AAPT UNITED STATES PHYSICS TEAM
AIP 2007

2007 Semi-Final Exam

INSTRUCTIONS
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems.
- After you have completed Part A, you may take a break.
- Then work Part B. You have 90 minutes to complete both problems.
- Show all your work. Partial credit will be given.
- Start each question on a new sheet of paper. Be sure to put your name in the upper right-hand corner of each page, along with the question number and the page number/total pages for this problem. For example,

  Doe, Jamie
  A1 – 1/3
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's, or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- Do not discuss the contents of this exam with anyone until after March 27th.
- Good luck!

Possibly Useful Information - (Use for both part A and for part B)

Gravitational field at the Earth's surface	g = 9.8 N/kg
Newton's gravitational constant	G = 6.67 x 10^{-11} N·m²/kg²
Coulomb's constant	k = $1/4\pi\varepsilon_o$ = 8.99 x 10^{9} $N{\cdot}m^2/C^2$
Biot-Savart constant	k_m = $\mu_o/4\pi$ = 10^{-7} T·m/A
Speed of light in a vacuum	c = 3.00 x 10^{8} m/s
Boltzmann's constant	k_B = 1.38 x 10^{-23} J/K
Avogadro's number	N_A = 6.02 x 10^{23} $(mol)^{-1}$
Ideal gas constant	R = $N_A k_B$ = 8.31 J/(mol·K)
Stefan-Boltzmann constant	σ = 5.67 x 10^{-8} $J/(s{\cdot}m^2{\cdot}K^4)$
Elementary charge	e = 1.602 x 10^{-19} C
1 electron volt	1 eV = 1.602 x 10^{-19} J
Planck's constant	h = 6.63 x 10^{-34} J·s = 4.14 x 10^{-15} eV·s
Electron mass	m = 9.109 x 10^{-31} kg = 0.511 MeV/c^2
Binomial expansion	$(1 + x)^n \approx 1 + nx$ for $\lvert x\rvert << 1$
Small angle approximations	$\sin\theta \approx \theta$
	$\cos\theta \approx 1 - {}^1/_2\,\theta^2$

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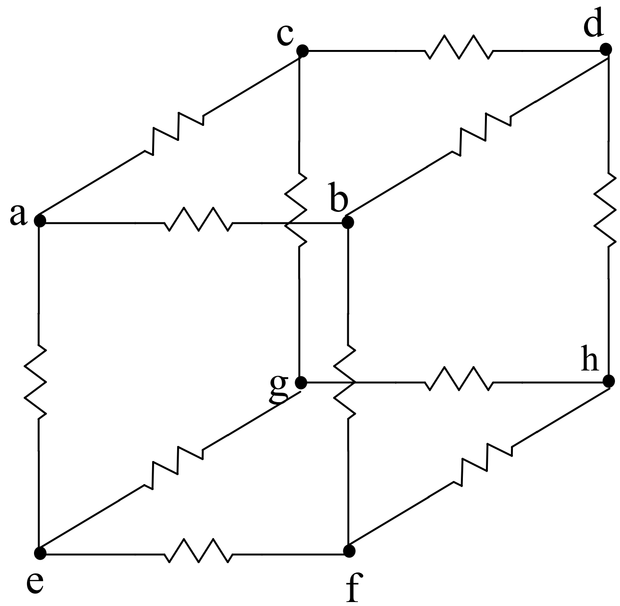
Part A

A1. A group of 12 resistors is arranged along the edges of a cube as shown in the diagram below. The vertices of the cube are labeled *a*-*h*.



a. (13 pts) The resistance between each pair of vertices is as follows:

$R_{ab} = R_{ac} = R_{ae} = 3.0\ \Omega$

$R_{cg} = R_{ef} = R_{bd} = 8.0\ \Omega$

$R_{cd} = R_{bf} = R_{eg} = 12.0\ \Omega$

$R_{dh} = R_{fh} = R_{gh} = 1.0\ \Omega$

What is the equivalent resistance between points *a* and *h*?

b. (12 pts) The three 12.0 Ω resistors are replaced by identical capacitors. $C_{cd} = C_{bf} = C_{eg} =$ 15.0 μF. A 12.0 V battery is attached across points *a* and *h* and the circuit is allowed to operate for a long period of time. What is the charge (Q_{cd}, Q_{bf}, Q_{eg}) on each capacitor after this long period of time?

A2. A simple gun can be made from a uniform cylinder of length L_0 and inside radius r_c. One end of the cylinder is sealed with a moveable plunger and the other end is plugged with a cylindrical cork bullet. The bullet is held in place by friction with the walls of the cylinder. The pressure outside the cylinder is atmospheric pressure, P_0. The bullet will just start to slide out of the cylinder if the pressure inside the cylinder exceeds P_{cr}.

a. There are two ways to launch the bullet: either by heating the gas inside the cylinder and keeping the plunger fixed, or by suddenly pushing the plunger into the cylinder. In either case, assume that an ideal monatomic gas is inside the cylinder, and that originally the gas is at temperature T_0, the pressure inside the cylinder is P_0, and the length of the cylinder is L_0.

(8 pts) i. Assume that we launch the bullet by heating the gas without moving the plunger. Find the minimum temperature of the gas necessary to launch the bullet. Express your answer in terms of any or all of the variables: $r_c, T_0, L_0, P_0,$ and P_{cr}.

(8 pts) ii. Assume, instead that we launch the bullet by pushing in the plunger, and that we do so quickly enough so that no heat is transferred into or out of the gas. Find the length of the gas column inside the cylinder when the bullet just starts to move. Express your answer in terms of any or all of the variables: $r_c, T_0, L_0, P_0,$ and P_{cr}.

b. (9 pts) It is necessary to squeeze the bullet to get it into the cylinder in the first place. The bullet normally has a radius r_b that is slightly larger than the inside radius of the cylinder; $r_b - r_c = \Delta r$, is small compared to r_c. The bullet has a length $h \ll L_0$. The walls of the cylinder apply a pressure to the cork bullet. When a pressure P is applied to the bullet along a given direction, the bullet's dimensions in that direction change by

$$\frac{\Delta x}{x} = \frac{-P}{E}$$

for a constant E known as Young's modulus. You may assume that compression along one direction does not cause expansion in any other direction. (This is true if the so-called Poisson ratio is close to zero, which is the case for cork.)

If the coefficient of static friction between the cork and the cylinder is μ, find an expression for P_{cr}. Express your answer in terms of any or all of the variables: $P_0, \mu, h, E, \Delta r,$ and r_c.

A3. A volume V_f of fluid with uniform charge density ρ is sprayed into a room, forming spherical drops. As they float around the room, the drops may break apart into smaller drops or coalesce into larger ones. Suppose that all of the drops have radius *R*. Ignore inter-drop forces and assume that $V_f \gg R^3$.

(10 pts) a. Calculate the electrostatic potential energy of a single drop. (Hint: suppose the sphere has radius *r*. How much work is required to increase the radius by *dr*?).

(4 pts) b. What is the total electrostatic energy of the drops?

Your answer to (b) should indicate that the total energy increases with *R*. In the absence of surface tension, then, the fluid would break apart into infinitesimally small drops. Suppose, however, that the fluid has a surface tension γ. (This value is the potential energy per unit surface area, and is positive.)

(4 pts) c. What is the total energy of the drops due to surface tension?

(7 pts) d. What is the equilibrium radius of the drops?

A4. A nonlinear circuit element can be made out of a parallel plate capacitor and small balls, each of mass m, that can move between the plates. The balls collide inelastically with the plates, dissipate all kinetic energy as thermal energy, and immediately release the charge they are carrying to the plate. Almost instantaneously, the balls then pick up a small charge of magnitude q from the plate; the balls are then repelled directly toward the other plate under electrostatic forces only. Another collision happens, kinetic energy is dissipated, the balls give up the charge, collect a new charge, and the cycle repeats. There are n_0 balls per unit surface area of the plate. The capacitor has a capacitance C. The separation d between the plates is much larger than the radius r of the balls. A battery is connected to the plates in order to maintain a constant potential difference V. Neglect edge effects and assume that magnetic forces and gravitational forces may be ignored.

(5) a. Determine the time it takes for one ball to travel between the plates in terms of any or all of the following variables: m, q, d, and V.

(5) b. Calculate the kinetic energy dissipated as thermal energy when one ball collides inelastically with a plate surface in terms of any or all of the following variables: m, q, d, and V.

(5) c. Derive an expression for the current between the plates in terms of the permittivity of free space, ε_0, and any or all of the following variables: m, q, n_0, C, and V.

(5) d. Derive an expression for the effective resistance of the device in terms of ε_0, and any or all of the following variables: m, q, n_0, C, and V.

(5) e. Calculate the rate at which the kinetic energy of the balls is converted into thermal energy in terms of ε_0, and any or all of the following variables: m, q, n_0, C, and V.

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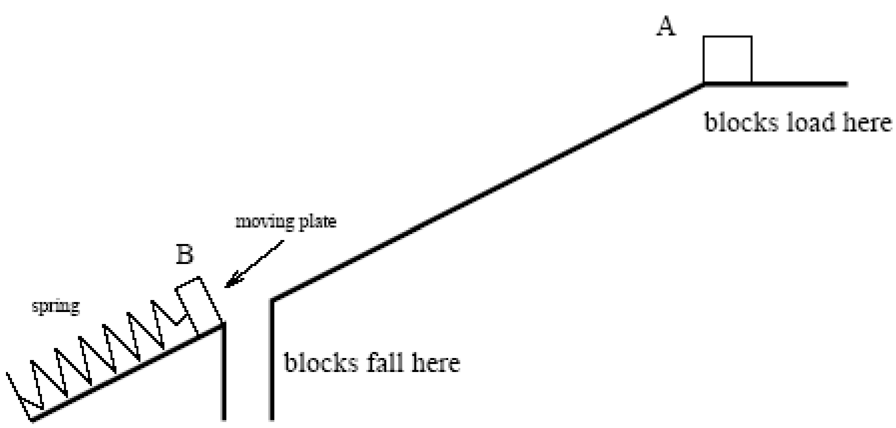
Part B

B1. A certain mechanical oscillator can be modeled as an ideal massless spring connected to a moveable plate on an incline. The spring has spring constant k, the plate has mass m, and the incline makes an angle θ with the horizontal. When the system is operating correctly, the plate oscillates between points A and B in the figure, located a distance L apart. When the plate reaches point A it has zero kinetic energy, but then trips a small lever that instantaneously loads a block of mass M onto the plate. The block and plate then move down the incline to point B, where the force from the spring stops the plate. At this point, the block falls through a hole in the incline, allowing the plate to move back up under the force of the spring. Upon returning to point A it collects another block, and the cycle repeats. Both the plate and the block have a coefficient of friction μ with the incline for both kinetic and static friction. It is reasonable that the motion in either direction is simple harmonic in nature.



(10 pts) a. Let μ_c be the critical value of the coefficient of friction where the block will just start to slide under the force of gravity on an incline (without the spring acting on it). Then let $\mu = \frac{\mu_c}{2}$. Find μ in terms of g, the acceleration of free fall, and any or all of the following variables: θ and M.

(14 pts) b. In order for this system to work correctly, it is necessary to have the correct ratio between the mass of the block and the mass of the plate. These masses are chosen so that the downward moving block and plate just stop at point *B* while the upward moving plate just stops at point *A*. Find the ratio $R = \frac{M}{m}$.

(13 pts) c. The system delivers blocks to point *B* with period T_0, until the blocks run out. After that, the plate alone oscillates with a period T'. Find the ratio $\frac{T_0}{T'}$.

(13 pts) d. The plate only oscillates a few times after delivering the last block. At what distance up the incline, measured from point *B*, does the plate come to a permanent stop?

B2. A model of the magnetic properties of materials is based upon small magnetic moments generated by each atom in the material. One source of this magnetic moment is the magnetic field generated by the electron in its orbit around the nucleus. For simplicity, we will assume that each atom consists of a single electron of charge –*e* and mass m_e, a single proton of charge +*e* and mass $m_p \gg m_e$, and that the electron orbits in a circular orbit of radius *R* about the proton.

a. **Magnetic Moments**.

Assume that the electron orbits in the *x*-*y* plane.

(3 pts) i. Calculate the net electrostatic force on the electron from the proton. Express your answer in terms of any or all of the following parameters: e, m_e, m_p, R, and the permittivity of free space, ε_0, where

$$\varepsilon_0 = \frac{1}{4\pi k}.$$

(*k* is the Coulomb's Law constant).

(5 pts) ii. Determine the angular velocity ω_0 of the electron around the proton in terms of any or all of the following parameters: e, m_e, R, and ε_0.

(8 pts) iii. Derive an expression for the magnitude of the magnetic field B_e due to the orbital motion of the electron at a distance $z \gg R$ from the *x*-*y* plane along the axis of orbital rotation of the electron. Express your answer in terms of any or all of the following parameters: e, m_e, R, ω_0, z, and the permeability of free space μ_0.

(4 pts) iv. A small bar magnet has a magnetic field far from the magnet given by

$$B = \frac{\mu_0}{2\pi}\frac{m}{z^3},$$

where *z* is the distance from the magnet on the axis connecting the north and south poles, *m* is the magnetic dipole moment, and μ_0 is the permeability of free space. Assuming that an electron orbiting a proton acts like a small bar magnet, find the dipole moment *m* for an electron orbiting an atom in terms of any or all of the following parameters: e, m_e, R, and ω_0.

b. **Diamagnetism**.

We model a diamagnetic substance to have all atoms oriented so that the electron orbits are in the *x*-*y* plane, exactly half of which are clockwise and half counterclockwise when viewed from the positive *z* axis looking toward the origin. Some substances are predominantly diamagnetic.

(3 pts) i. Calculate the total magnetic moment of a diamagnetic substance with N atoms. Write your answer in terms of any or all of the following parameters: e, m_e, R, N, and μ_0.

(6 pts) ii. An external magnetic field $\vec{B}_0 = B_0\hat{z}$ is applied to the substance. Assume that the introduction of the external field doesn't change the fact that the electron moves in a circular orbit of radius R. Determine $\Delta\omega$, the change in angular velocity of the electron, for both the clockwise and counterclockwise orbits. Throughout this entire problem you can assume that $\Delta\omega \ll \omega_0$. Write your answer in terms of e, m_e, and B_0 only.

(6 pts) iii. Assume that the external field is turned on at a constant rate in a time interval Δt. That is to say, when $t = 0$ the external field is zero and when $t = \Delta t$ the external field is $\vec{B}_0$. Determine the induced emf $\mathcal{E}$ experienced by the electron. Write your answer in terms of any or all of the following parameters: e, m_e, R, N, B_0, ω_0, and μ_0.

(6 pts) iv. Verify that the change in the kinetic energy of the electron satisfies $\Delta K = e\,\mathcal{E}$. This justifies our assumption in (ii) that R does not change.

(6 pts) v. Determine the change in the total magnetic moment Δm for the N atoms when the external field is applied, writing your answer in terms of e, m_e, R, N, μ_0 and B_0.

(3 pts) vi. Suppose that the uniform magnetic field used in the previous parts of this problem is replaced with a bar magnet. Would the diamagnetic substance be attracted or repelled by the bar magnet? How does your answer show this?