**A ring on a string**

A small marble of charge $q$ and mass $m$ can slide without friction along a long, thin vertical rod passing through the center of a horizontal conducting ring of radius $r$, mounted on an insulating support. What is the magnitude of the minimum charge $Q$ placed on the ring that would allow the marble to oscillate along the rod?

**Solution:**

In order for the mass $m$ to oscillate along the rod at some point $y$, it is necessary that the following conditions are met (figure):

1. The equilibrium condition, i.e.:
   
   $$ F_{\text{total} - y} = \sum F_y = 0 $$
   
   $$ F_{\text{total} - y} = E_y q - mg $$

   
   where $E_y$ is the electric field of the ring at the location of the mass:

   $$ E_y = \frac{1}{4 \pi \varepsilon_0} \frac{Qy}{(y^2 + R^2)^{3/2}} $$

   $$ E_y = \frac{1}{4 \pi \varepsilon_0} \frac{qQy}{(y^2 + R^2)^{3/2}} = mg. \tag{1} $$

2. It is also necessary to have a stable equilibrium, i.e.:

   $$ \frac{dF_{\text{total} - y}}{dy} = 0 $$

   $$ \frac{dE_y}{dy} = 0. \tag{2} $$

From Eq. (2) we obtain: $-2y^2 + R^2 = 0$. \tag{3}

From Eq. (3) we obtain the value of $y$

$$ y = \pm R \sqrt{\frac{1}{2}}. $$

We use the obtained value of $y$ in Eq. (1) and finally we obtain

$$ Q = \pm \frac{mg2\pi\varepsilon_0 R^2 \sqrt{27}}{q}. $$

So, we have two possible solutions:

- if $y > 0$ then $Q = -\frac{mg2\pi\varepsilon_0 R^2 \sqrt{27}}{q}$.
- if $y < 0$ then $Q = -\frac{mg2\pi\varepsilon_0 R^2 \sqrt{27}}{q}$.

The final solution can be written in the form:

$$ Q_{\text{min}} = -\frac{mg2\pi\varepsilon_0 R^2 \sqrt{27}}{|q|}. $$

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– We ask that all solutions, preferably in Word format, be submitted to the dedicated email address challenges@aapt.org. Each message will receive an automatic acknowledgment.
– The subject line of each message should be the same as the name of the solution file.
– The deadline for submitting the solutions is the last day of the corresponding month.
– Each month, a representative selection of the successful solvers’ names will be published in print and on the web.
– If your name is—for instance—Donald Duck, please name the file “Duck16May” (do not include your first initial) when submitting the May 2016 solution.
– If you have a message for the Column Editor, you may contact him at korsunbo@post.harvard.edu; however, please do not send your solutions to this address.

Many thanks to all contributors; we hope to hear from many more of you in the future. We also hope to see more submissions of the original problems – thank you in advance!

Boris Korsunsky, Column Editor