

(3) Rayleigh scattering would limit how far we could see horizontally even if the Earth were flat. Estimate the limit, given that liquid nitrogen has a refractive index of 1.2 for visible light and a density of  $0.8 \text{ gm/cm}^3$ .

The number density  $N$  of molecules in liquid nitrogen is  $(0.8/28) \times 6 \times 10^{23}$ , or  $1.7 \times 10^{22} \text{ cm}^{-3}$ . The dielectric constant  $\epsilon = (1.2)^2 = 1.44$  is related to the molecular polarizability  $\alpha$  by  $3(\epsilon - 1)/(\epsilon + 2) = 4\pi N\alpha$ , from which we find

$\alpha = 1.8 \times 10^{-24} \text{ cm}^3$ . The cross section per molecule for Rayleigh scattering is  $(8\pi/3) \alpha^2/\lambda^4$ . (That factor  $8\pi/3$  you may recall from Thomson scattering, for which  $\alpha$  is just  $r_0^3$ .) For  $\lambda = 10^{-5} \text{ cm}$  we get the cross section  $\sigma = 2.7 \times 10^{-27} \text{ cm}^2$ . Assume  $\alpha$  is the same for an oxygen molecule. There are  $2.5 \times 10^{19}$  molecules per  $\text{cm}^3$  in sea-level air. The distance to extinction by  $1/e$  through Rayleigh scattering is therefore 150 km. Distant vision would be impaired not merely by extinction of light from the object viewed, but by loss of contrast owing to the sunlight scattered into the line of sight. What one could discern through the "Rayleigh haze" would depend, of course, on the characteristics of the distant scene. But it seems fair to predict that one could not "see farther" than a few hundred kilometers.