AAPT UNITED STATES PHYSICS TEAM
AIP 2021

USA Physics Olympiad Exam

Information About The 2021 USAPhO

- The 2021 USAPhO consists of two 90-minute parts taking place on two separate days:
  - Part A: Monday, April 19 from 4:00 pm to 5:30 pm Eastern Time
  - Part B: Wednesday, April 21 from 4:00 pm to 5:30 pm Eastern Time

  The exam is hosted by AAPT on the platform provided by Art of Problem Solving.
- This year, we require the exam to be proctored by either your high school science/physics teacher, or your parent/guardian.
- Before you start the exam, make sure you are provided with blank paper, both for your answers and scratch work, writing utensils, a hand-held scientific calculator with memory and programs erased, and a computer for you to log into the USAPhO testing page. Then agree to the Honor Policy, and download the USAPhO exam papers.
- At the end of the exam, you have 20 minutes to upload solutions to all of the problems for that part. For each problem, scan or photograph each page of your solution, combine them into a single PDF file, and upload them on the testing platform.
- USAPhO graders are not responsible for missing pages or illegible handwriting. No later submissions will be accepted.

Thank you for participating in the USAPhO this year under such extraordinary circumstances. We hope that you and your family stay safe, and that you continue to encourage more students like you to study physics and try out the $F = ma$ exam hosted by AAPT.

We acknowledge the following people for their contributions to this year's exam (in alphabetical order):

JiaJia Dong, Mark Eichenlaub, Abijith Krishnan, Kye W. Shi, Brian Skinner, Mike Winer, and Kevin Zhou.

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Part A

Question A1

Toffee Pudding

A box of mass m is at rest on a horizontal floor. The coefficients of static and kinetic friction between the box and the floor are μ_0 and μ (less than μ_0), respectively. One end of a spring with spring constant k is attached to the right side of the box, and the spring is initially held at its relaxed length. The other end of the spring is pulled horizontally to the right with constant velocity v_0. As a result, the box will move in fits and starts. Assume the box does not tip over.

a. Calculate the distance s that the spring is stretched beyond its rest length when the box is just about to start moving.

b. Let the box start at $x = 0$, and let $t = 0$ be the time the box first starts moving. Find the acceleration of the box in terms of x, t, v_0, s, and the other parameters, while the box is moving.

The position of the box as a function of time t as defined in part (b) is

$$x(t) = \frac{v_0}{\omega}(\omega t - \sin \omega t) + (1 - r)s(1 - \cos \omega t),$$

where $\omega = \sqrt{k/m}$ and $r = \mu/\mu_0$. This expression applies as long as the box is still moving, and you can use it in the parts below. Express all your answers in terms of v_0, ω, s, and r.

c. Find the time t_0 when the box stops for the first time.

d. For what values of r will the spring always be at least as long as its rest length?

e. After the box stops, how long will it stay at rest before starting to move again?

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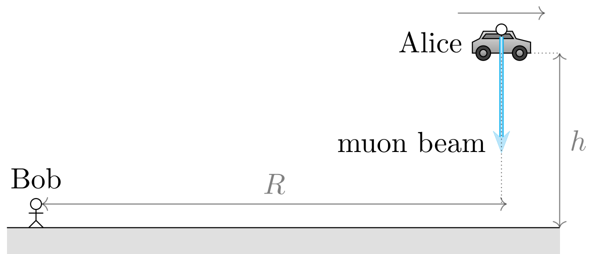
Question A2

Flashlight

Alice the Mad Scientist, travelling in her flying car at height h above the ground, shoots a beam of muons at the ground. Bob, observing from the ground at distance $R \gg h$ from Alice's car, decides to check some facts about special relativity. Assume the muons travel extremely close to the speed of light in Alice's frame.



(side view)

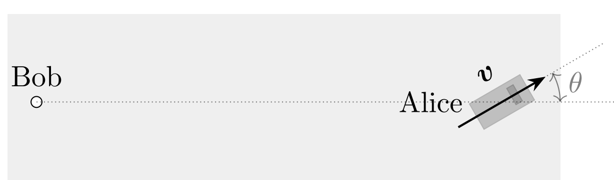
a. Alice's car flies at horizontal speed $v = \beta c$. Alice shoots her muon beam straight down, in her reference frame. Express your answers in terms of β, h, R and fundamental constants.

   i. What is the horizontal velocity of the muons in Bob's reference frame?

   ii. What is the vertical velocity of the muons in Bob's reference frame?

   iii. How long does it take the muons to reach the ground in Bob's reference frame?

Alice's velocity v is directed an angle θ away from Bob. For the rest of the problem, you may additionally express your answers in terms of θ.



(view from above)

b. In Bob's reference frame, how much time is there between when he sees Alice first fire the beam, and when he sees the beam first hit the ground? (Hint: remember to account for the travel time of light to Bob's eyes.)

c. In this part, suppose that $\beta = 1/2$. Does there exist a value of θ so that the time it takes the muons to hit the ground in Alice's frame is equal to the time taken according to Bob's eyes, in Bob's frame? If so, find the value of θ in degrees. If not, briefly explain why not.

d. Suppose Alice is carrying a radio transmitter set to frequency f. To what frequency would Bob have to set his radio receiver in order to receive Alice's transmission?

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Question A3

Electroneering

An electron is a particle with charge $-q$, mass m, and magnetic moment μ. In this problem we will explore whether a classical model consistent with these properties can also explain the rest energy $E_0 = mc^2$ of the electron.

Let us describe the electron as a thin spherical shell with uniformly distributed charge and radius R. Recall that the magnetic moment of a closed, planar loop of current is always equal to the product of the current and the area of the loop. For the electron, a magnetic moment can be created by making the sphere rotate around an axis passing through its center.

a. If no point on the sphere's surface can travel faster than the speed of light (in the frame of the sphere's center of mass), what is the maximum magnetic moment that the sphere can have? You may use the integral:

$$\int_0^\pi \sin^3\theta \, d\theta = \frac{4}{3}.$$

b. The electron's magnetic moment is known to be $\mu = q\hbar/2m$, where $\hbar$ is the reduced Planck constant. In this model, what is the minimum possible radius of the electron? Express your answer in terms of m and fundamental constants.

c. Assuming the radius is the value you found in part (b), how much energy is stored in the electric field of the electron? Express your answer in terms of $E_0 = mc^2$ and the fine structure constant,

$$\alpha = \frac{q^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}.$$

d. Roughly estimate the total energy stored in the magnetic field of the electron, in terms of E_0 and α. (Hint: one way to do this is to suppose the magnetic field has roughly constant magnitude inside the sphere and is negligible outside of it, then estimate the field inside the sphere.)

e. How does your estimate for the total energy in the electric and magnetic fields compare to E_0?

In parts (a) and (b), you can also give your answers up to a dimensionless multiplicative constant for partial credit.

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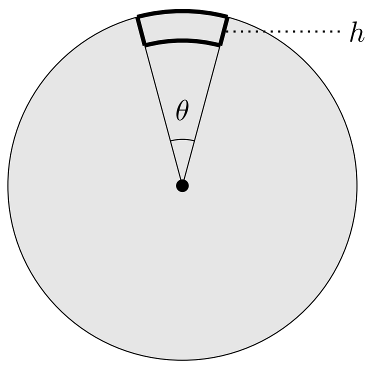
Part B

Question B1

Disk Jockey

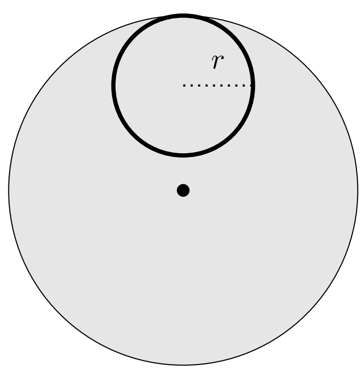
A disk of uniform mass density, mass M, and radius R sits at rest on a frictionless floor. The disk is attached to the floor by a frictionless pivot at its center, which keeps the center of the disk in place, but allows the disk to rotate freely. An ant of mass $m \ll M$ is initially standing on the edge of the disk; you may give your answers to leading order in m/M.

a. The ant walks an angular displacement θ along the edge of the disk. Then it walks radially inward by a distance $h \ll R$, tangentially through an angular displacement $-\theta$, then back to its starting point on the disk. Assume the ant walks with constant speed v.



Through what net angle does the disk rotate throughout this process, to leading order in h/R?

b. Now suppose the ant walks with speed v along a circle of radius r, tangent to its starting point.



Through what net angle does the disk rotate?

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Question B2

Hot Pocket

This question consists of two independent parts.

a. It's winter and you want to keep warm. The temperature is $T_0 = 263$ K outside and $T_1 = 290$ K in your room. You have started a fire, which acts as a hot reservoir at temperature $T_2 = 1800$ K.

You want to add a small amount of heat dQ_1 to your room. The simplest method would be to extract heat $-dQ_{2,\text{dump}} = dQ_1$ from the fire and directly transfer it to your room. However, it is possible to heat your room more efficiently. Suppose that you can exchange heat between any pair of reservoirs. You cannot use any external source of work, such as the electrical grid, but the work extracted from running heat engines can be stored and used without dissipation.

i. What is the minimum heat extraction $-dQ_{2,\text{min}}$ required by the laws of thermodynamics to heat up the room by dQ_1?

ii. Let the "efficiency gain" be the ratio $G = dQ_{2,\text{dump}}/dQ_{2,\text{min}}$. Assuming T_1 is fixed at 298 K, make a graph whose axes are T_0 and T_2, where T_0 varies from 230 K to 290 K, and T_2 varies from 300 K to 2000 K. On the graph, sketch curves corresponding to gain $G = 2$, 5, and 12.

b. When the air at the bottom of a container is heated, it becomes less dense than the surrounding air and rises. Simultaneously, cooler air falls downward. This process of net upward heat transfer is known as convection.

Consider a closed, rectangular box of height h filled with air initially of uniform temperature T_0. Next, suppose the bottom of the box is heated so that the air there instantly reaches temperature $T_0 + \Delta T$. The hot parcel of air at the bottom rises upward until it hits the top of the box, where its temperature is instantly reduced to T_0.

You may neglect any heat transfer and friction between the parcel of air and the surrounding air, and assume that the temperature difference is not too large. In addition, you may assume the height h is small enough so that the pressure P_0 and density ρ_0 of the surrounding air are very nearly constant throughout the container. More precisely, assume that $\rho_0 g h/P_0 \ll \Delta T/T_0 \ll 1$. Express your answers in terms of P_0, g, h, ΔT, and T_0.

i. As a parcel of air moves upward, it accelerates. Find a rough estimate for the average speed v_0 during its upward motion.

ii. In the steady state, warm parcels of air are continuously moving upward from the bottom, and cold parcels of air are continuously moving downward from the top. Find a rough estimate for the net rate of upward energy transfer per area.

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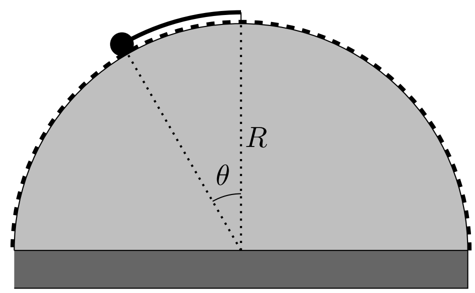
Question B3

The Mad Hatter

A frictionless hemisphere of radius R is fixed on top of a flat cylinder. One end of a spring with zero relaxed length and spring constant k (i.e. the force from the spring when stretched to length ℓ is $-k\ell$) is fixed to the top of the hemisphere. Its other end is attached to a point mass of mass m.

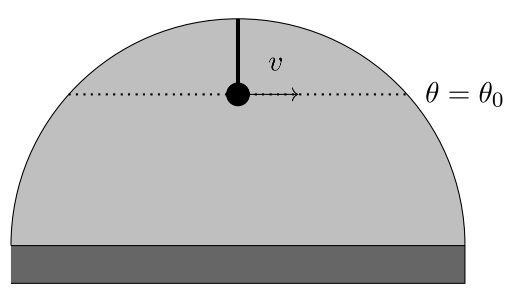


a. The number and nature of the equilibrium points on the hemisphere depends on the value of the spring constant k. Consider the semicircular arc shown above as a dashed line, which is parameterized by angles in the range $-\pi/2 \le \theta \le \pi/2$. Make a table indicating the number of equilibrium points on the arc, and the number that are stable, for each range of k values. A blank table for your reference is given below. (You may need more or fewer rows than shown.)

Range of k ($k_{\text{min}} < k < k_{\text{max}}$)	# of Equilibria	# of Stable Equilibria
$0 < k < ?$		
$? < k < \infty$		

For the rest of the problem, suppose the value of k is such that the mass begins at stable equilibrium on the surface of the hemisphere at angle θ_0. The mass can move on the two-dimensional surface of the hemisphere, but a radially-inward external force prevents it from jumping off the surface.

b. At $t = 0$, the mass is given a speed v along a line of constant latitude $\theta = \theta_0$.



i. Indicate which of the following trajectories the mass takes for a short time after $t = 0$ and briefly explain your reasoning. The differences between the paths are exaggerated.

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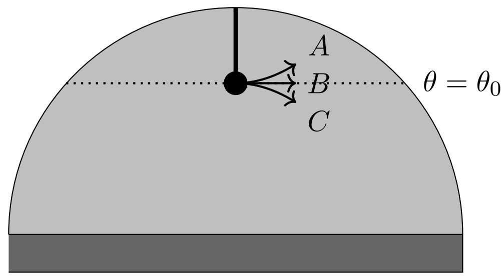
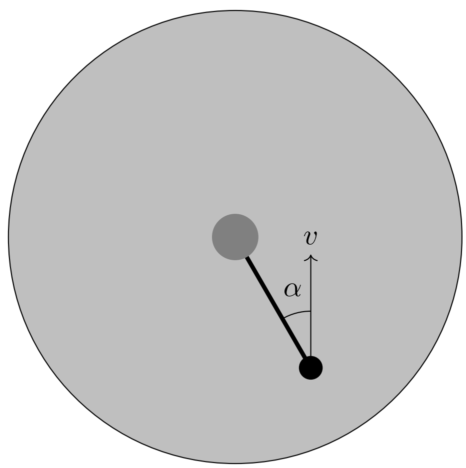


ii. What is the total radial force (i.e., normal to the surface of the hemisphere) on the mass at $t = 0$? Express your answer in terms of m, v, R, g, and θ_0.

c. A cylinder of radius $r \ll R\theta_0$ is placed on top of the sphere. Suppose the mass is launched at an angle α away from the direction of the spring's displacement with kinetic energy K, as shown. What is the maximum angle $\alpha_{\max}$ at which the mass can be launched such that it can still hit the cylinder? Express your answer in terms of K, m, g, θ_0, r, and R. You may assume K is large enough for the mass to reach the cylinder for $\alpha = 0$.



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