(2) In the near-perfect interstellar vacuum a proton has captured an electron in a circular orbit of 1-micrometer radius. How soon will this atom emit an ultraviolet photon?

We can treat the orbit classically. The energy E is $-e^2/2r$ and the rate at which energy is lost by radiation, $-dE/dt$, is $2e^2(r\omega^2)^2/3c^3$. The frequency of revolution ω is $(e^2/mr^3)^{1/2}$. Combining these gives $r^2\,dr = -(4/3)(e^4/m^2c^3)dt = -(4/3)r_0^2c\,dt$, where r_0 is the classical electron radius. Integrating, we find that the time to collapse from an initial radius r_2 to zero radius is $(1/4)r_1^3/r_0^2c$. For $r_1 = 10^{-4}$ cm this is 100 seconds. Our classical calculation remains correct over practically all of this time. (It is only in the last 100 microseconds that r is less than 100 Å.) We may expect a Lyman α photon very close to 100 seconds after the electron was captured.