QUANTUM GRAVITY WITH UNDERGRADUATES

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Abstract. This essay presents a personal perspective on working on quantum gravity research with undergraduates. One project, "Astrophysical constraints on Modified Dispersion Relations", is described. There are observations on the nature of successful projects and on the role of research with undergraduates.

1. Introduction

"Hands are moving in an upwards direction," Ben Auerbach said. It was early in the summer. Ben and I were discussing models of particle propagation in a discrete space. There followed a flood of questions from Ben, one for each "hand". What is a direction field? Why couldn't it change continuously on a discrete space? What does his have to do with a random walk? So Ben's research experience in quantum gravity started.

I've worked with a number of undergraduates over the last six years on projects related to quantum gravity. This experience may offer some guidance for folks interested in working with undergraduates in theoretical physics. I hasten to add that that this essay is by no means a study of undergraduate research in theoretical physics or even quantum gravity, rather for this essay I summarize my observations on working with undergraduates on research. Nearly all of these experiences have begun with a summer research experience. (I have found that students simply do not have time during the semester to dig into a project.) I have found that one can find a good deal of fun in working with the talented, quirky, motivated undergraduate who populate college physics departments. It is rewarding and can be surprisingly productive.

Of course, this type of work has its costs. Research accessible to undergraduates pulls one away from less accessible material and projects. Working with undergraduates in theoretical physics projects is a continual balancing act between work in areas accessible to the students and more sophisticated projects.

Since these projects are largely based on quantum gravity, in the next section there is a little introduction. This is followed by a brief description of one such project. In the final sections, I reflect on what has worked for me and on the role of undergraduate research. Finally, I list the students and projects.

2. (loop) Quantum Gravity

Quantum gravity is the field devoted to finding the microstructure of spacetime. Is space continuous? If the spacetime continuum is replaced, how does the continuum approximation arise? Does spacetime geometry make sense near the initial singularity? Deep inside a black hole? These are questions that the theory of

Date: June 2006.

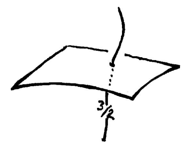
quantum gravity is expected to answer. The root of our search for the theory is a exploration of the quantum foundations of spacetime. At the very least, quantum gravity ought to describe physics at the smallest possible length. This naive length, found by dimensional analysis and the speed of light, Planck's constant, and Newton's constant, is the Planck length, $\ell_P \sim 10^{-35}$ m. The Planck scale is also characterized in terms of energy, $E_P \sim 10^{27}$ eV. The Planck scale is very remote and it is not hard to picture many fields lying between the scales of relativity well-known physics ($\sim 10^{-17}$ m) and the Planck scale. After all, chemistry lies between scales on the order of 1 m and the atomic scale. Whether quantum gravity will yield new fields, or a revolutionary shift in quantum theory, general relativity, or both, remains to be seen.

While the history of quantum gravity is almost as old a quantum mechanics, it is much more sparse. There is no theoretical framework that supporters claim to be complete. Nor is there a generally agreed-upon approach to the problem. Much worse for the field than this, there has been a complete absence of experimental or observational results. These are required for the development of a new physical theory. Partly due to the lack of observations, there are several approaches to quantum gravity. My work with undergraduates has been set in the context of one approach to quantum gravity called loop quantum gravity.

This approach starts with general relativity as Einstein conceived it, introduces new variables, and then quantizes the resulting theory using techniques of the Hamiltonian formulation. It rests on the key principles of non-perturbative quantization and background independence. While the project is by no means complete, the kinematics is established (space!) and dynamics (although incomplete) has an accepted formulation that is used in studies of models systems including cosmology. For reviews of loop quantum gravity, see [2], written for a general audience, or [1], written for physicists.

The kinematics of loop quantum gravity reveals a new, quantum, picture of spatial geometry. Motivated by the need for geometric observables in quantum gravity and placed on mathematically rigorous foundations, the framework for quantum geometry is an echo of an older, combinatorial definition of spacetime. States of spatial geometry are represented as "spin networks". Spin networks are labeled, embedded graphs such as

As electric field lines are flux lines of electric field, spin network edges are flux lines of area. The labeling, or colors in the above spin network, of the edges of the graph are "good quantum numbers" of area. For instance if an edge passes through a surface,



then the measured area has a contribution of $\ell_P^2\sqrt{j(j+1)}$ [3] or, in this example, since $j = 3/2$, $\ell_P^2\sqrt{15}/2$. Vertices are "atoms" of volume and angle. The spectra of area, volume, and angle operators are discrete. For more on spin networks and geometry see [4]. So far, the picture that has been uncovered describes the deep geometry of space as a discrete network of one-dimensional excitations of geometry. It is clear that, at some scale, such a radical transformation of the framework of space will yield observable consequences.

3. Modified dispersion relations with Tomasz Konopka

Early in the spring semester of 2001 Tomasz came by to talk about a possible summer project. He was then in my mathematical methods course and had already done research with one of my colleagues on the solenoid design for aCORN (a cold neutron experiment which will study angular correlation in beta decay). We talked about possible projects and settled on a tentative initial direction: "The problem which this project will address is, 'How can one test the physical ramifications of the discrete spectra of geometric quantities?'" The initial direction was in exploring atomic tests. However during spring break in 2001, I visited the Institute for Gravitational Physics and Geometry. Hugo Morales-Técotl was also visiting at the time and he told me about a series of calculations he was working on with collaborators. They were calculating the effective equations of motion for matter in a state of semiclassical quantum geometry[1]. They could not specify the semiclassical state precisely due to several gaps in the theory of loop quantum gravity. Nevertheless, based on reasonable assumptions they were finding some interesting results. They found modifications to the dispersion relations of special relativity. For instance, the Maxwell's equations enjoyed (suffered?) modifications such that the photon's energy, E_γ, was related to its momentum, p, to leading order in the Planck energy E_P, as (taking $c = 1$) [5]

$$E_\gamma^2 = p^2 + 2\alpha p^{4+2\Upsilon} E_P^{-(2+2\Upsilon)} \pm 4\beta \frac{p^3}{E_P} \tag{1}$$

for unspecified parameters α, β (expected to be of order unity), and Υ. Lorentz invariance was broken! If this was so then there are observable consequences. I took an early draft of the paper home with me and started to puzzle over these possible consequences.

In the beginning of the summer Tomasz and I soon started to explore the consequences of modified dispersion relations and discovered that many people had proposed various effects such as changing particle process thresholds. Over the summer we decided to make a compendium model of all the effects we could find. To place all the particles on the same footing we built a phenomenological model in which the modification to the usual dispersion relations was parameterized by a parameter κ_a for particle species a

$$E_a^2 = p_a^2 + m_a^2 + \kappa_a \ell_p p_a^3. \tag{2}$$

[1] More precisely, a *class* of semiclassical states

In this equation p is the magnitude of the particle 3-momentum, a quantity greater than 0. We used astrophysical data to constrain the parameters of cubic modification (the κ_a's in the above equation) of the model. In the fall semester Tomasz and I were finalizing our list of constraints and identifying allowed regions of parameter space. Tomasz had already written much of his thesis. However, there was an extra bit of drama waiting for us. In the first week of October a group at the University of Maryland, Jacobson, Liberati and Mattingly, posted a conference proceedings [6] on the arXiv on essentially the same model! The calculations were eerily familiar up to a point. But their constraints were completely different. The authors showed that the momentum was distributed asymmetrically among outgoing particles. Tomasz and I had missed this key observation. Quickly - Tomasz's thesis was due in a few weeks - we re-computed the constraints, confirmed the results in the paper and applied them to the other processes we were working on. I wrote up the paper over the Christmas holidays and posted the results in late January [7]. Although this was the end of our engagement in the study of cubic modifications to dispersion relations, the field blossomed with papers appearing in *Phys. Rev. Lett., Nature, Phys. Rev. D* and many other journals.

4. Observations on a successful research experience

For a successful research experience in theoretical physics, there seem to be two necessary characteristics. The students should have the ability to calculate independently and have curiosity about the subject. With these traits, I have found that they have the motivation and independence to make progress on their own. In addition, I found that following contribute to fruitful projects.

Know your student The less successful experiences I have had shared one characteristic: I did not know the student well before the project. On the other hand, successful projects generally proceed well when the student and I already know one another, at least in a small classroom setting.

Meet frequently For many students these projects involve the first full-blown calculations they have attempted. Students may get lost or feel at sea in this new setting. Frequent meetings help them keep on course. When there are more than one student involved, group meetings are helpful since students trade ideas, methods and encouragement.

Let them have a say in the choice of project. This allows the student to take ownership of the project. The most spectacular example of this in my experience was work with Kevin Setter. During the academic year, Kevin often came by on Friday afternoons to talk about quantum gravity. We had extended conversations on Hawking radiation, black hole thermodynamics, and discrete models of spacetime. When he began studying statistical mechanics in the spring he immediately applied his new knowledge to a model of discrete spacetime. I remember well a series of weeks when he would burst into my office to present a new result. Having just learned about he microcanonical ensemble, he calculated the entropy of space in a bounded region. The next week after learning about the canonical ensemble, he applied this new framework to the problem. As the spring semester progressed, his analysis became more sophisticated. By summer he was already well into a research project.

A colleague of mine in mathematics contrasts her two, very different, experiences as an undergraduate. One summer she tackled a challenging problem suggested by a professor. After working "for days" on a small part of the problem, she would meet with the professor who would say, "Oh well we could do this" and in five minutes he repeated and extended her work. Despite the fact that this work resulted in a publication, this experience was discouraging. In her other experience she started a project on her own initiative, extending a class problem. She worked on examples and then moved onto general results in an independent study. She called these general results "conjectures". In the end of the semester her advisor said, "Ill give you an A but I want you to re-write this. What you call conjectures are called theorems and they go in the beginning of a paper." The key aspect to the difference was that on the latter project she could do something, get somewhere on her own.

Plan on teaching Whether teaching general relativity, brushing up on special relativity or statistical mechanics, every one of these projects has involved a significant amount of tutorial-style instruction. This is time directly taken away from one's own work so it feels like - and is - a cost.

Work long term In part to recoup this loss of time and in part to allow the student time to gain deeper understanding and make progress, it is helpful to work on projects over longer times. One way to do this is to extend the project from the summer into the academic year. Since Hamilton physics majors complete a thesis, this is an effective goal that lengthens the students' engagement with their project. A theoretical physics colleague at Occidental signs students up for two years. In the second summer the students help teach the next generation.

Ask "Where am I?" regularly. Working with undergraduates focuses one's time on certain types of projects. It is challenging to strike a balance between time spent on these projects and ones that are inaccessible to those with an undergraduate background. In the ideal world, of course, the students' research is your own. But in quantum gravity, there are a large array of tempting problems, a fraction of which are appropriate for undergraduates.

Meet with collaborators, visit research centers This is true for anyone but especially so for faculty at primarily undergraduate institutions. This came up repeatedly during the Theorists at Undergraduate Institutions conference at KITP in Santa Barbara [8].

5. Reflections on the role of undergraduate research

In the last two decades, there has been a transformation in the role of research at colleges. There generally has been an increased expectation of scholarly activity and undergraduate research. Many colleges now have established programs and dedicated funds to support summer research.

Student-faculty research can be a wonderful experience for both student and faculty. I have had the pleasure to experience both sides of the research experience first hand. I chose my first undergraduate institution partly based on the advertised availability of research experiences. I worked for a bit over a year in an STM lab. The experience was valuable chiefly for a chance to jump into the water. Work in a lab - programming, preparing samples, designing circuit boards, building circuits, imaging, attending conferences - showed me what "research" is. While it was by no means the pinnacle of my undergraduate experience it gave me a chance to observe how science is "done" and gave me an opportunity to work closely with a physicist. The experience showed me both the excitement and, significantly, the frustrations of a laboratory researcher.

As a faculty advisor I have enjoyed working with talented students on problems related to quantum gravity. It may surprise some that these undergraduate research experiences are productive. College physics majors often have the ability and motivation to work on a project with a significant degree of independence. Once underway, students may become "colleagues" in their particular research area rather than simply "students". In one measure of the productivity, among the six completed projects five resulted in publications, the majority of my scholarly output in this time period. In another measure, I have seen excellent students go to graduate school in general relativity, high energy physics, quantum gravity, and string theory. Despite this, some important subtleties should be considered, especially some drawbacks and the role of undergraduate research in faculty hiring and promotion.

I have noticed three drawbacks in my experience. First and foremost, time. The most precious currency in my life is time. There is a prevailing attitude that "real work" fills the gaps in the academic schedule.[2] As mentioned above, introducing new students to quantum gravity, as fun as it is, means less time directly working on research. Second, while students are often paid a stipend (typically $3.5k), since it is difficult for professors at primarily undergraduate institutions to secure funding, faculty frequently work "out of the goodness of their hearts." It is not clear to me that appropriate funding sources exist for this type of research.[3] Third, it is usual in my field for faculty to travel, meet with colleagues, and visit research centers during the summer. The ability to do this is limited by a ten week summer research experience.

There are thorny issues surrounding undergraduate research and faculty promotion. On college campuses there is often little discussion (never mind conclusions!) on what research with undergraduates is. Is it teaching? Is it research? To my mind, it is by no means clear that research with undergraduates is always one or the other. At one college with which I am familiar, student-faculty research in physics comes down solidly on the scholarship side. At another, work with students in the summer is discouraged unless the faculty supervisor can demonstrate that the experience will further their own research effort. Given the variety in these experiences it is perhaps not surprising that no consensus has emerged. The result is that there is a great deal of uncertainty on the role of such work among faculty, especially junior faculty.

Meaningful student involvement does not naturally occur in all fields but only *in certain research areas, at certain times.* Obviously there are many interesting problems that take years of study before one can begin to answer them. These "established" problems are simply not accessible to undergraduates. Physics also has problems that can be readily grappled by an undergraduate. These often occur in emerging fields (though faculty must still "rephrase" the work). Even so, fields may be accessible to undergraduates for only for a limited amount of time. This raises a question, would it be a mistake for a researcher in a field such as quantum computing (which was quite accessible in the early days) to switch out

[2]As Frank Moscatelli of Swarthmore points out, physics may be unique among professions in that once vacations arrive we think, "Great! Now we can *really* get some work done."

[3]Research Corporation is an exception. This funding agency is a strong supporter of work involving undergraduates. The finding is modest, typically $20k - $35k for two years, and they will not pay overhead. Information can be found on their web site.

of the field when the research moves beyond undergraduates? If faculty members made these transitions, it would be difficult to establish themselves in (any of) their chosen field(s). Because of these issues it is difficult to see how research with undergraduates fits in the research rubric.

So, what is the point of research with undergraduates? Are we educating students for graduate school? Should all able students have first-hand experience with research science? Very quickly the question of the role of undergraduate research becomes ensnared with wider questions on the goals of the educational institution. I have found that each experience is quite different. It is not obvious, even for this small number, whether there is a typical project, or clear educational or research goal. Nevertheless I draw two conclusions: The experience was rewarding and my research has been different in terms of both direction and output. While I think these research experiences should be nurtured, there are drawbacks, mainly in the time required. It may be time to make room for this thing that is neither teaching as done at a college nor research as done at an institute. A simple way to do this would be to trade course teaching loads for research work with undergraduates. Leaving aside these wider issues, in my experience working with undergraduates has been rewarding, fun, and even productive.

6. List of students

- Michael Gregg – Hamilton '08
  Summer 2006
- Yubo Lu – Hamilton '07
  Summer 2006
- Julia MacDougall – Hamilton '08
  Summer 2006
- Rob Silversmith – Clinton High School
  Summer 2006
- Alice Francis – Hamilton '06
  Quantum Cosmological Effects on the Primordial Stochastic Gravitational Wave Background
  Senior Project 2005-06
- Benjamin Auerbach – Hamilton '05 (Rochester)
  Anisotropic Mass Effects on a Foucault Pendulum
  Summer and Senior Project 2004
- Sean McGovern – Hamilton '07
  Numerical Experiments in Spin Network Dynamics
  Summer 2004 and Spring 2005
- Nancy Shaw – Hamilton '07
  Energy and Momentum in Modified Special Relativity
  Summer 2004
- Dan Heyman – Hamilton '02
  Is Doubly Special Relativity Consistent with the Relativity Principle?
  Summer and Senior Project 2002
- Julien LeBrun – Paris VI (French exchange student)
  Summer 2002

- Tomasz Konopka – Hamilton '02 (Waterloo and Perimeter Institute)
  Quantum Gravity Effects on Ultra High Energy Particles
  Summer and Senior Project 2001
- Kevin Setter – Swarthmore '02 (Cambridge and Caltech)
  On the Statistical Mechanics of Quantum Geometry
  Summers 2000, 2001 and Honors Thesis
- Michael Seifert – Swarthmore '01 (Chicago)
  Angle and Volume Studies in Quantized Space (Apker Award finalist)
  Summer 2000, Honors Thesis

References

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