

(3) The ball player Monty Carlo is a genuine .250 hitter. In every official time at bat (walks, etc. not counted) the probability that he gets a hit is .250. In Carlo's record for the season, which included 300 official times at bat, what is likely to be the length of his longest slump, that is, run of consecutive hitless official times at bat? What is the probability that his actual batting average for the season exceeded .300?

Consider a very long string of N statistically independent times at bat of which hN are hits. A slump starts after every hit. The probability that its length is s is $h(1-h)^s$, for $s = 0, 1, 2, \dots$. The total number of slumps, including slumps of length zero, is the same as the number of hits hN . Hence the expected number of slumps of length k or greater is

$$hN \sum_{s=k}^{\infty} h(1-h)^s = Nh(1-h)^k \quad \text{or } 75(0.75)^k$$

in the case of Carlo. This equals one for $k \approx 15$. Carlo will be fortunate if his worst slump is not longer than that. Of course we have neglected the finiteness of N , which limits slumps near the end of the season, but that will be a small correction if $k \ll N$, as in this case.

For N official times at bat the number of hits n is the sum of N terms, each of which is 1, with probability h , or zero. Averaged over many players with the same h , \bar{n} is hN and $\overline{n^2}$ is $nH + N(N-1)h^2$. Hence $\overline{n^2} - \bar{n}^2 = Nh(1-h)$. This is σ^2 for the approximately normal distribution of n around its mean \bar{n} . In our case with $N = 300$ and $h = 0.25$ we have $\bar{n} = 75$ and $\sigma = 7.5$. For a batting average $n/N = .300$ a player needs $n = 90$, which is just 2σ above the mean. The area above 2σ is 2.3% of the whole, so we may expect that about one .250 batter in 44 will be lucky enough to post a season's average of .300 or higher. About one in six should do better than .275.

To test these answers I ran one thousand .250 hitters through a season 300 times at bat, recording the worst slump each experienced during the season. The median of the thousand "worst slumps" was of length 16. For more than half the players, the worst slump was greater than 12 and less than 19. One unlucky batter had a slump of length 36. No player was lucky enough to escape a slump of at least nine consecutive hitless times at bat. Twenty players finished the season with a batting average above .300.