Accurate Determination of the Volume of an Irregular Helium Balloon
Jack Blumenthal, Rafaela Bradvica, and Katherine Karl

Citation: The Physics Teacher 51, 93 (2013); doi: 10.1119/1.4775529
View online: http://dx.doi.org/10.1119/1.4775529
View Table of Contents: http://scitation.aip.org/content/aapt/journal/tpt/51/2?ver=pdfcov
Published by the American Association of Physics Teachers

Articles you may be interested in
Uncertainties in RECIST as a measure of volume for lung nodules and liver tumors
Med. Phys. 39, 2628 (2012); 10.1118/1.3701791

A comment regarding the Coriolis effect
Phys. Teach. 48, 212 (2010); 10.1119/1.3361981

How Does It Sound? Young Interferometry Using Sound Waves
Phys. Teach. 46, 410 (2008); 10.1119/1.2981287

MicroReviews by the Book Review Editor: Don't Try this at Home: The Physics of Hollywood Movies — Your favorite action movie sequences Deconstructed, Demystified & Debunked: Adam Weiner
Phys. Teach. 46, 320 (2008); 10.1119/1.2909766

Book Reviews MicroReviews by the Book Review Editor: The Medicine of ER: Or, How We Almost Die: Harlan Gibbs, M.D., and Alan Duncan Ross
Phys. Teach. 45, 190 (2007); 10.1119/1.2709695
Accurate Determination of the Volume of an Irregular Helium Balloon

Jack Blumenthal, Rafaela Bradvica, and Katherine Karl, Mayfield Senior School, Pasadena, CA

In a recent paper, Zable\textsuperscript{1} described an experiment with a near-spherical balloon filled with impure helium. Measuring the temperature and the pressure inside and outside the balloon, the lift of the balloon, and the mass of the balloon materials, he described how to use the ideal gas laws and Archimedes’ principal to compute the average molecular mass and density of the impure helium. This experiment required that the volume of the near-spherical balloon be determined by some approach, such as measuring the girth. The accuracy of the experiment was largely determined by the balloon volume, which had a reported uncertainty of about 4%.

We describe here a simple experiment to determine the volume of an irregularly shaped helium-filled balloon to an uncertainty of less than 2%. Our method also relies on the use of Archimedes’ principle and the ideal gas laws. Its data analysis can be carried out at different levels, depending on the depth of the error analysis, and on the assumptions made in using the gas laws. For example, in the first approximation the ideal gas constant for dry air can be used, while in a more accurate calculation the gas constant for the specific conditions of relative humidity and ambient temperature can be determined.

The practical methodology of taking the uncertainty in each of the measurements and determining the uncertainty in the final calculations is a skill that is often overlooked in high school.

We attach the 100-g weight to the balloon with the ribbon as shown in Fig. 1, place the weight on the balance, and note the reading of the balance. The temperature of the air next to the balloon is also recorded. This completes the measurements needed to calculate the balloon volume. For a more accurate calculation of the volume, one needs to take into account the mole fraction of moisture in the air. For this, we either need to measure the relative humidity of the air or use the value reported at a nearby weather station.

Calculation of balloon volume

Let $W_{r}$, $W_{bag}$, $W_{100}$, and $W_{scale}$ be the weights of the ribbon, the balloon material, the 100-g mass, and the balance reading when the inflated balloon is attached (converted to newtons by multiplying by $g$), respectively. In our experiments these were, in that order: 0.00265 N, 0.1202 N, 0.9823 N, and 0.9526 N.
Denote by \(W_{\text{He}}\) and \(W_{\text{buoy}}\) the weight of the helium in the balloon and the buoyancy force, respectively. The latter is the weight of the surrounding air displaced by the balloon. We calculate these as follows:

Assume that air at ambient conditions will accurately follow an ideal gas equation of state:

\[
P_{\text{air}} V = m_{\text{air}} R_{\text{air}} T, \tag{1}
\]

where \(P_{\text{air}}\) is the atmospheric pressure in kPa, \(V\) the balloon volume, \(R_{\text{air}}\) the gas constant for dry air in kJ/kgK, and \(T\) is the temperature in kelvin. The dry air gas constant is 0.2870 kJ/kgK. The ambient temperature was 298.0 K, and the air pressure was 101.26 kPa (as estimated from nearby weather stations).

Inserting all the data, we get for \(W_{\text{buoy}}\):

\[
W_{\text{buoy}} = (11.62V) \text{ N.}
\]

A similar application of the gas law for helium, with \(R_{\text{He}} = 2.077\) kJ/kgK, gives:

\[
W_{\text{He}} = (1.605V) \text{ N.}
\]

The force balance in Fig. 1 is given by:

\[
W_{\text{scale}} = W_r + W_{\text{bag}} + W_{100} + W_{\text{He}} - W_{\text{buoy}}. \tag{2}
\]

Inserting all of the data into Eq. (2) and solving for \(V\), we get the volume of the balloon as 0.0152 m\(^3\) or 15.2 L.

**Uncertainty analysis**

**Systematic error**

In the above calculation of the balloon volume, we used the gas constant for dry air and ignored the effect of moisture in the air. This is probably fine for first-year physics or chemistry classes, since the systematic error is small (about 1.3% at 70% relative humidity and 298 K ambient temperature) and the classes probably haven’t been introduced to the concept of ideal gas partial pressure and mole fraction. For more advanced classes, a correction to the ideal gas constant for dry air can be made as follows:

The gas constant \(R\) for a particular ideal gas or gas mixture is inversely proportional to the molecular mass of the gas. Thus, we must first find the molecular mass for moist air.

\[
P_{\text{H}_2\text{O}} = P_T X_{\text{H}_2\text{O}},
\]

\[
P_T = \text{atmospheric pressure} = 101.26 \text{ kPa}
\]

\[
P_{\text{H}_2\text{O}} = \text{partial pressure of water vapor at 298 K and 70% relative humidity} = 2.22 \text{ kPa.}
\]

\[
X_{\text{H}_2\text{O}} = \text{mole fraction of water vapor in the air.}
\]

The partial pressure of moisture in the air is determined by multiplying the vapor pressure of water at 298 K (3.165 kPa) times the relative humidity (70%) to get 2.22 kPa.

From the above equation, the mole fraction of moisture in the air is 0.0219. Thus the moist air is 2.19% moisture and 97.81% dry air. From this we can calculate that the average molecular mass of the moist air is 28.73 g/mole when the molecular mass of dry air is 28.97. This results in the moist air gas constant being 0.2894 kJ/kgK rather than the dry air value of 0.2870. If we use all of the measured values for temperature, pressure, etc. and include the \(R\) value for moist air, then the calculated volume of the balloon is 15.4 L rather than 15.2 L, which was obtained assuming dry air. Thus using the dry air gas constant under these conditions introduces a systematic error of about 1.3%.

**Random error**

The uncertainty in the measured parameters of this experiment are assumed as follows:

\[
T = \pm 0.5 \degree C
\]

\[
P = \pm 0.1 \text{ kPa (not measured on site)}
\]

\[
W_{\text{bag}} = \pm 0.0001 \text{ N}
\]

\[
W_{\text{ribbon}} = \pm 0.0001 \text{ N}
\]

\[
W_{100} = \pm 0.0001 \text{ N}
\]

\[
W_{\text{scale}} = \pm 0.0005 \text{ N (reading jumps)}
\]

If the actual temperature is 0.5 K higher than measured, then the actual balloon volume will be higher than the calculated volume. Similarly if the actual mass values of the bag, ribbon, and weight are greater than measured, then the actual balloon volume will be higher than calculated. For pressure, if the actual pressure is 0.1 kPa lower than measured, then the actual volume will be higher than calculated.

The uncertainty in the \(W_{\text{scale}}\) measurement was much larger than the uncertainty in the other mass measurements because when the balloon is attached to the weight sitting on the balance, the air currents cause the readings to jump around. A smaller value of \(W_{\text{scale}}\) leads to a larger balloon volume.

When we apply the maximum uncertainty to all of the measurements that will result in a bigger balloon volume and also use the value of \(R\) for moist air, then we get the biggest possible balloon volume when plugging these values into the above equations. This results in a maximum balloon volume of 15.5 L as compared to the original calculated value of 15.2 L. Thus, the maximum volume uncertainty in the example experiment and calculation, using the \(R\) value for dry air, is about 1.9% (15.2 ± 0.3 L). If we make the correction for moisture in the air, then the maximum uncertainty is about 0.7% (15.4 ± 0.1 L).

**Conclusions**

The experiment and calculations described herein would be valuable to beginning experimental science classes, excluding the section pertaining to uncertainty analysis and the
calculation of the gas constant for moist air. It provides a simple, interactive way for students to learn about Archimedes’ principle and the ideal gas laws. More advanced classes can perform the detailed uncertainty analysis and make the corrections to the gas constant for moisture in the air. At either level, this technique provides a way to determine the volume of an irregular shape with an uncertainty of less than 2%.

**Instructor’s Note:** The experiment described herein and the accompanying calculations and error analysis were developed by two seniors and their instructor at Mayfield Senior School in Pasadena, CA, as part of a special study program introducing engineering thermodynamics.

**Reference**


**Jack Blumenthal** has been a math and science teacher at Mayfield Senior School in Pasadena, CA, for the past 13 years. Prior to that he was an engineer and executive at TRW for 36 years. Blumenthal is the only member of the National Academy of Engineering currently teaching in high school.

jack.blumenthal@mayfieldsenior.org

**Rafaela Bradvica** currently is a senior at Mayfield Senior School and is taking the most advanced math class available to her grade level. She is president of the Mayfield Respect Life Club and the secretary of the National Honor Society. She has been a member of the varsity basketball team for the past four years. She is also involved with the advising council and is a newspaper journalist.

rafaela.bradvic@mayfieldsenior.org

**Katherine Karl** is a senior currently attending Mayfield Senior School. She is an avid learner particularly in the math and sciences and is interested in pursuing a career in medicine. She is also president of the Modern Movie Club. She has been a dedicated water polo player and is captain of the varsity team for her high school, and has played for the Rose Bowl Water Polo team for three years.

kmkarl2012@yahoo.com