Using Think-Pair-Share (TPS) to Promote Quantitative Problem Solving

Edward E. Prather
Center for Astronomy Education (CAE),
Steward Observatory
University of Arizona
eprather@as.arizona.edu

Colin S. Wallace
Department of Physics and Astronomy
University of North Carolina at Chapel Hill
cswphys@email.unc.edu
Think of all the ways you might complete these sentences.

When doing problem solving with my students in class what I like to do is ____________.

When doing problem solving with my students in class its my experience that they ____________.
Title: Using Think-Pair-Share (TPS) to Promote Quantitative Problem Solving

Overview: In this session we will discuss how to create TPS question sequences to motivate and structure student collaborative group quantitative problem solving.

Session Learning Outcomes:
Participants will be able to:
• Identify fundamental problem solving issues to target with TPS questions.
• Describe how TPS question sequences can be used to promote student problem solving abilities.
• Describe how TPS problem solving techniques can be implemented in the lecture portion of the course.
Welcome

Transforming Undergraduate STEM Education

Our Central Goals:

The UA-AAU STEM Education Project seeks to provide thousands of science and engineering majors at the University of Arizona with solid understanding in core STEM disciplines. For this purpose, we are engaged in the redesign of three foundational science courses (general chemistry, introductory biology, and introductory physics/mechanics) and two introductory engineering courses (elements of chemical engineering II and computer programming for engineering applications). The course redesigns are using student-centered and active learning pedagogy to enhance discipline knowledge and conceptual understanding. Three common themes cut across all redesign efforts: 1) promotion of information and quantitative literacy, 2) use of real-life applications in problem solving, and 3) use of models to develop conceptual understanding. The topics covered in the courses are being critically examined to emphasize core disciplinary ideas, problem-solving abilities, critical thinking, and teamwork, to ensure students are provided with a solid foundational understanding.
Insights from the Univ. of Arizona AAU STEM reform effort in Physics

**Traditional Class**
- Three 50 minute lectures per week
  - Focused on introducing concepts and on instructor-led modeling of problem solving
  - Many derivations of equations
- Instructor experienced in teaching PHYS 141 and widely regarded by faculty and students as an excellent lecturer

**Reformed Class**
- Two 50 minute lectures per week
  - Focused on introducing concepts using active engagement instructional strategies and on collaborative group problem solving
  - Minimal derivations of equations
- Each student also attends a 50 minute recitation sections per week
  - Led by graduate TA with assistance from undergraduate peer instructors
  - Students work on collaborative tutorials, which promote conceptual understanding and reasoning abilities
- Instructor experienced in astronomy and physics education research, but teaching PHYS 141 for the first time
Idealized (& shorthand) Implementation of Think-Pair-Share

• Create a cognitively engaging multiple choice question that challenges students thinking and has the ability to foster deep discussion amongst your students.
• Present question to students.
• Ask students to "think" individually about the question; read the question to yourself slowly and silently and go through the reasoning process needed to get the right answer.
• Ask “Do you need more time?”
• Have students anonymously provide their answer to the question simultaneously as a class at the count of three.
• Decide if students should "share" their answers with each other. Is so then…
• “Turn to your neighbor and try to convince them that you are right. Just because both of you have the same answer doesn’t mean you are both right, so be sure to explain your reasoning.”
• Give students a time limit, tell them “Go!”, and start counting down.
• Again have students anonymously provide their answer to the question simultaneously as a class.
• Project the results and correct answer to your students (“And the correct answer is?”).
Guiding Principles for creating TPS Quantitative Problem Solving:

• Find or create “normal” multi-step problems that serve to highlight areas that students struggle with and which model thinking and skills that will be exemplified on the Exams.

• Problem needs to appropriate for students to start after a “brief” lecture on the topic.

• “Unpack” and then “chunk” the problem solution path, creating TPS questions that ask students to “solve” or “generate” important parts of the solution path.

• Try to create a model of what happens in “Help session” but now with all your students, every day in class.

• Students must first attempt the problems on their own, separately, before any group TPS begins.

• Solution path should emphasize the classes “problem solving method” whenever possible.
A 1500 kg car is traveling north through an intersection when it is hit by a 2200 kg SUV traveling east. The two vehicles become locked together during the impact and slide together as one after the collision. The cars slide to a halt at a point 5.39 m east and 6.43 m north of the impact point. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.75$. How fast was each car traveling just before the impact?
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**Solution pathway:**
1) Apply the Work-Energy Theorem to the combined SUV and car after the collision in order to find $v_{S+C}$.
2) Apply the Conservation of Momentum to the $x$- and $y$-components to find $v_S$ and $v_C$.

Is this collision elastic or inelastic? **inelastic**

We **cannot** use work and energy principles to analyze the collision. We **can** use work and energy principles to analyze what happens after the collision.
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**Solution pathway:**
1) Apply the Work-Energy Theorem to the combined SUV and car after the collision in order to find $v_{SC}$.
2) Apply the Conservation of Momentum to the x- and y-components to find $v_x$ and $v_y$.

Write an expression for the Work done by friction in terms of the change in kinetic energy.
A 1500 kg car is traveling north through an intersection when it is hit by a 2200 kg SUV traveling east. The two vehicles become locked together during the impact and slide together as one after the collision. The cars slide to a halt at a point 5.39 m east and 6.43 m north of the impact point. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.75$. How fast was each car traveling just before the impact?

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1) Apply the Work-Energy Theorem to the combined SUV and car after the collision in order to find $v_{S+C}$.
2) Apply the Conservation of Momentum to the $x$- and $y$-components to find $v_x$ and $v_y$.

If $W_f$ is the work done by friction, then which of the following is true?

A) $W_f = \left( \frac{1}{2} m_s v_s^2 + \frac{1}{2} m_C v_C^2 \right) - \frac{1}{2}(m_s + m_C)(v_{S+C})^2$
B) $W_f = \left( \frac{1}{2} m_s v_s^2 + \frac{1}{2} m_C v_C^2 \right) - 0$
C) $W_f = 0 - \frac{1}{2}(m_s + m_C)(v_{S+C})^2$
D) $W_f = \left( \frac{1}{2} m_s v_s^2 + \frac{1}{2} m_C v_C^2 \right) + \frac{1}{2}(m_s + m_C)(v_{S+C})^2 - 0$
E) $W_f = \left( \frac{1}{2} m_s v_s^2 + \frac{1}{2} m_C v_C^2 \right) - \frac{1}{2}(m_s + m_C)(v_{S+C})^2 - 0$
Write an expression for the Work done by friction in terms of the force of friction and displacement of the cars.

Solution pathway:
1) Apply the Work-Energy Theorem to the combined SUV and car after the collision in order to find $v_{S+C}$.
2) Apply the Conservation of Momentum to the $x$- and $y$-components to find $v_s$ and $v_c$.

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What is the work done by friction?
A) $W_f = -\mu_k (m_S + m_C) \cdot g \cdot ((6.43 \text{ m})^2 + (5.39 \text{ m})^2)^{1/2}$
B) $W_f = -\mu_k (m_S + m_C) \cdot g \cdot ((6.43 \text{ m})^2 + (5.39 \text{ m})^2)$
C) $W_f = -\mu_k (m_S + m_C) \cdot g \cdot (6.43 \text{ m} + 5.39 \text{ m})$
D) $W_f = -\mu_k (m_S + m_C) \cdot g \cdot (6.43 \text{ m})$
E) $W_f = -\mu_k (m_S + m_C) \cdot g \cdot (5.39 \text{ m})$
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Solution pathway:
1) Apply the Work-Energy Theorem to the combined SUV and car after the collision in order to find $v_{S+C}$:

$$-\mu_k (m_S + m_C) g \sqrt{(6.43 \text{ m})^2 + (5.39 \text{ m})^2}$$

$$= 0 - \frac{1}{2} (m_S + m_C) (v_{S+C})^2$$

2) Apply the Conservation of Momentum to the $x$- and $y$-components to find $v_s$ and $v_c$. 
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Which of the following is the conservation of momentum applied to the x-components of the momenta:

A) $m_s v_s + m_c v_c = (m_c + m_s) v_{s+c}$

B) $m_s v_s + m_c v_c = (m_c + m_s) v_{s+c} \tan(\theta)$

C) $m_s v_s = (m_c + m_s) v_{s+c} \cos(\theta)$

D) $m_c v_c = (m_c + m_s) v_{s+c} \sin(\theta)$

E) More than one of the above.
A 1500 kg car is traveling north through an intersection when it is hit by a 2200 kg SUV traveling east. The two vehicles become locked together during the impact and slide together as one after the collision. The cars slide to a halt at a point 5.39 m east and 6.43 m north of the impact point. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.75$. How fast was each car traveling just before the impact?

Which of the following is the conservation of momentum applied to the $y$-components of the momenta:

A) $m_s v_s + m_c v_c = (m_c + m_s) v_{s+c}$

B) $m_s v_s + m_c v_c = (m_c + m_s) v_{s+c} \tan(\theta)$

C) $m_s v_s = (m_c + m_s) v_{s+c} \cos(\theta)$

D) $m_c v_c = (m_c + m_s) v_{s+c} \sin(\theta)$

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**Conservation of Momentum for x-components:**

$$m_s v_s = (m_c + m_s) v_{s+c} \cos(\theta)$$

**Conservation of Momentum for y-components:**

$$m_c v_c = (m_c + m_s) v_{s+c} \sin(\theta)$$

What is $\cos(\theta)$?

A) $\frac{5.39}{6.43}$
B) $\frac{6.43}{5.39}$
C) $\frac{5.39}{(5.39 + 6.43)}$
D) $\frac{6.43}{(5.39 + 6.43)}$
E) $\frac{5.39}{(5.39^2 + 6.43^2)^{1/2}}$
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**Solution:**

1) Work-Energy Theorem:

$$v_{S+C}^2 = 2\mu_k g \sqrt{(6.43m)^2 + (5.39m)^2} = 123 \frac{m^2}{s^2}$$

$$v_{S+C} = 11.1 \frac{m}{s}$$

2) Conservation of Momentum:

$$v_C = \frac{(m_C + m_S)v_{S+C} \sin(\theta)}{m_C} = \frac{(1500 \text{ kg} + 2200 \text{ kg})(11.1 \frac{m}{s})}{1500 \text{ kg}} \frac{6.43}{\sqrt{6.43^2 + 5.39^2}} = 21.0 \frac{m}{s}$$

$$v_S = \frac{(m_C + m_S)v_{S+C} \cos(\theta)}{m_S} = \frac{(1500 \text{ kg} + 2200 \text{ kg})(11.1 \frac{m}{s})}{2200 \text{ kg}} \frac{5.39}{\sqrt{6.43^2 + 5.39^2}} = 12.0 \frac{m}{s}$$
Let’s look at some other examples...
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• Try to create a model of what happens in “Help session” but now with all your students, every day in class.

• Students must first attempt the problems on their own, separately, before any group TPS begins.

• Solution path should emphasize the classes “problem solving method” whenever possible.
Exam 1

Percentage of students

Grade on Exam 1 (points)

Reformed (N = 206)

Traditional (N = 234)
Exam 2

Exam 2 results show a comparison between reformed (N = 206) and traditional (N = 226) methods. The average percentage for each item is indicated, with error bars to show variability. The reformed method generally leads to higher average scores, except for Item 2 and Item 3 where traditional methods perform better.

Exam Item: Item 1, Item 2, Item 3, Item 4, Item 5, Item 6, Entire Exam

Average (%)

Reformed (N = 206)
Traditional (N = 226)
Exam 2

Reformed-Trad. Scores (%)

Exam Item

-20.00 -15.00 -10.00 -5.00 0.00 5.00 10.00 15.00 20.00

Item 1  Item 2  Item 3  Item 4  Item 5  Item 6  Entire Exam
Exam 2

Percentage of students

Grade on Exam 2 (points)

Reformed (N = 206)

Traditional (N = 226)
Exam 3

Reformed (N = 203)
Traditional (N = 230)
Exam 3

Reformed - Trad. Scores (%)

Exam Item

- Item 1
- Item 2
- Item 3
- Item 4
- Item 5
- Item 6
- Entire Exam
Exam 3

Percentage of students

Grade on Exam 3 (points)

- Reformed (N = 203)
- Traditional (N = 230)
A Unique Instructional Framework for Elevating Students’ Quantitative Problem Solving Abilities

Edward E. Prather
Center for Astronomy Education (CAE), Steward Observatory
University of Arizona

Colin S. Wallace
Department of Physics and Astronomy
University of North Carolina at Chapel Hill