

(2) Let's define "the top of the atmosphere" as the height at which a nitrogen molecule moving upward with more than escape velocity has a probability 0.5 of leaving the Earth for good. About how high is that?

Ignoring any difference between nitrogen and oxygen, let  $\sigma$  be the  $N_2$ - $N_2$  collision cross section (the cross section that would be  $4\pi a^2$  for hard spheres of radius  $a$ ). At the height  $h$  that we seek to determine, the number of molecules above unit area must be about  $0.5/\sigma$ . The mass above unit area is then  $(0.5/\sigma) \times 28 \times 1.6 \times 10^{-24}$  g. Let's assume a value of  $10^{-15}$  cm<sup>2</sup> for  $\sigma$ , corresponding roughly to  $a = 1$  Å. Then the mass above 1 cm<sup>2</sup> at height  $h$  is  $2 \times 10^{-8}$  g, which is  $2 \times 10^{-11}$  of the mass above 1 cm<sup>2</sup> at sea level. The ratio is approximately  $e^{-24}$ , so  $h$  would be 24 scale heights in a uniformly exponential atmosphere. If the scale height in the lower atmosphere, approximately 8 km, persisted to extreme altitudes,  $h$  would be around 200 km. In fact, the scale height itself begins to increase rapidly above 100 km, owing to the increasing temperature above the stratosphere, and the height  $h$  above which the sky is "half-clear" of molecules is actually close to 400 km, rather than 200 km.