Physics Challenge for Teachers and Students

Solution to the November 2016 Challenge Boards of education

(The problem adapted from Russian Physics Olympiads, S. Kozel, V. Slobodyanin, Eds., Verbum-M, Moscow, 2002)

Description of the System

Three very long boards are stacked upon each other, with respective masses $m_1 = m_2 = m_3 = m$. The position of the left ends of each are $x_1, x_2, \text{ and } x_3$, respectively. Initially, they are positioned directly atop each other: $x_1 = x_2 = x_3 = 0$. They are at rest: $v_{10} = v_{20} = v_{30} = 0$. We may define their relative velocities: $v_{12} = v_1 - v_2$ and $v_{23} = v_2 - v_3$.

Frictional forces are at work between the surfaces of the boards. Both the static and kinetic coefficients of friction between boards $m_1$ and $m_2$ are given by $\mu_{s12} = \mu_{k12} = \mu$. Both the static and kinetic coefficients of friction between boards $m_2$ and $m_3$ are given by $\mu_{s23} = \mu_{k23} = 2\mu$. Both the static and kinetic coefficients of friction between board $m_3$ and the ground are given by $\mu_{s3} = \mu_{k3} = 3\mu$.

First, an instantaneous impulsive force is applied to the top board $m_1$: $F_1 = mv_0 \delta(t)$. As a result, $m_1$, and only $m_1$, starts to move with an initial speed $v_0$ to the right. After a time $\tau_1$ (the stated problem describes this time as $t$, but I prefer to employ $t$ as my dynamical time variable and $\tau_1$ for time constants), the entire system comes to rest.

Later, the system is restored to its initial state. Then an instantaneous impulsive force of the same magnitude as the first one is applied to $m_3$: $F_3 = mv_0 \delta(t)$. As a result, $m_3$, and only $m_3$, starts to move with the same initial speed $v_0$ to the right. After a time $\tau_2$, the entire system again comes to rest.

Statement of the Problem

Find the value of $\tau_2$ in terms of $\tau_1$. 
**Proposed Solution**

We will solve for \( \tau_1 \) and \( \tau_2 \) in turn, and then express \( \tau_2 \) in terms of \( \tau_1 \).

**General Considerations**

To do this we will start by drawing the free-body diagrams for the three boards. This will allow us to calculate normal and frictional forces as well as accelerations. Knowing the kinematical equations for uniform acceleration, we may write down the equations of motion for each of the three boards. We must break the motion down into steps, or regimes, when the frictional forces take different forms. Whenever there is relative motion, \( v_{ij} \neq 0 \), kinetic friction will operate: \( |f_i| = \mu_{ki}N_i \). Whenever there is no relative motion, \( v_{ij} = 0 \), static friction is available to operate, producing as much force as is necessary to maintain the absence of relative motion, but not exceeding its maximum value, \( |f_i| \leq \mu_{si}N_i \).
We immediately note that we have given a direction to the frictional forces. In point of fact, the direction will depend upon the relative velocities of the two surfaces in contact. We will accommodate this fact by ascribing a sign, $\pm$, to the frictional forces, in accord with the arrows as we have drawn them.

We proceed first to consider the vertical forces. All motion is in the horizontal, $x -$ direction, and there is no motion in the vertical, $y -$ direction. Therefore, there is no acceleration in the vertical: $a_{iy} = 0$. By Newton’s second law, the sum of the vertical forces on any mass must therefore be zero, $\sum_k F_{iyk} = 0$.

\[
\begin{align*}
\sum_k F_{1yk} &= 0 \\
N_{12} - mg &= 0 \\
N_{12} &= mg \\
\sum_k F_{2yk} &= 0 \\
N_{23} - N_{12} - mg &= 0 \\
N_{23} - mg - mg &= 0 \\
N_{23} - 2mg &= 0 \\
N_{23} &= 2mg \\
\sum_k F_{3yk} &= 0 \\
N_3 - N_{23} - mg &= 0 \\
N_3 - 2mg - mg &= 0 \\
N_3 - 3mg &= 0 \\
N_3 &= 3mg
\end{align*}
\]

We may now make statements about the possible magnitudes of the frictional forces.

\[
\begin{align*}
|f_{12}| &\leq \mu_{s12} N_{12} = (\mu)(mg) = \mu mg, \text{ if } v_{12} = 0 \\
|f_{12}| &= \mu_{k12} N_{12} = (\mu)(mg) = \mu mg, \text{ if } v_{12} \neq 0 \\
|f_{23}| &\leq \mu_{s23} N_{23} = (2\mu)(2mg) = 4\mu mg, \text{ if } v_{23} = 0 \\
|f_{23}| &= \mu_{k23} N_{23} = (2\mu)(2mg) = 4\mu mg, \text{ if } v_{23} \neq 0
\end{align*}
\]
\[ |f_3| \leq \mu_3 N_3 = (3\mu)(3mg) = 9\mu mg, \text{ if } v_3 = 0 \]
\[ |f_3| = \mu_k N_3 = (3\mu)(3mg) = 9\mu mg, \text{ if } v_3 \neq 0 \]
**Case One**

We now proceed to calculate the accelerations of the three masses for the first case.

Initial conditions:

\[ v_1 = v_0 \]
\[ v_2 = 0 \]
\[ v_3 = 0 \]

With relative velocities:

\[ v_{12} = v_0 \neq 0 \]
\[ v_{23} = 0 \]
\[ v_3 = 0 \]

Thus, if there is sufficient static friction, \( m_2 \) and \( m_3 \) may remain motionless, and the friction between \( m_1 \) and \( m_2 \) is necessarily kinetic and directed toward the left.

\[
\sum_{k} F_{1k} = ma_1 \\
-f_{12} = ma_1 \\
-\mu mg = ma_1 \\
a_1 = -\mu g
\]

We now assume that there is sufficient friction to maintain zero relative velocity between \( m_1 \) and \( m_2 \) and check to see if the required frictional forces are less than the maximal static frictional forces.

\[
\sum_{k} F_{2k} = ma_2 \\
f_{12} - f_{23} = ma_2 \\
\mu mg - f_{23} = ma_2
\]

We remember that \( |f_{23}| \leq 4\mu mg \), if \( v_{23} = 0 \), and we know that \( |f_{23}| \) is large enough to overcome \( f_{12} \). If \( m_3 \) remains stationary, \( m_2 \) will remain stationary as well.

\[
\sum_{k} F_{3k} = ma_3 \\
f_{23} - f_3 = ma_3
\]

Since \( |f_3| \leq 9\mu mg \), if \( v_3 = 0 \), and \( |f_{23}| \leq 4\mu mg \) (whether static or kinetic), there is sufficient frictional force between \( m_3 \) and the ground to prevent \( m_3 \) from starting to move relative to the ground.

\[
0 = ma_3 \\
a_3 = 0
\]
Since $m_3$ is stationary, it follows by our previous reasoning that $m_2$ will remain stationary as well.

Thus, only $m_1$ will move, slowing down with the negative acceleration we found, $a_1 = -\mu g$. From the kinematical equations, we find:

$$v_1 = v_0 - \mu g t$$

We may solve this equation for the time required for $m_1$, and thus the entire system, to come to rest:

$$0 = v_0 - \mu g \tau_1$$

$$\tau_1 = \frac{v_0}{\mu g}$$

**Second Case**

We may now proceed to the second case, in which it is the bottom board, $m_3$ which is struck. Nothing has changed about the forces, as we found them in the first case, except that the directions of friction and their magnitudes, if they are static, may change. The normal forces are completely unchanged.

These are the initial conditions for the second case:

$$v_1 = 0$$
$$v_2 = 0$$
$$v_3 = v_0$$

With relative velocities:

$$v_{12} = 0$$
$$v_{23} = -v_0$$
$$v_3 = v_0$$

Since there is motion between $m_3$ and the ground, we know that kinetic friction is at work between them. Similarly, we know that kinetic friction is at work between $m_2$ and $m_3$. Thus:

$$|f_{23}| = 4\mu mg$$
$$|f_3| = 9\mu mg$$

Since only $m_3$ is moving, and moving to the right, we may infer that both $f_{23}$ and $f_3$, acting on $m_3$, are directed toward the left. Initially, $v_{12} = 0$. We need to find out if there is enough static frictional force on $m_1$ for it to accelerate at the same rate as $m_2$, $a_1 = a_2$. We start by assuming that they accelerate together, and then calculate the necessary static friction.

$$\sum F_{1k} = ma_1$$
\[-f_{12} = m a_1\]
\[-(-|f_{12}|) = m a_1\]
\[|f_{12}| = m a_1\]

\[
\sum_k F_{2k} = m a_2
\]
\[f_{12} - f_{23} = m a_2\]
\[-|f_{12}| - (-|f_{23}|) = m a_2\]
\[-|f_{12}| + |f_{23}| = m a_2\]
\[-|f_{12}| + 4\mu m g = m a_2\]
\[-|f_{12}| + 4\mu m g = m a_1\]

Substituting for \(m a_1\) from our equation for \(m_1\):

\[-|f_{12}| + 4\mu m g = |f_{12}|\]
\[2|f_{12}| = 4\mu m g\]
\[|f_{12}| = 2\mu m g\]

However, we already know that \(|f_{12}| \leq \mu m g\). Therefore, we must be in the regime of kinetic friction:

\[|f_{12}| = \mu m g\]
\[|f_{12}| = m a_1\]
\[\mu m g = m a_1\]
\[a_1 = \mu g\]

\[-\mu m g + 4\mu m g = m a_2\]
\[3\mu m g = m a_2\]
\[a_2 = 3\mu g\]

\[
\sum_k F_{3k} = m a_3
\]
\[f_{23} - f_3 = m a_3\]
\[-4\mu m g - 9\mu m g = m a_3\]
\[-13\mu m g = m a_3\]
\[a_3 = -13\mu g\]

We may now invoke the kinematical equations to obtain equations for the velocities of the three boards:

\[v_1 = \mu g t\]
\[v_2 = 3\mu g t\]
\[v_3 = v_0 - 13\mu g t\]
Initially, $m_3$ is moving fastest and slowing down. Both $m_1$ and $m_2$ start from rest and are speeding up. However, $m_2$ is accelerating faster and gaining both speed and distance relative to $m_1$. Unless something changes, $m_1$ will never catch up to $m_2$.

The next event of interest will be when the velocities of $m_2$ and $m_3$ become equal. Then, there is a momentary possibility that they will start to move with the same velocity, should static friction be sufficient.

Let us calculate the time, $t_1$, at which this occurs.

\[ v_2 = v_3 \]
\[ 3\mu g t_1 = v_0 - 13\mu g t_1 \]
\[ 16\mu g t_1 = v_0 \]
\[ t_1 = \frac{v_0}{16\mu g} \]

At this point, $v_{12} = v_1 - v_2 = \mu g t_1 - 3\mu g t_1 = 2\mu g t_1 = 2\mu g \left( \frac{v_0}{16\mu g} \right) = \left( \frac{1}{8} \right) v_0 \neq 0$, so we are in the regime of kinetic friction:

\[ |f_{12}| = \mu mg \]

Similarly, $v_3 = v_0 - 13\mu g t_1 = v_0 \left( \frac{16}{16} \right) - 13\mu g t_1 \left( \frac{v_0}{16\mu g} \right) = \left( \frac{16}{16} \right) v_0 - \left( \frac{13}{16} \right) v_0 = \left( \frac{3}{16} \right) v_0 \neq 0$, so we are again in the regime of kinetic friction:

\[ |f_3| = 9\mu mg \]

We now need to see if there is sufficient static friction between $m_2$ and $m_3$ for the two of them to move together, with the same acceleration, $a_2 = a_3$.

If they do move together, they will both be slowing down. The frictional force between them will act to the left on $m_2$ and to the right on $m_3$. Since $m_1$ is still moving slower than the two of them, the force of friction between $m_1$ and $m_2$ will continue be kinetic friction, directed to the right on $m_1$ and to the left on $m_2$.

Let us see what size of static friction, $|f_{23}|$, would be required between $m_2$ and $m_3$ for them to move together.

\[
\sum_k F_{2k} = m a_2 \\
f_{12} - f_{23} = m a_2 \\
-|f_{12}| - |f_{23}| = m a_2 \\
-\mu mg - |f_{23}| = m a_2 \\
\sum_k F_{3k} = m a_3
\]
\[
\begin{align*}
  f_{23} - f_3 &= m a_3 \\
  |f_{23}| - |f_3| &= m a_3 \\
  |f_{23}| - 9\mu mg &= m a_3 \\
  |f_{23}| - 9\mu mg &= m a_2 \\
  m a_2 &= m a_3 \\
  -\mu mg - |f_{23}| &= |f_{23}| - 9\mu mg \\
  2|f_{23}| &= 8\mu mg \\
  |f_{23}| &= 4\mu mg \\
\end{align*}
\]

Static friction is able to produce this much force \(|f_{23}| \leq 4\mu mg\). Therefore, we are in the regime of static friction between \(m_2\) and \(m_3\) and \(|f_{23}| = 4\mu mg\).

We may now solve for the new accelerations of the three boards.

\[
\begin{align*}
  \sum_{k} F_{1k} &= ma_1 \\
  -f_{12} &= ma_1 \\
  -(|f_{12}|) &= ma_1 \\
  |f_{12}| &= ma_1 \\
  \mu mg &= ma_1 \\
  a_1 &= \mu g \\
  -\mu mg - |f_{23}| &= ma_2 \\
  -\mu mg - 4\mu mg &= ma_2 \\
  -5\mu mg &= ma_2 \\
  a_2 &= -5\mu g \\
  4\mu mg - 9\mu mg &= ma_3 \\
  -5\mu mg &= ma_3 \\
  a_3 &= -5\mu g \\
\end{align*}
\]

We may now write down the equations of motion for the three boards:

\[
\begin{align*}
  v_1 &= \mu gt_1 + \mu g(t - t_1) = \mu gt_1 + \mu gt - \mu gt_1 = \mu gt \\
  v_2 &= 3\mu g \left( \frac{v_0}{16\mu g} \right) - 5\mu g(t - t_1) = \left( \frac{3}{16} \right) v_0 - 5\mu gt + 5\mu gt_1 \\
  &= \left( \frac{3}{16} \right) v_0 - 5\mu gt + 5\mu g \left( \frac{v_0}{16\mu g} \right) = \left( \frac{8}{16} \right) v_0 - 5\mu gt + \left( \frac{5}{16} \right) v_0 \\
  &= \left( \frac{8}{16} \right) v_0 - 5\mu gt = \left( \frac{1}{2} \right) v_0 - 5\mu gt \\
\end{align*}
\]
\[ v_3 = \left( \frac{3}{16} \right) v_0 - 5\mu g(t - t_1) = \left( \frac{3}{16} \right) v_0 - 5\mu g t + 5\mu g t_1 = \left( \frac{3}{16} \right) v_0 - 5\mu g t + \left( \frac{5}{16} \right) v_0 \]
\[ = \left( \frac{1}{2} \right) v_0 - 5\mu g t \]

At this point, \( m_1 \) is speeding up, while \( m_2 \) and \( m_3 \) are slowing down. There will come a point when all three velocities are equal. At this point, one or more pairs of boards might be able to move together, if there is sufficient static friction.

Let us find the time, \( t_2 \), when all three are at the same speed. Since \( m_2 \) and \( m_3 \) are moving together, it is sufficient to find out when

\[ v_1 = v_2 \]
\[ \mu g t_2 = \left( \frac{1}{3} \right) v_0 - 5\mu g t_2 \]
\[ 6\mu g t_2 = \left( \frac{1}{2} \right) v_0 \]
\[ t_2 = \left( \frac{1}{12} \right) \left( \frac{v_0}{\mu g} \right) \]

After this event, frictional forces \( f_{12} \) on \( m_1 \), \( f_{23} \) on \( m_2 \), and \( f_3 \) on \( m_3 \) are devoted to slowing down the three moving boards and are therefore directed towards the left. By Newton’s third law, \( f_{12} \) on \( m_2 \) and \( f_{23} \) on \( m_3 \) are directed towards the right. We must learn if there is enough frictional force for them all to move together, or any pair of them to move together.

\[ \sum F_{3k} = m a_3 \]
\[ f_{23} - f_3 = ma_3 \]
\[ |f_{23}| - |f_3| = ma_3 \]
\[ |f_{23}| - 9\mu mg = ma_3 \]

\[ \sum F_{2k} = m a_2 \]
\[ f_{12} - f_{23} = ma_2 \]
\[ |f_{12}| - |f_{23}| = ma_2 \]

\[ \sum F_{1k} = m a_1 \]
\[ -f_{12} = ma_1 \]
\[ -|f_{12}| = ma_1 \]

If
\[ a_2 = a_3 \]
\[ ma_2 = ma_3 \]
\[ |f_{12}| - |f_{23}| = |f_{23}| - 9\mu mg \]
\[ 2|f_{23}| = |f_{12}| + 9\mu mg \]
\[ |f_{23}| = \frac{1}{2} |f_{12}| + \frac{9}{2} \mu mg \]
\[ |f_{23}| \geq \frac{9}{2} \mu mg \]
\[ |f_{23}| > 4 \mu mg \]

But we know \( |f_{23}| \leq 4 \mu mg \), so \( a_2 = a_3 \) is impossible. The friction between \( m_2 \) and \( m_3 \) is kinetic and \( |f_{23}| = 4 \mu mg \).

If

\[
\begin{align*}
a_1 &= a_2 \\
ma_1 &= ma_2 \\
-|f_{12}| &= |f_{12}| - |f_{23}| \\
2|f_{12}| &= |f_{23}| \\
2|f_{12}| &= 4 \mu mg \\
|f_{12}| &= 2 \mu mg \\
|f_{12}| &= \mu mg
\end{align*}
\]

But we know \( |f_{12}| \leq mg \), so \( a_1 = a_2 \) is impossible. The friction between \( m_1 \) and \( m_2 \) is kinetic and \( |f_{12}| = \mu mg \).

We may now solve for the accelerations of the three boards.

\[
\begin{align*}
ma_1 &= -|f_{12}| \\
ma_1 &= -\mu mg \\
a_1 &= -\mu g \\
ma_2 &= |f_{12}| - |f_{23}| \\
ma_2 &= \mu mg - 4 \mu mg \\
ma_2 &= -3 \mu mg \\
a_2 &= -3 \mu g \\
ma_3 &= |f_{23}| - 9 \mu mg \\
ma_3 &= 4 \mu mg - 9 \mu mg \\
ma_3 &= -5 \mu mg \\
a_3 &= -5 \mu g
\end{align*}
\]

Again, we may employ the kinematical equations to obtain the time dependence of the velocities of the three boards.

\[
\begin{align*}
v_1 &= \mu gt_2 - \mu g(t - t_2) = \mu gt_2 - \mu g + \mu gt_2 = 2 \mu gt_2 - \mu gt = 2 \mu g \left( \frac{1}{12} \right) \left( \frac{v_0}{\mu g} \right) - \mu gt \\
&= \left( \frac{1}{6} \right) v_0 - \mu gt
\end{align*}
\]
\[ v_2 = \left( \frac{1}{2} \right) v_0 - 5\mu g t_2 - 3\mu g(t - t_2) = \left( \frac{1}{2} \right) v_0 - 5\mu g t_2 - 3\mu g t + 3\mu g t_2 \]

\[ = \left( \frac{1}{2} \right) v_0 - 2\mu g t_2 - 3\mu g t = \left( \frac{3}{6} \right) \left( \frac{1}{2} \right) v_0 - 2\mu g \left( \frac{1}{12} \right) \frac{v_0}{\mu g} - 3\mu g t \]

\[ = \left( \frac{3}{6} \right) v_0 - \left( \frac{1}{6} \right) v_0 - 3\mu g t = \left( \frac{2}{6} \right) v_0 - 3\mu g t = \left( \frac{1}{3} \right) v_0 - 3\mu g t \]

\[ v_3 = \left( \frac{1}{2} \right) v_0 - 5\mu g t_2 - 5\mu g(t - t_2) = \left( \frac{1}{2} \right) v_0 - 5\mu g t_2 - 5\mu g t + 5\mu g t_2 = \left( \frac{1}{2} \right) v_0 - 5\mu g t \]

All three boards start with the same velocity and have negative accelerations. Since the magnitudes of the acceleration decrease from \( m_3 \) to \( m_2 \) to \( m_1 \), we expect them to come to a stop (\( v = 0 \)) in the order \( m_3 \), then \( m_2 \), and then \( m_1 \).

Since \( m_3 \) will stop first, let us calculate \( t_3 \), when \( m_3 \) stops.

\[ v_3 = 0 \]

\[ \left( \frac{1}{2} \right) v_0 - 5\mu g t_3 = 0 \]

\[ t_3 = \left( \frac{1}{10} \right) \left( \frac{v_0}{\mu g} \right) \]

We should check to see if there is sufficient static frictional force to keep \( m_3 \) motionless. Since \( v_{23} \neq 0 \), the frictional force between \( m_2 \) and \( m_3 \) is still kinetic and \( |f_{23}| = 4\mu mg \).

\[ ma_3 = |f_{23}| - |f_3| \]

\[ m(0) = 4\mu mg - |f_3| \]

\[ 0 = 4\mu mg - |f_3| \]

\[ |f_3| = 4\mu mg \]

\[ |f_3| \leq 9\mu mg \]

So, there is sufficient static friction to keep \( m_3 \) motionless.

The frictional forces between \( m_2 \) and \( m_3 \) and between \( m_1 \) and \( m_2 \) remain kinetic. All the forces on \( m_1 \) and \( m_2 \) remain as they were, so the equations of motion for \( m_1 \) and \( m_2 \) remain the same.

The next event will be the stopping of \( m_2 \). Let us calculate \( t_4 \) when \( m_2 \) stops.

\[ v_2 = 0 \]

\[ \left( \frac{1}{3} \right) v_0 - 3\mu g t_4 = 0 \]

\[ t_4 = \left( \frac{1}{9} \right) \left( \frac{v_0}{\mu g} \right) \]
We should check to see if there is sufficient static frictional force to keep \( m_3 \) and \( m_2 \) motionless. Since \( v_{12} \neq 0 \), the frictional force between \( m_1 \) and \( m_2 \) is still kinetic and \( |f_{12}| = \mu mg \).

\[
ma_2 = |f_{12}| - |f_{23}|
\]
\[
m(0) = \mu mg - |f_{23}|
\]
\[
0 = \mu mg - |f_{23}|
\]
\[
|f_{23}| = \mu mg
\]
\[
|f_{23}| \leq 4\mu mg
\]

\[
ma_3 = |f_{23}| - |f_3|
\]
\[
m(0) = \mu mg - |f_3|
\]
\[
0 = \mu mg - |f_3|
\]
\[
|f_3| = \mu mg
\]
\[
|f_3| \leq 9\mu mg
\]

The required frictional forces are less than the maximal values that may be provided by static friction, so \( m_2 \) and \( m_3 \) will remain motionless.

The force upon \( m_1 \) remains the same (\( \mu mg \) to the left), so the equation of motion for \( m_1 \) remains the same.

We may now solve for \( t_5 \), the time when \( m_1 \) comes to rest.

\[
v_1 = 0
\]
\[
\left(\frac{1}{6}\right) v_0 - \mu g t_5 = 0
\]
\[
t_5 = \left(\frac{1}{6}\right) \left(\frac{v_0}{\mu g}\right)
\]

At time \( t_5 \), all three boards have come to rest. Therefore \( t_5 \) is the time when the system comes to rest, \( \tau_2 \).

\[
\tau_2 = t_5
\]
\[
\tau_2 = \left(\frac{1}{6}\right) \tau_1
\]

To employ the language of the problem as stated:

If \( t \) is the time required for the system to come to rest after an impulsive force is exerted on the top board, \( \frac{t}{6} \) would be the time required for the system to come to rest if the same impulsive force were to be exerted on the bottom board instead.

\( \text{(Submitted by Thomas Olsen, Dar Al Uloom University, College of Medicin, Riyadh, Saudi Arabia)} \)
We also recognize the following successful contributors:

Salvatore Basile (Università degli Studi di Palermo, Palermo, Italy)
Phil Cahill (The SI Organization, Inc., Rosemont, PA)
Supriyo Ghosh (Kolkata, India)
Daniel Mixson (Naval Academy Preparatory School, Newport, RI)
Jason L. Smith (Richland Community College, Decatur, IL)

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Many thanks to all contributors and we hope to hear from many more of you in the future! Many thanks to all contributors; we hope to hear from many more of you in the fall. We also hope to see more submissions of the original problems – thank you in advance!

--Boris Korsunsky, Column Editor