Apparatus Competition

2017 Summer Meeting of the American Association of Physics Teachers

AAPT PHYSICS EDUCATION 2017 SUMMER MEETING JULY 22-26 Cincinnati, OH

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Table of Contents

Apparatus Competition

2017 AAPT Summer Meeting

Cincinnati, OH

Apparatus Title: Candle Oscillator (Candelator)

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Abstract (50-75 words): This is a very simple yet elegant demonstration of an oscillatory system with chaos and periodic orbits. The system consist of two candles connected by a sleeve with porcelain bearing rings that allows them to rotate around the center of mass with very reduced friction. Friction is the key parameter that can damp the oscillations and when almost eliminated, the dynamics can become chaotic and display measurable periodic orbits.

Background: The candle see-saw is a simple, cost effective and highly instructive hands-on physics experiment that dates back to the early 19th century. It is composed of a candle which is free to undergo vertical oscillations about a perpendicular rod running through its center. When the candle is lit at both ends, wax begins to drip unevenly from each end (at a rate that depends on the angle of inclination) and the candle begins to oscillate. ***What is new in our system is the designed sleeve and mount that minimizes friction; this allows larger oscillations. In the case of large inclination angles, the mass loss becomes nonlinear and the candle see-saw can display chaotic motion, which had not been shown before.***

In order to study the chaotic dynamics of the candle-see saw, extreme care must be taken in setting up the experiment. Repeated trial and error to find an optimal setup can be time consuming and expensive. Over the course of our research, we have developed a hands-on laboratory experiment kit that allows the user to quickly and easily get started with the experiment.

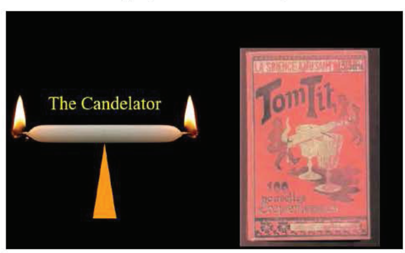

Figure 1. Picture of the Candelator and of the book That describes it: Tom Tit. Arthur Good. La Science Amusante Librairie Larousse, Paris, 1890, Vol. 1 to 3.

Our kit is very inexpensive to make and includes two major components: a candle sleeve and a candle track. Each is briefly described below.

Construction of Apparatus: The candle track consists of two parallel sheets of clear plexiglass held in place by two blocks that form its base. The rod running through the candle sleeve is placed on the top of the track, and when lit, the candles are free to oscillate vertically between the plexiglass sheets. Because friction can't be completely eliminated by the ceramic bearings, the candles are free to roll along the top of the plexiglass uninhibited. The use of Plexiglas allows clear view of the full range of dynamics.

The system consists of five pieces (as shown in Figure 2), two using a 3D printer (item 3 and 4) and the rest are easy to obtain with a total cost of less than $30.00

1) A metallic rod
2) Two porcelain bearing rings
3) A sleeve to hold the candles with a hole in the center where the two bearing rings are inserted
4) Two support blocks with slits to insert the clear plexiglass sheets (only one shown)
5) Two transparent plexiglass sheets (not shown)

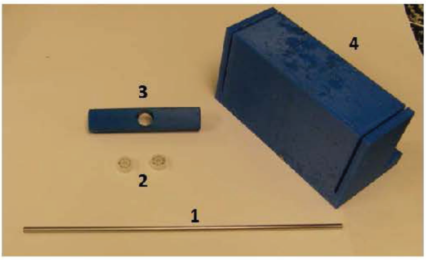

Figure 2. Elements of the kit.

The setup once it is assembled looks as in the illustration in Figure 3.

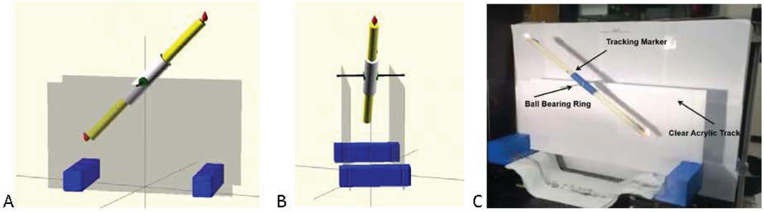

Figure 3. Shows the system with an illustration at a (A) semi-frontal view, (B) semi-lateral view, and (C) a real picture of the experimental system.

Use of Apparatus: Balance the candlator horizontally and then burn both ends. As the wax melts, at some point one candle will lose a little bit more wax than the other and will the Candelator will tilt. Since the flame is always perpendicular to the table due to gravity, the heavier side (pointing down) will have more contact with the flame than the lighter one (pointing up), thus the heavier side will lose faster wax than the lighter one and they will reverse position. This sets up an oscillation that can grows in time (see figure 4A). The oscillations remain actually relatively small if there is friction in the rod, however as friction is minimized (via the bearing rings) the oscillations can grow large and the candle can actually rotate 360 degrees and then come back or not, leading to chaotic dynamics (see figure 4B)

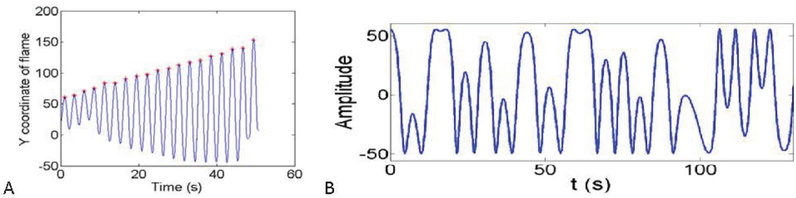

Figure 4A. Oscillations measured with a camera and a tracking algorithm of a point that can be identified in the candle. The oscillations can be seen to grow in amplitude. B) Amplitude will grow and can become chaotic as shown, once friction is reduced.

Figure 5 illustrates the beginning of the dynamics shown in Figure 4A with a time lapse of an experiment showing in A, the starting point (when the candlelator is horizontal and both ends have been set on fire). B and C shows half of an oscillation where the candle with the dot oscillates from -30 to +30 degrees from the horizontal. This rhythmic oscilations become chaotic once one of the candles goes over and tips to the other side making one full 360 rotation as shown in figure 6.

Figure 5. Time lapse of an experiment. Note the black dot at the beginning of one of the candles, used to track the candle's angle/position. A) Initial horizontal position, notice the dot on the right candle. B-C) oscillation of the candle showing two snap shots half oscillation apart. Notice dot going from -30 to +30 degrees.

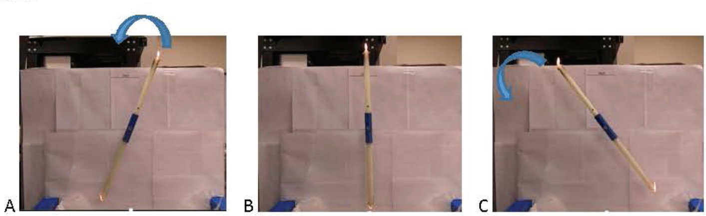

Figure 6. Time lapse showing in one of the oscillations the candle doing one over the top rotation.

This Physics Demo for oscillations and chaos can be used at many levels of teaching. From middle and high school where it is possible to explain how a system will produce oscillations and how chaos looks like. To undergraduate/graduate level where students can actually calculate Lyapunov exponents (figure 7, obtained using the algorithm from H. Kantz, Phys. Lett A 185, 77, 1994) and search for periodic orbits (as shown in Figure 8 using recurrence plots).

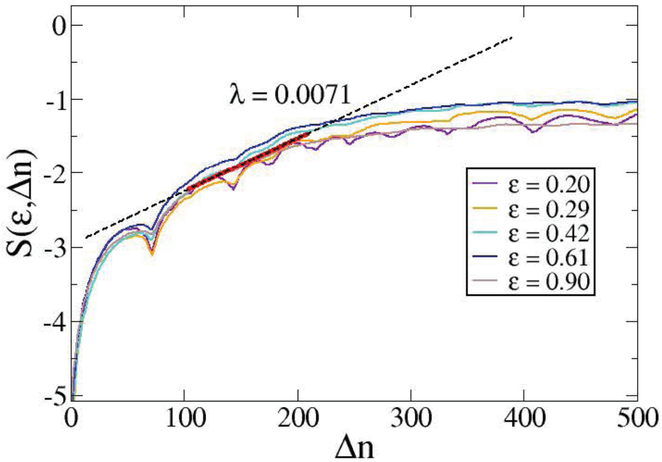

Figure 7. Calculation of the Lyapunov Exponent (positive) demonstrate that the system is chaotic.

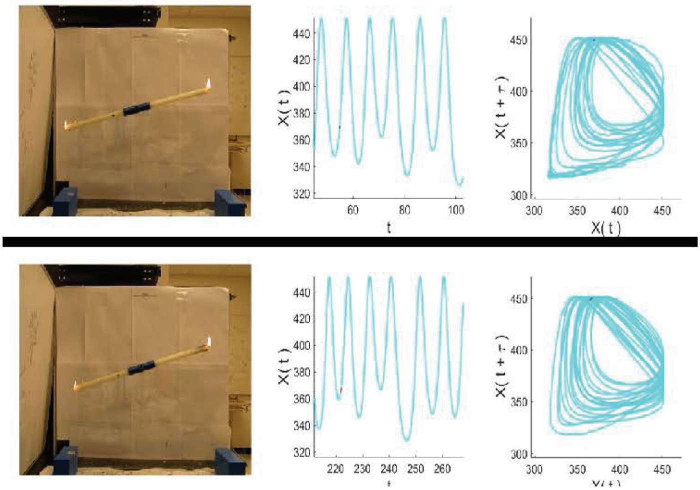

Figure 8. Embedded periodic orbits. Chaotic systems are characterized by the existence of periodic orbits where the system can transiently visit as shown here with two trajectories that occurred in this experiment from time 55s to 97 and then again from 222s to 264s. Where the two solutions where close to a periodic orbit of about 42 seconds and about 4 oscillations.

We have used the Candlelator in demonstrations at several middle and high schools around Atlanta, as well as the Atlanta Science Festival and in a couple of workshops organized for undergraduates as shown in Figure 9. And as a problem in a my computational physics course for undergraduates, where students observe the dynamics and try to simulate the dynamics of the system using basic principles and study the effect of friction and the mass loss as a function of angle.

Figure 9. Undergraduates studying the dynamics of the candle oscillator.

Apparatus Title: Current Balance
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Abstract: Moving charge carriers (e.g. electrical currents) have a force applied to them by magnetic fields. This invisible force can change the reading on an inexpensive scale. Drilled and countersunk rare earth magnets can be inexpensively placed on a scale to measure the force.

Current Balance

(https://tinyurl.com/y8c9w9m3)

Live the dream. Change the weight of an object without even "touching" it.

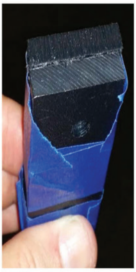
Moving charge carriers (e.g. electrical currents) have a force applied to them by magnetic fields. This invisible force can change the reading on an inexpensive scale.

Materials

- Two 1-inch (25 mm) x 1-inch (25 mm) x 3/16-inch (4.5 mm) rare earth magnets with dual countersunk holes like this one from Magnets4Less. ($5 at $2.50 each)
- Flat head screws and nuts to match (The above magnet has two #6 countersunk holes, so you'll need four #6-32 1 ¼ inch machine screws and four #6-32 nuts.)
- Two approximately 1 ¼-inch (32 mm) x 3-inch (75 mm) x ¼-inch (6 mm) sturdy pieces of plastic made of Delrin or other sturdy plastic
- Two ¼-20 x 1 3/4-inch (50 mm) x ¼ inch (6 mm) nylon thumb screws or similar
- Six nylon nuts to match the thumb screws
- Masking tape
- Drill and drill bits (#6 machine screw needs 5/32 inch bit and a ¼ inch thumb screws need a ¼ inch bit)
- 1 m (or more) ⅛ inch (3 mm) aluminum armature wire cut into pieces approximately 30 cm long
- 1 roll electrical tape or heat shrink tubing
- 3 test leads with alligator clips
- 2 alkaline D-cell batteries (do NOT use rechargeable batteries) and battery holder OR current-limited power supply
- 10 A multimeter

Magnet Assembly

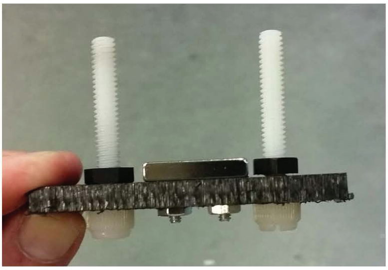
1. Place the two pieces of plastic one on top of each other and tape firmly together. By drilling both pieces at once, the holes will line up better afterwards.



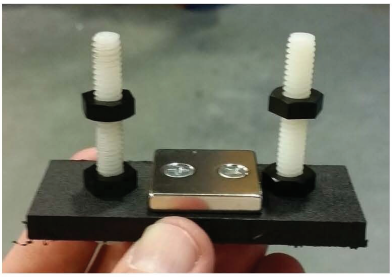
2. Set one of the magnets on top of the and mark the position of the holes on the tape.

3. Approximately one quarter of the way from each of the ends mark a hole for the nylon screw.
4. Drill all the holes with appropriate bits.
5. Pull off the tape. Attach the magnets with the #6 machine screws and nuts so that the faces of the magnets will attract each other.

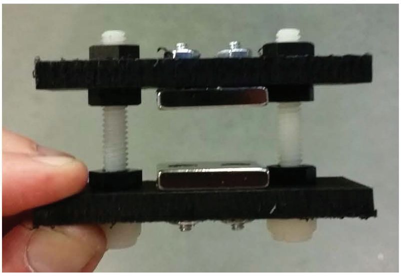
6. Push the nylon screws all the way through the holes on one piece. The heads should be on the opposite side of the piece from the magnet. Tighten nuts to secure the screws.



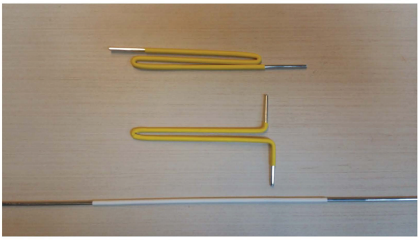
7. Take another nut and twist it down the screw so that it is approximately ½ inch (12 mm) from the other nut. Repeat with the other nut on the other screw.



8. Place the other piece of plastic so that the magnets face each other. Add the remaining nuts to hold everything in place.



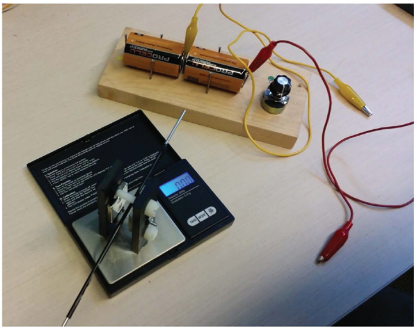
9. Cover all but the ends of several pieces of armature with either shrink wrap or electrical tape.
10. Leave one wire straight, bend another wire into a tight U, bend a third wire into a zig-zag, and so on.



11. I've used a very simple battery holder, but obviously you can use some other design.

To Do and Notice

Set the magnet assembly on its short side on the scale and tare. Run the straight armature wire between the faces of the magnet. Attach the multimeter in series with an alkaline D-cell or current limited power supply set to measure current. Be sure that the multimeter is set to an appropriate range. It is good practice to start with a higher range and then work to lower ranges. If you are using batteries, you will probably want the 10 A scale, given D-cells usual internal resistance. (The multimeter isn't shown. The potentiometer shown is optional. You will need a high power pot or only use it for short periods of time.)

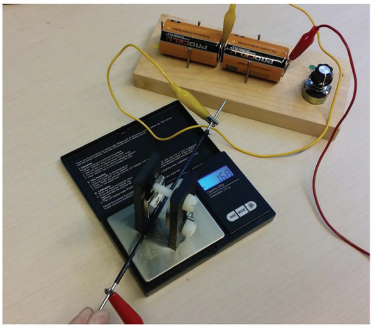


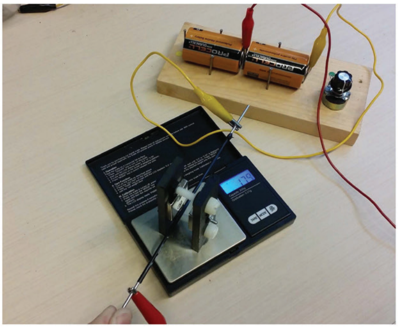
Connect the leads, measure the change in mass recorded by the scale, and record the current. Change the amount of current by either adding another battery in series or adjust the power supply.

Place other configurations of the wire between the poles of the magnet and record the change in mass recorded on the scale and the current.

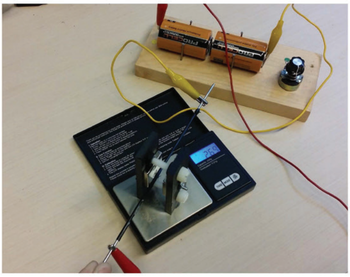
What's Happening?

Moving charges in a magnetic field have a force applied to them that changes their direction at a right angle to both the field and their direction of motion. Depending on the arrangement of the wire and the magnet in your arrangement, this force either presses down or up on the charges. When *A* applies a force to *B*, an equal in magnitude but opposite in direction force is applied to *A*, as described by Newton's Third Law. So, if the electrons in the wire have a force applied to them by the magnets, an equal force is applied to the magnets. This changes the weight (well, mass) displayed on the scale. Changing the direction of the current will change the sign of the change in weight and so will changing the direction of the magnetic field.

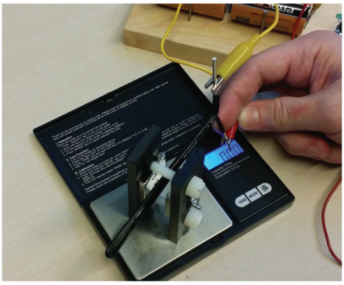




Increasing the current increases the number of electrons per second which also increases the force.



Two wires with the same current in opposite directions will have little force since the opposite force of each wires will cancel the other out.



Like most scales, this scale’s metric setting displays units of mass instead of force. The scale of course can only measure force, but it’s electronics assume that it is in a typical gravitational field. To convert the mass reading back to force, first convert the reading to kilograms (if necessary) and then multiply by 9.8 N/kg.

Cost

Item	Number	At	Total
Countersunk magnets	2	$2.50	$5.00
#6-32 x 1 ¼ inch flat head screws	4	$0.05	$0.20
#6-32 nuts	4	$0.02	$0.08
Delrin	1	$9.95	$9.95
¼-20 x 1 ¾ nylon thumb screw	2	$0.10	$0.20
Nylon ¼-20 nuts	6	$0.07	$0.42
Aluminum armature wire	1	$6.95	$6.95
Electrical tape	1	$3.00	$3.00
Test leads with alligator clips set	1	$6.59	$6.59
D-cells	2	$4.00	$4.00
Battery holder	1	$2.00	$2.00
Mulitmeter	1	$8.00	$8.00
Total			**$44.39**

Apparatus Title: Kelvin Current Balance

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Abstract: We have developed a compact and low-cost 3D-printed version of the Kelvin current balance. Two current coils, each wound with a large number of turns 30 AWG enameled-magnet wire, are positioned parallel and coaxially to each other on a low-cost digital scale using 3-D printed stand. The measured magnetic force and theory agree within 3%. In addition, the mutual inductance can also be determined and agrees with theory to within 2%.

I. Introduction:

Figure 1: (a) Digital scale and lower current circle of Kelvin current balance

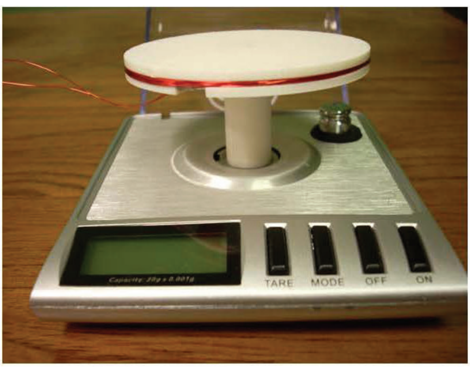
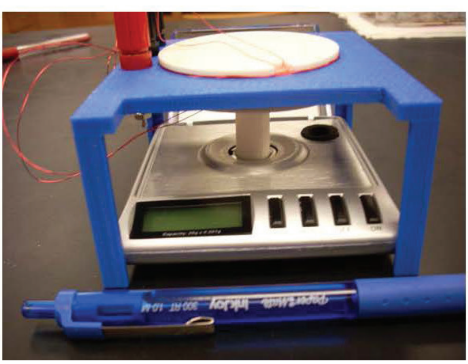


Figure 1: (b) Compact Kelvin Current Balance



(This report is produced in MathCad). Almost every physics major does an experiment with a current balance. With most current balances, students measure the force of repulsion between two straight and parallel current-carrying wires, one fixed, and one moving, by measuring the gravitational force which just balances the magnetic force. The magnetic force is proportional to the product of the currents in the two conductors, and depends on the permeability of free space.

Typical currents in such balances often exceeds ten amperes which is far too large to be safe and accessible for multiple laboratory stations in survey (introductory) courses. Due to the complexity and expense of such commercially available current balances however, not many introductory courses include a current balance experiment with the corresponding laboratory component.

Lord Kelvin's design dating from 1893 (see "*Absolute Measurements In Electricity And Magnetism*", Volume II, pg 407-412, Andrew Gray, M.A., MacMillan and Co., 1893) and later, the Rayleigh design, took advantage of the significantly enhanced (for the same current) force between parallel and coaxial current circles (circular loops) due to superposition of the magnetic field.

We have developed a modern, compact and low-cost version of the Kelvin current balance that is well-suited for use in any instructional physics laboratory. We use two current circles, each wound with a large number of turns (typically 90, with 30 AWG enameled-magnet wire) and positioned coaxially and lying in parallel planes. As shown in Figure 1a above, one such current circle rests upon a miniature low-cost digital scale which measures the force of repulsion (or attraction). The current balance is completed with the other current circle positioned above the first on a supporting stand as shown above in Figure 1b above.

A precise spacing is achieved using low-cost 3-D printing technology. Two separations (9.0 and 16.0 mm) have been used thus far to measure the force (chosen to be either repulsive or attractive) as a function of modest currents (one ampere or less is sufficient for accurate results). ***The measured forces are found to be in excellent agreement (within 3%) with a classical electromagnetism analytical expression.*** The particular digital scale we have used thus far, is the low-cost *AWS Gemini-20* with a capacity of 20 gm in 0.001 gm increments, although the 3D-printed rings and stand can can be easily adapted for other use with other digital miniature scales.

In addition, the mutual inductance between the circles can also be conveniently measured using a signal generator and a two channel oscilloscope, and compared to theory. **Again, as will be shown below, excellent agreement (within 1.5%) is also found between the measured mutual inductance and the theoretical value, for the spacing of 6.2 mm**. Measurements of the mutual inductance as a function of separation could also be obtained using insertable spacers into the top supporting stand. As discussed below, there is an intrinsic relationship between the mutual inductance and the current balance force.

II. Kelvin Current Balance

From Sec. 7.19, W.R. Smythe, "Static and Dynamic Electricity" (Hemisphere Publishing, NY, 1989), 3rd edition, provides an analytical expression for the force between two identical current loops. For two identical current circles of common radius **a** and separated by a distance **c**, the force is expressed in the complete elliptic functions of the 1st and 2nd kind with modulus k. The complete elliptic integrals are defined as (and readily computed using Excel, MathCad, etc.):

$$K(k) := \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1 - k^2 \cdot \sin(\theta)^2}}\, d\theta$$

The complete elliptic integral of the first kind.

$$E(k) := \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \cdot \sin(\theta)^2}\, d\theta$$

The complete elliptic integral of the second kind.

The force (SI units) between current circles (each wound with N turns), each carrying current I is then given by :

$$F(a,c,I,N) := 4\cdot\pi\cdot10^{-7}\cdot(N\cdot I)^2\cdot\frac{c}{\sqrt{4a^2+c^2}}\cdot\left[-K\left(\sqrt{\frac{4\cdot a^2}{4a^2+c^2}}\right)+\left(\frac{2a^2+c^2}{c^2}\right)\cdot E\left(\sqrt{\frac{4\cdot a^2}{4a^2+c^2}}\right)\right]$$

For convenience, one can convert to an effective mass in grams.

$$m_{eff}(a,c,I,N) := \frac{F(a,c,I,N)\cdot1000}{9.8}$$

Figure 2 below shows a plot of our data and theory for the magnetic force versus current between two circles having an average radius of 2.86 cm, a separation of 1.6 cm, and 90 turns of AWG 30 magnet wire wound on each circle. The circles are wired in series but one may change the relative direction in the two to achieve either a repulsion (opposite circulations) or attraction (same circulations). The forces were measured with the *Gemini-20***.** At 1 A, the power dissipated in each circular loop of resistance 6 ohms is estimated to be 6 watts. It is advisable to take a single measurement with the current turned on, for durations less than 30 seconds to avoid overheating of the plastic rings. **We note the excellent agreement between theory and data. 102415 - average radius = (2.+2.5)/4"=2.86 cm**

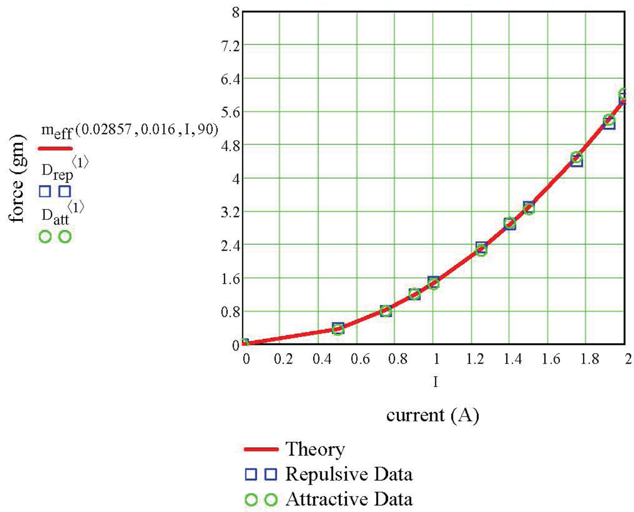
Data matrices:

(Current (A), Force (gm))

$$D_{rep} := \begin{pmatrix} 0 & 0 \\ 0.5 & .39 \\ 0.75 & 0.8 \\ 0.9 & 1.2 \\ 1 & 1.5 \\ 1.25 & 2.33 \\ 1.4 & 2.89 \\ 1.5 & 3.3 \\ 1.75 & 4.4 \\ 1.92 & 5.3 \\ 2 & 5.9 \end{pmatrix} \qquad D_{att} := \begin{pmatrix} 0 & 0 \\ 0.5 & 0.35 \\ 0.75 & 0.801 \\ 0.9 & 1.21 \\ 1 & 1.45 \\ 1.25 & 2.25 \\ 1.4 & 2.92 \\ 1.5 & 3.25 \\ 1.75 & 4.5 \\ 1.92 & 5.4 \\ 2 & 6.02 \end{pmatrix} \qquad \frac{2.25\cdot in}{2} = 2.857\cdot cm$$

$$I := D_{att}^{\langle 0 \rangle}$$

Figure 2: Magnetic force (gm) versus current (A)



We also have performed similar measurements for the repulsive force versus current for a smaller separation of 9 mm. These are shown in Figure 3 below. (The minimum separation possible is estimated to be 5 mm given the constraints of the plastic grooved rings and the supporting stand thickness). We note that for smaller separation, the simpler model of parallel current-carrying wires alsc becomes applicable, this result is typically introduced to students in survey courses. To apply the mode of long parallel current-carrying wires, one can simply assume the loops of the current circles are effectively unwound to equal the length of the parallel wires of the model. Figure 2 also shows this theoretical model's predicted force versus current. Agreement with this model on linear parallel current for a spacing of 9 mm is found to be accurate to within 10%. At a spacing of 5 mm, the expected agreement would improve to 4%. (*Comment: We have realized there has been a 2 mm offset between the axes of the coils in this data, and new data will be taken shortly with this error corrected and with even closer separation. We expect better agreement to result and also (an important consideration for the introductory physics student) with the simpler theory resulting from assumed parallel and straight current-carrying wires.*)

For parallel wires, the force (in grams) versus current is given by the simpler expression::

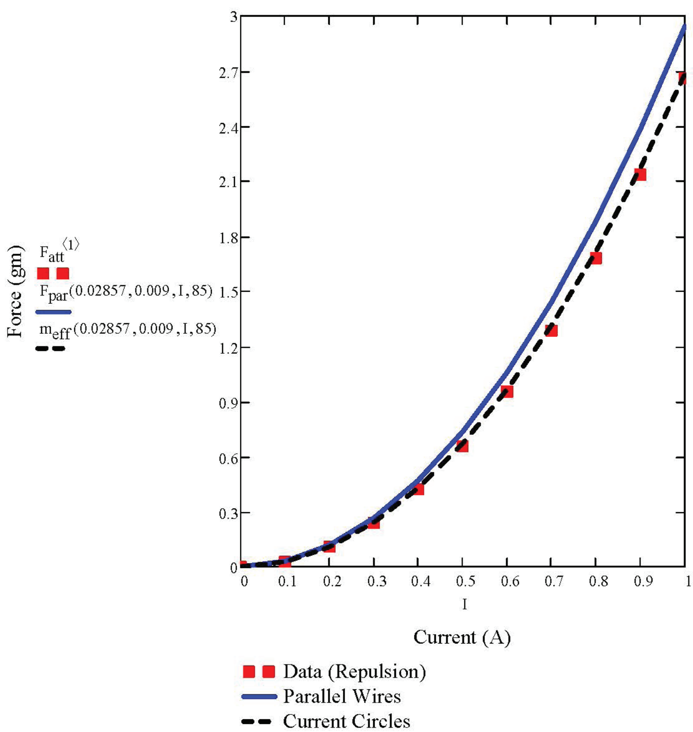
$$F_{par}(r,c,I,N) := \frac{4\cdot\pi\cdot10^{-7}\cdot r I^2 N^2}{c}\cdot\frac{1000}{9.8}$$

$$F_{rep} := \begin{pmatrix} 0 & 0 \\ 0.1 & 0.025 \\ 0.2 & 0.100 \\ 0.3 & 0.228 \\ 0.4 & 0.409 \\ 0.5 & 0.614 \\ 0.6 & 0.920 \\ 0.7 & 1.250 \\ 0.8 & 1.65 \\ 0.9 & 2.08 \\ 1. & 2.6 \end{pmatrix} \qquad F_{att} := \begin{pmatrix} 0 & 0 \\ 0.1 & 0.027 \\ 0.2 & 0.109 \\ 0.3 & 0.238 \\ 0.4 & 0.422 \\ 0.5 & .656 \\ 0.6 & .953 \\ 0.7 & 1.285 \\ 0.8 & 1.68 \\ 0.9 & 2.135 \\ 1. & 2.66 \end{pmatrix}$$

$$I := F_{rep}^{\langle 0 \rangle}$$

PRECISON CURRENT BALANCE

Plot of magnetic force (gm) versus current (A) for 9.0 mm coil separation



Excellent agreement is again found for the model of current circles (although as mentioned above, there is a 2 mm offset between concentricity of the circles). Good agreement is also found for the model of parallel wires. Percent differences are computed below for currents of 1 ampere.

Parallel Wires Model: $\frac{2.941 - 2.65}{2.941} \cdot 100 = 9.895$

Current Circles Model: $\frac{|2.681 - 2.65|}{2.681} \cdot 100 = 1.156$

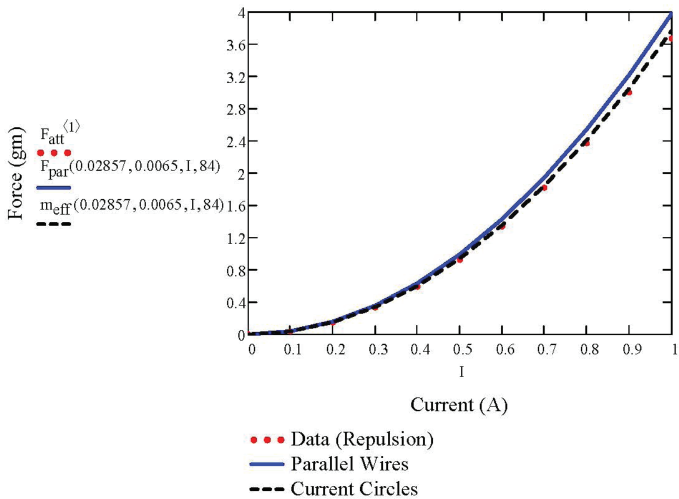
New data with redesigned mount for concentric alignment and at closer spacing of 6.5 mm. At this spacing, the data agrees with the simpler model of magnetic force between long and parallel current carryng wires to within 8%.

$$F_{att} := \begin{pmatrix} 0 & 0 \\ 0.1 & 0.038 \\ 0.2 & 0.147 \\ 0.3 & 0.333 \\ 0.4 & 0.59 \\ 0.5 & .922 \\ 0.6 & 1.34 \\ 0.7 & 1.820 \\ 0.8 & 2.370 \\ 0.9 & 3.000 \\ 1. & 3.670 \end{pmatrix}$$

Column one is current (A) and colunm 2 is the measured force (grams)

$$I := F_{rep}^{\langle 0 \rangle}$$

Figure 4: Magnetic force (gm) versus current (A) for 6.5 mm separation

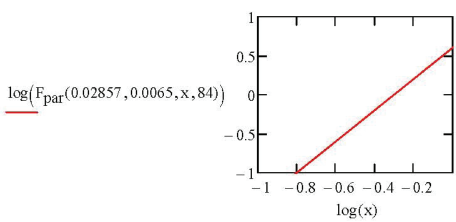


Parallel Wires Model: $\frac{F_{par}(0.02857, 0.0065, 1., 84) - 3.67}{F_{par}(0.02857, 0.0065, 1., 84)} \cdot 100 = 7.716$

Current Circles Model: $\frac{\left|m_{eff}(0.02857, 0.0065, 1., 84) - 3.67\right|}{m_{eff}(0.02857, 0.0065, 1, 84)} \cdot 100 = 2.595$

$\log(10) = 1$

$\log\left(F_{par}(0.02857, 0.0065, 1, 84)\right) = 0.6$



$$\mu_0 := 10^{0.6} \cdot \frac{0.0065 \cdot 9.807}{0.02857 \cdot 84^2 \cdot 10^3} = 1.259 \times 10^{-6}$$

$$4 \cdot \pi \cdot 10^{-7} = 1.257 \times 10^{-6}$$

III. Mutual Induction

The magnetic field energy stored in two current circles, each carrying the same current in N loops is equal to

$$U = (L + M) \cdot (N \cdot I)^2$$

where L is the self-inductance and M is the mutual inductance of the circles. The force between the two coaxial (the axis is taken as the z-axis) current circles lying in parallel planes will then be given by the z-component of the spatial gradient of the *mutual* inductance.

$$F_z(I) = (N \cdot I)^2 \cdot \frac{d}{dz} M(z)$$

An expression for the mutual induction for conducting circles that are coaxial and lying in parallel planes (as per the Kelvin current balance) was given already by James Clerk Maxwell in 1873 ("'A Treatise on Electricity and Magnetism, Dover Publications Inc, New York, 1954 (reprinted from the original in 1873)). As above, we first define the modulus k (a is the circle radius and c is the separation between circles as used above in Section II).

$$k(a,c) := \frac{2a}{\sqrt{4a^2 + c^2}}$$

For circles with 85 turns (for the data presented below), the mutual inductance is given by:

$$M_{Maxwell}(a,k) := \mu_0 \cdot a \cdot \left[\left(\frac{2}{k} - k\right) \cdot K(k) - \frac{2}{k} E(k)\right] 85^2$$

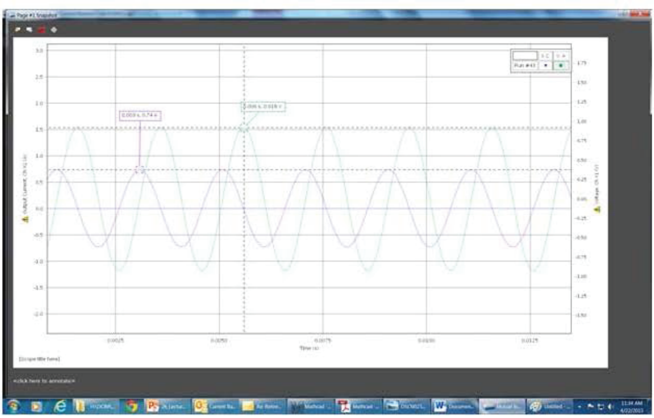
(We note the obvious similarity to the expression for the force of interaction discussed in Section II in terms of the complete elliptic integrals).

The mutual inductance is given by the ratio of the amplitude of the induced emf around one circle to amplitude of the time-rate of change of the current in the other.

$$M = \frac{emf_2}{\frac{dI_1}{dt}}$$

In order to measure the mutual inductance, we have placed the conducting coils, having the same mean radius as above, i.e., 2.86 cm, and in direct contact with good cencentric alignment with a mean separation of 6.2 mm. We have driven one coil at 500 Hz and measured the induced emf of the other. In Figure 4, we show our results as captured by a *Pasco ScienceWorkshop 750 Interface*.

Figure 4: Induced emf and driving ac-current at 500 Hz



Using the Pasco amplitudes for 500 Hz sinusoidal current and emf, we find the measured mutual inductance to be:

$$M_{meas} := \frac{0.918 \cdot V}{2 \cdot \pi \cdot 500 \cdot 0.74 \cdot \frac{A}{s}} = 3.949 \times 10^{-4} H$$

However, we discovered the Pasco sensors were not well calibrated. Consequently, we also took rms readings with a good quality multimeter (BK TookKit 2706A). We calculate the mutual inductance:

$$M_{meas} := \frac{0.661 \cdot V}{2 \cdot \pi \cdot 500 \cdot 0.493 \cdot \frac{A}{s}} = 4.268 \times 10^{-4} H$$

We compare to the theoretical value from the theory of Maxwell. We first evaulate the appropriate modulus:

$$k(0.0286, 0.00623) = 0.994$$

and the theoretical mutual inductance is found to be:

$$M_{Maxwell}(0.0286 \cdot m, 0.994) = 4.221 \times 10^{-4} \frac{A^2 \cdot s^2}{m \cdot kg} \cdot H$$

This is in excellent agreement with our measurement, i.e.,

$$\%error := \frac{\left| M_{meas} - M_{Maxwell}(0.0286 \cdot m, 0.994) \right| \cdot 100}{M_{Maxwell}(0.0286 \cdot m, 0.994)} = \blacksquare$$

IV. Summary::

We have developed a novel, compact (4" x 4" x 2"), low-cost Kelvin current balance that should be well received in the educational community for laboratory experiments investigating the force between current-carrying coils and the mutual inductance of the coils. The recent availability of low-cost, compact and sensitive miniature digital scales and 3D-printing technology (for custom wire-wound circles and supporting structures) allows this device to perform as well or better than other more expensive, bulkier, and difficult-to-align (requiring optical lever arrangement with a HeN laser) current balances. These current balances, requiring 10 amps or more for reliable readings, also often rely upon the use of a mechanical balance with an easily-damaged knife-edge.

Our device will enable the current balance to become an accessible experiment *for both advanced undergraduate physics majors as well as s*tudents in a survey course in undergraduate or secondary school physics.

One end goal of such a laboratory is the extraction of the permeability of free space. The magnetic force can be easily switched from attractive to repulsive, and measured values of the magnetic force are found to be in excellent agreement with theory (for advanced students, the latter can be easily computed with Excel or MathCad software or even from a pre-calculated table that could be supplied with the device). The force as functions of both current and circle (coil) separation can als be investigated.

In addition, our device also is applicable for a separate laboratory on mutual inductance using routine instructional laboratory equipment. The measured and theoretical values agree within 2%. The mutual inductance could also be measured as a function of circle separation. The mutual inductance also agrees quite well with the related theory provided by Maxwell.

For the 040115 parts, the disc wall thicknesses are

$top := 0.078 \cdot in = 1.981 \cdot mm$

$bot := 0.078 \cdot in = 1.981 \cdot mm$

$gap := 0.050 \cdot in = 1.27 \cdot mm$

$edgegap := 0.078 \cdot in + 0.025 \cdot in = 2.616 \cdot mm$

Pedastal base to gap center:

$ht := 28.396 \cdot mm$

Stand shelf thickness

$shelf := 0.067 \cdot in = 1.702 \cdot mm$

Minimum separation for these parts would be:

$2 edgegap + shelf = 6.934 \cdot mm$

For induction with touching disks, separation should be

$2 \cdot edgegap = 5.232 \cdot mm$

Measured table floor to pedastal *top* when positioned on Gemini-20 is:

$floor := 46.6 \cdot mm$

Stand floor to lower shelf from Inventor:

$stand := 47.727 \cdot mm$

The separation is

$gap := stand - floor = 1.127 \cdot mm$

in agreement with visual observation when assembled..

For induction with touching disks, separation should be

$2 \cdot edgegap = 5.232 \cdot mm$

If have new thinner discs made, let's see what gap might result

$$top := \frac{0.078\cdot in}{2} = 0.991\cdot mm$$

$$bot := 0.078\cdot\frac{in}{2} = 0.991\cdot mm$$

$$gap := 0.050\cdot in = 1.27\cdot mm$$

$$edgegap := top + \frac{1}{2}\cdot gap = 1.626\cdot mm$$

$$2\cdot edgegap = 3.251\cdot mm$$

First test 052115, needed to place stand on small washers (thickness = 1.55 mm) for a 0.5-1 mm gap.

The measured total width of disc is 3.5 mm

The measured detpth from table surface to insert is 3.9 mm

The table thickness: 5.08 mm

The top of upper disc is flush with surface of table.

The calculated/measured shelf thickness is:

$$5.08\cdot mm - 3.9\cdot mm = 1.18\cdot mm$$

Therefore gap for measurements 052015 is:

$$3.5\cdot mm + 1.18\cdot mm + 1\cdot mm = 5.68\cdot mm$$

I get 6.23 with a micrometer from top of table to top of lower disc when positioned (~ 1/2 to 1 mm gap)

Therefore, distances between tops of discs is also 6.23 mm since top disc is flush with table surface. In this manner, I estimate a center-center separation of 6.23 mm also.

Pedastal base to gap center:

$$ht := 28.396\cdot mm$$

reduce stand shelf thickness

$$shelf := \frac{0.067\cdot in}{2} = 0.851\cdot mm$$

Minimum separation for these parts would be:

$$2edgegap + shelf = 4.102\cdot mm$$

Could just make 5 mm gap I believe.

Would need to increase pedastal length by extra 1/2 top

Asked Ron to make new parts 051315 designation.

Inventor assembly indeed shows a gap of 5.1 mm (1 mm air gap)