(8)}{(2)} = 4 = (2)^2$$

$$\beta = \frac{(4)-1}{(4)+1} = \frac{3}{5}$$

The "***4***"!

Bob says:

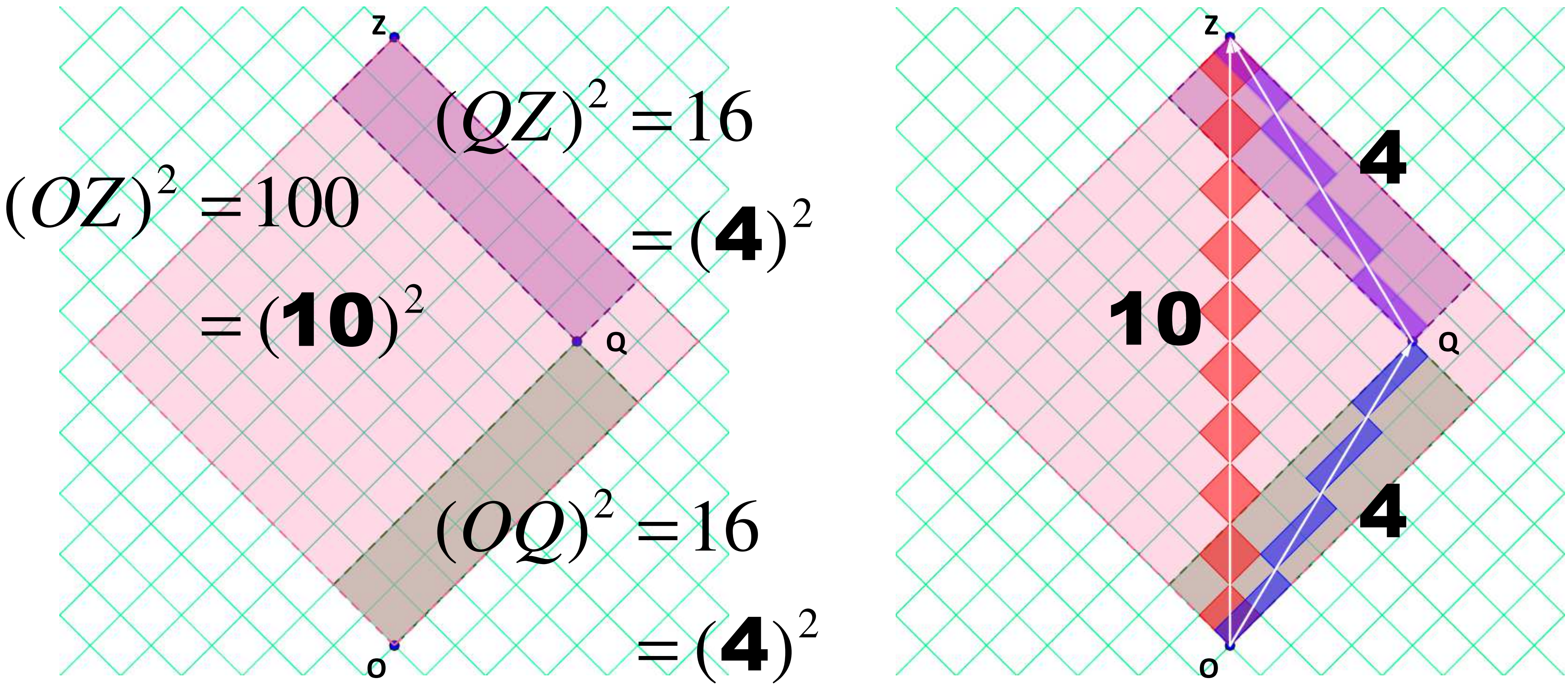


$$s^2 = (4)(4) = 16 = (\mathbf{4})^2$$

$$k^2 = \frac{(4)}{(4)} = 1 = (1)^2$$

$$\beta = \frac{(1)-1}{(1)+1} = 0$$

The Clock Effect/Twin Paradox



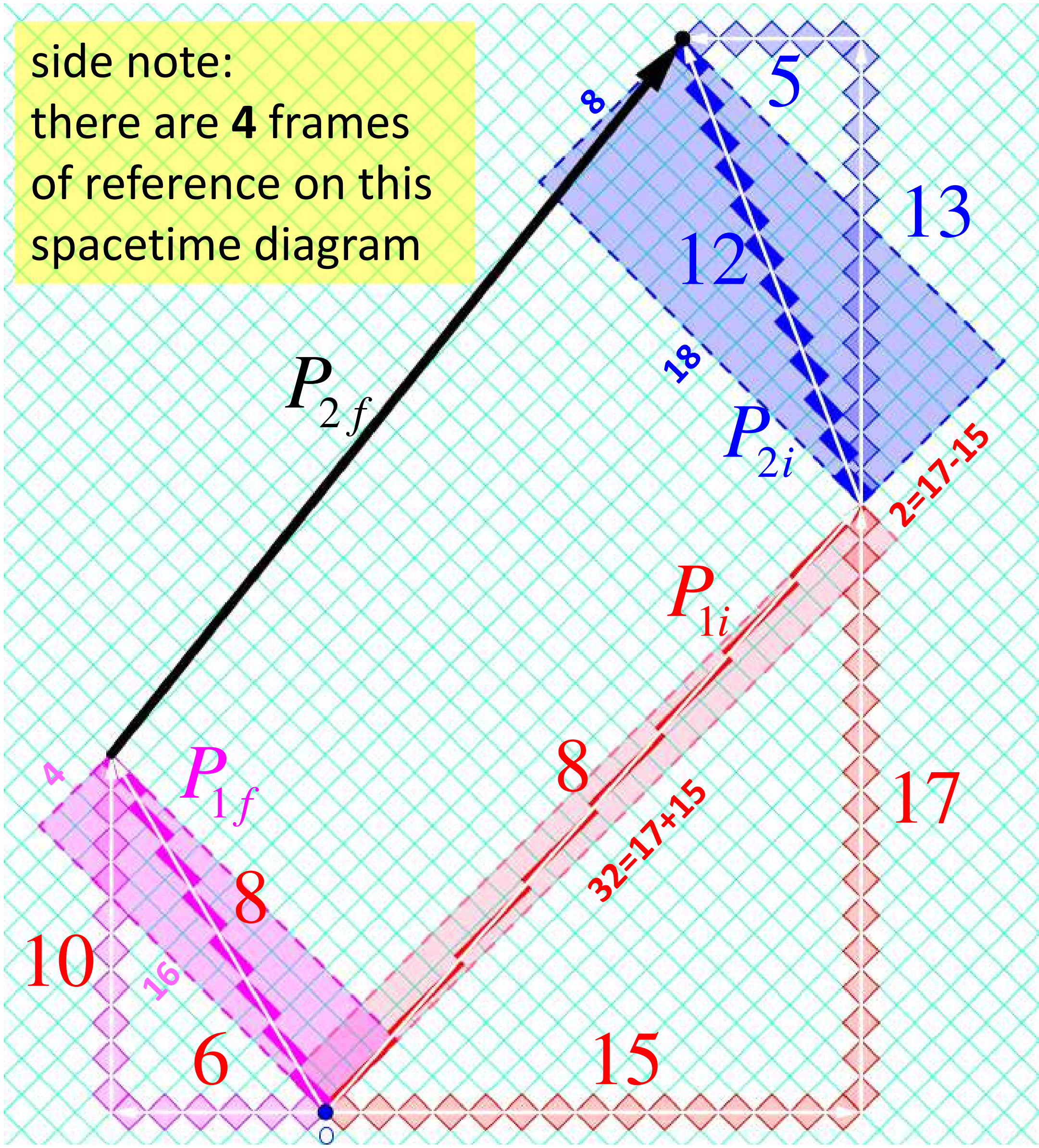
Collision
(in Energy-Momentum Space)

$m_1 = 8 \quad \beta_{1i} = \dfrac{15}{17} \quad \beta_{1f} = -\dfrac{3}{5}$

$m_2 = 12 \quad \beta_{2i} = -\dfrac{5}{13} \quad \beta_{2f} = ?$

- verify $m_{2f} = 12$
- compute β_{2f}

Moore (Six Ideas that Shaped Physics) R10S.3



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$m_2 = 12 \quad \beta_{2i} = -\frac{5}{13} \quad \beta_{2f} = ?$

- verify $m_{2f} = 12$

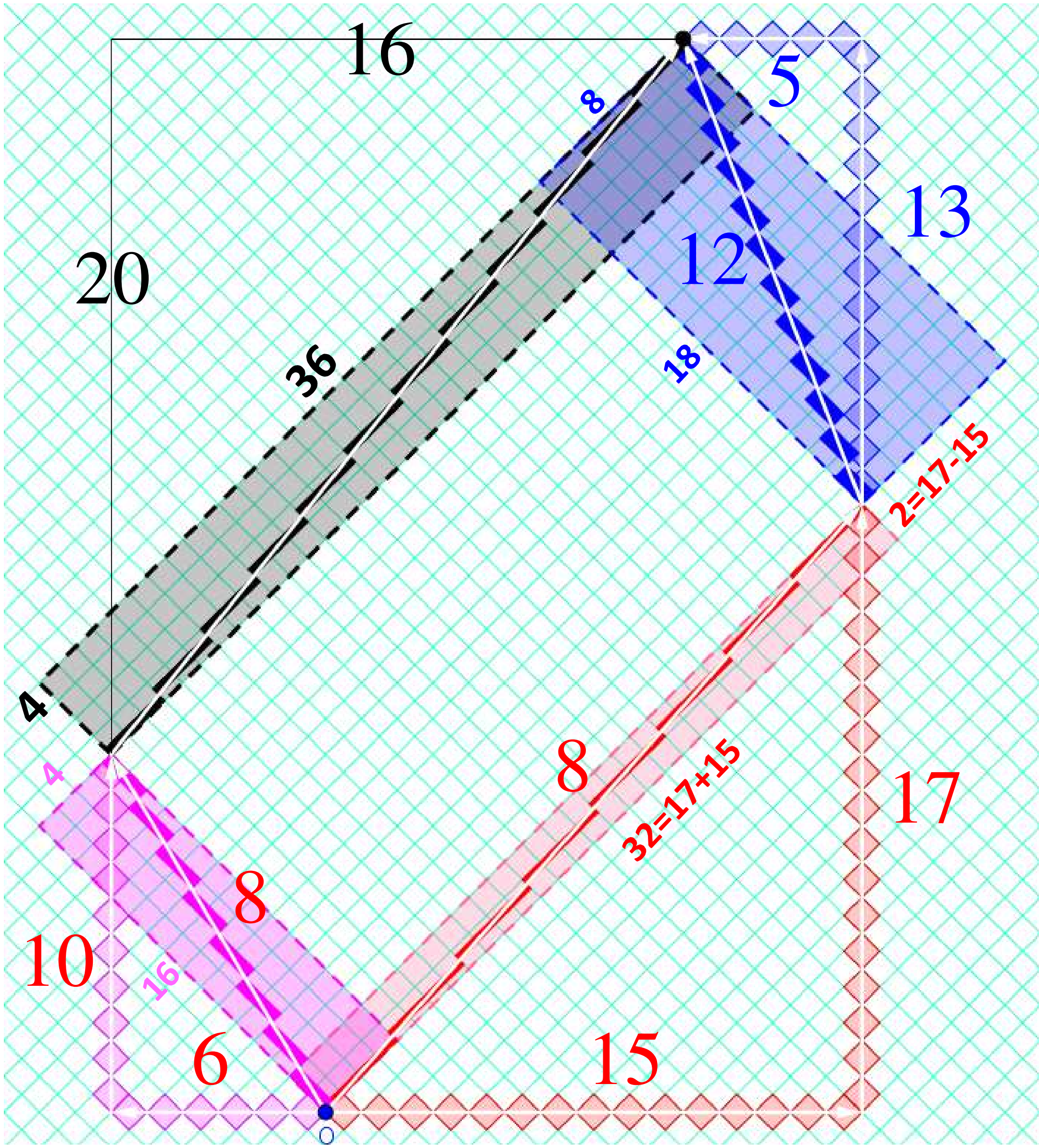
$$s^2 = (36)(4) = 144 = (\mathbf{12})^2$$

- compute β_{2f}

$$k^2 = \frac{(36)}{(4)} = 9$$

$$\beta = \frac{(9)-1}{(9)+1} = \frac{8}{10}$$

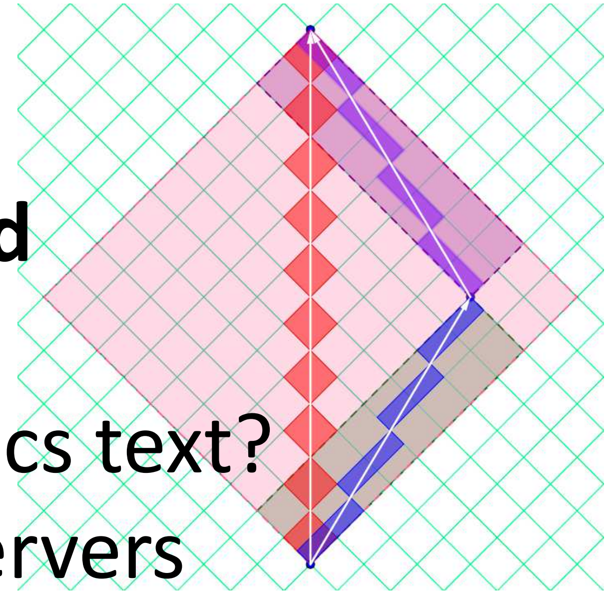
Moore (Six Ideas that Shaped Physics) R10S.3



Relativity on Rotated Graph Paper

- **get folks to use the method**

- chapter for a Modern Physics text?
- uniformly-accelerated observers
- other mathematical properties



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