

(2) At room temperature in still air, how long can a pencil remain balanced on its point? How long at absolute zero?

A pencil hanging by its point has a frequency of about 1.5 s^{-1} . (A bit of scotch tape and a pencil are all you need to determine that.) Therefore, $\omega \approx 10 \text{ s}^{-1}$. Turning the pendulum upside down makes ω imaginary so that when the pencil topples, both θ and $\dot{\theta}$ grow like $e^{\omega t}$. The kinetic energy thus grows like $e^{2\omega t}$ to a final value Mgb , where M is the pencil mass and b is the height of its center of mass.

The initial energy can't be much less than kT , due to Brownian motion of the pencil, so $2\omega t \approx \ln(Mgb/kT)$. For $Mb = 100 \text{ g cm}$, $t \approx 2.1 \text{ s}$. At absolute zero the best you can do is to start with a wave packet corresponding to the zero point energy $\frac{1}{2} \hbar\omega$. Then $t = (1/20) \ln(2Mgh/\hbar\omega) \approx 3.6 \text{ s}$.

The answers 2.1 and 3.6 s are only as accurate as the estimates of ω . A factor of 10 in Mb adds only 0.1 s to t .