Upper Division

Homework Questions: Geometrical Optics & Angular Momentum

Description: These open-ended homework prompts encourage students to reveal their thinking about the geometry of eclipses and the duration of the "eclipse season."

1. Geometrical optics

A. Understanding the relative positions of the Sun, Earth, and Moon in order for the umbra or penumbra in a solar eclipse to fall onto the Earth.

Requisite concepts:

Students should be able to understand how shadows are formed by an obstacle placed between an extended light source and a viewing screen. In particular, they need to recognize that an extended light source can be treated as an array of many closely-spaced point-source bulbs, each emitting light in all directions. The students should also be familiar with the definitions of umbra and penumbra, particularly in the context of solar eclipses. Students should also be familiar with creating ray diagrams from light sources.

1.1 The side-view diagram shown here (Fig. 1) illustrates a situation in which a small obstacle is placed between two very small light bulbs and a small viewing screen. (In this problem, you may treat each very small bulb as a point source of light.) Treat this figure as being drawn *to scale.*

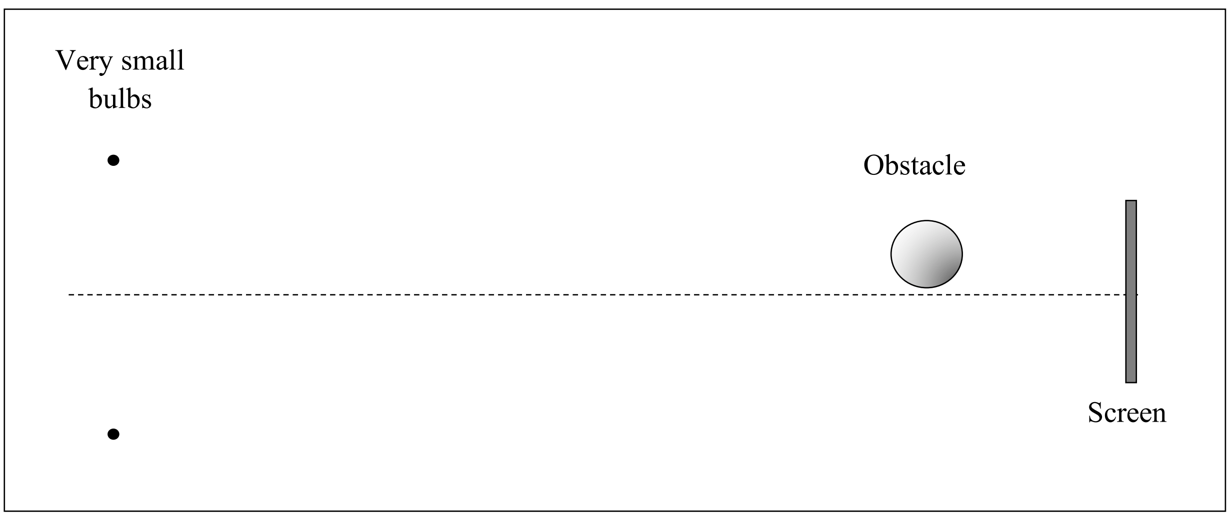

Fig. 1: Side view diagram for Problem 1.1. Diagram is drawn to scale.

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<u>Using a straightedge</u>, trace light rays directly onto Fig. 1 to determine the following:

(a) For which parts of the screen, if any, does the obstacle block *all* of the light that would reach those parts of the screen from the two point sources? Draw rays on the diagram to identify which portion(s) of the screen receive no light from the bulbs.

(b) For which parts of the screen, if any, does the obstacle block *some* (but not all) of the light that would reach those parts of the screen from two point sources? Draw rays on the diagram to identify which portion(s) of the screen receive light from one bulb but not the other.

1.2 The side-view diagram shown here (Fig. 2) illustrates a situation in which a small obstacle is placed between a long-filament bulb and a small viewing screen. As you did in Problem 1.1, treat this figure as being drawn *to scale.*

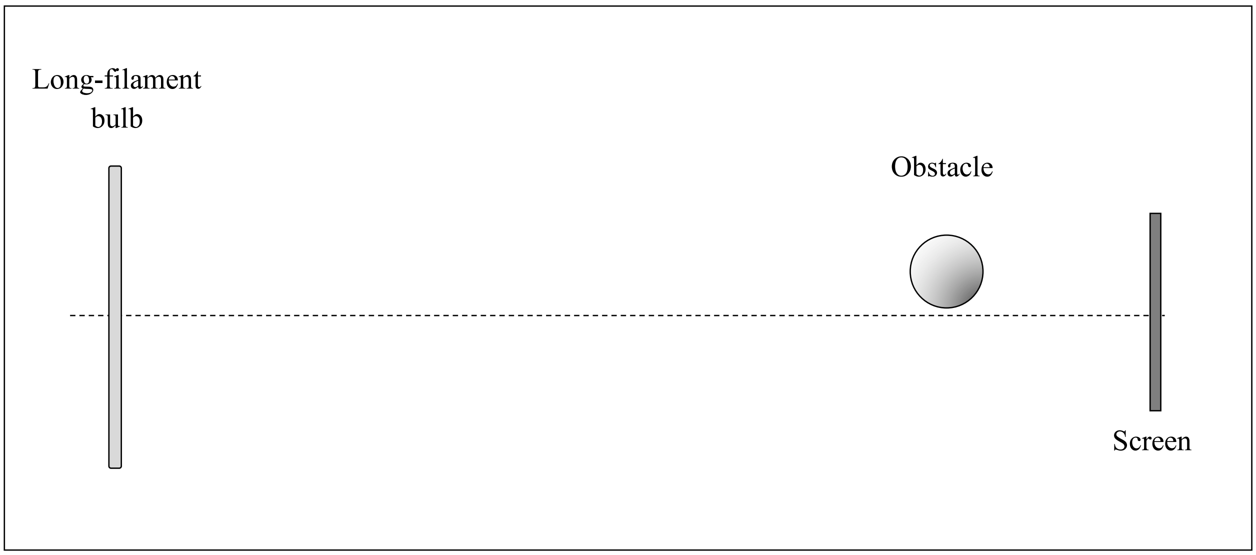

Fig. 2: Side view diagram for Problem 1.2. Diagram is drawn to scale.

As you did in Problem 1.1, use a <u>straightedge</u> and ray tracing techniques to determine the following:

(a) For which parts of the screen, if any, does the obstacle block *all* of the light that would reach those parts of the screen from the long-filament bulb? Draw rays on the diagram to identify which portion(s) of the screen receive no light from any part of the long-filament bulb.

(b) For which parts of the screen, if any, does the obstacle block *some* (but not all) of the light that would reach those parts of the screen from the long-filament bulb? Draw rays on the diagram to identify which portion(s) of the screen receive light from some (but not all) parts of the long-filament bulb.

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1.3 The situation from Problem 1.2 is now changed by moving the obstacle to a lower position relative to the long-filament bulb and screen. (See Fig. 3 below.) As before, treat this figure as being drawn *to scale.*

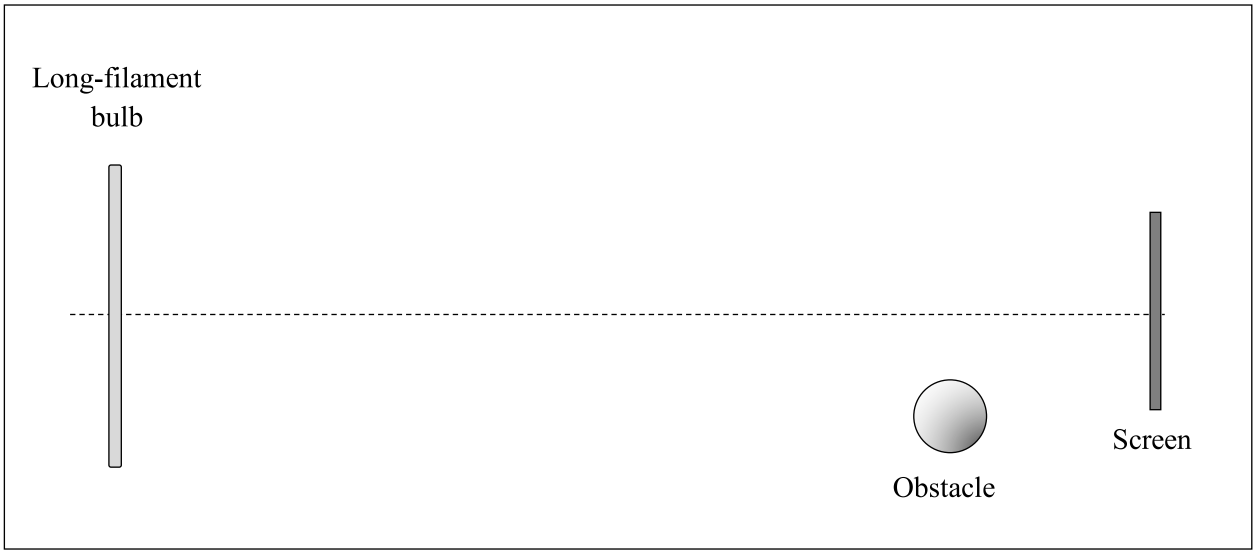

Fig. 3: Side view diagram for Problem 1.3. Diagram is drawn to scale.

As you did in Problem 1.2, use a straightedge and ray tracing techniques to determine the following:

(a) For which parts of the screen, if any, does the obstacle block *all* of the light that would reach those parts of the screen from the long-filament bulb? Draw rays on the diagram to identify which portion(s) of the screen—if any—receive no light from any part of the long-filament bulb.

(b) For which parts of the screen, if any, does the obstacle block *some* (but not all) of the light that would reach those parts of the screen from the long-filament bulb? Draw rays on the diagram to identify which portion(s) of the screen receive light from some (but not all) parts of the long-filament bulb.

Concepts tested:

Problems 1.1 through 1.3, designed to be used in sequence, test whether students recognize how to trace and interpret light rays from an extended source (particularly, the edges of that source) in order to determine which parts of a viewing screen (i) receive no light from any part of the source and (ii) receive light from some (but not all) parts of the source.

(Completing these problems, students can make sense of the terms *umbra* and *penumbra* before they are formally introduced in class.)

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B. Accounting for the duration of the "eclipse season" that occurs when Moon is near node

Requisite concepts:

- Determining the extent of the umbra and penumbra formed by an obstacle placed between a screen and an extended light source *(emphasized in the preceding problems, 1.1 through 1.3)*
- Right-triangle trigonometry
- Small-angle approximations for $\sin\theta$ and $\cos\theta$
- As Earth revolves around the Sun, the nodes (intersections between the ecliptic and the Moon's orbit) rotate at the same rate relative to the Sun-Earth line

1.4. Solar eclipses seem to be rare events, however there must be two solar eclipses in any given year. In fact, it is possible for consecutive solar eclipses to occur within a month apart from one another! In this problem you will be guided through the process to make sense as to why two consecutive solar eclipses—though they may be partial ones—could occur less than 30 days apart.

The side-view diagram below (Fig. 4) illustrates a situation in which a spherical obstacle (Moon) is placed between (though not *directly* in line with) a large, round, frosted lamp (Sun) and a globe (Earth). We will examine this situation as an analog to the Sun-Earth-Moon system (despite the fact that it is obviously not configured to be to scale with the actual Sun-Earth-Moon system).

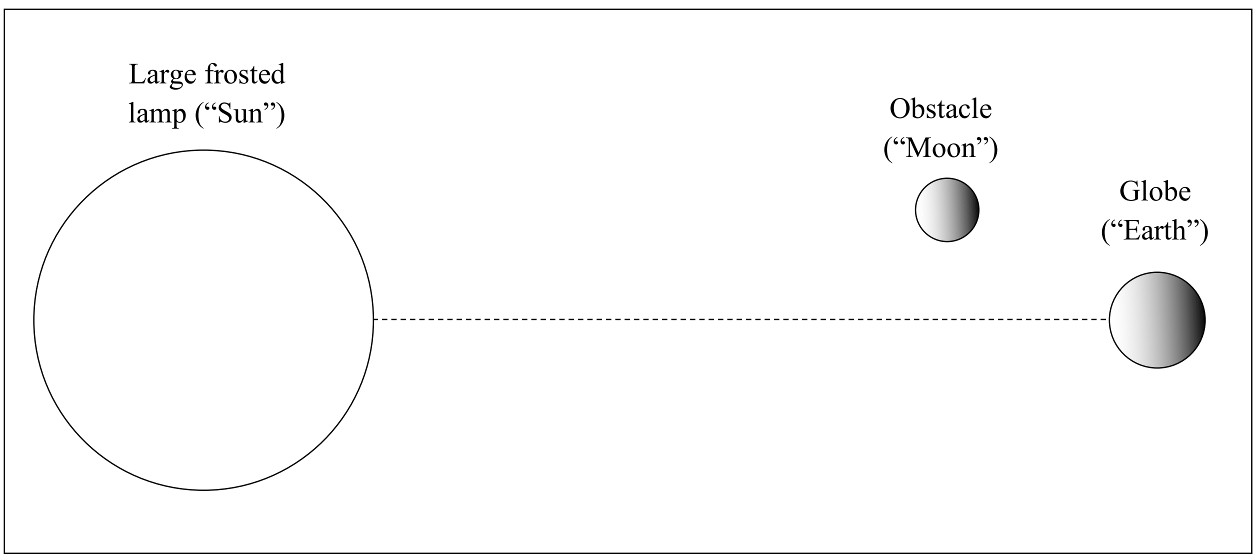

Fig. 4: Side view diagram for Problem 1.4.

As demonstrated in the preceding problems (Problems 1.1 through 1.3), an obstacle placed near an extended light source can create different types of shadows. A region of space that receives no light from any part of the source is called the umbra; a region that receives light from some (but not all) parts of the light source is called the penumbra.

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(a) On Fig. 4 (above), use a straightedge and ray tracing techniques to show that (i) the umbra produced by the obstacle (“Moon”) does not land on the globe (“Earth”), and (ii) that the penumbra *barely misses* the globe (“Earth”) by just grazing the North Pole. Clearly label the umbra and penumbra on your completed diagram.

(b) In this part of the problem, use the following variables names:

R_E: Mean distance between the Sun and Earth (center to center)
R_M: Mean distance between Earth and Moon (center to center)
r_S: Mean radius of the Sun
r_E: Mean radius of the Earth
r_M: Mean radius of the Moon
θ: Angle subtended by the Sun-Earth line (ecliptic) and Moon-Earth line

Note that because the plane of the Moon’s revolution around the Earth makes an angle of 5.145° (on average) relative to the ecliptic, the angle θ (measured above or below the ecliptic) cannot exceed this value. However, the question here is: what is the ***critical (smallest) value*** θ_c that this angle can attain in order for the Moon’s penumbra to completely miss Earth?

We can find the value of the angle θ_c by analyzing the two right triangles drawn in Fig. 4a (below) with the gray dashed line segments. (One triangle extends from the Sun to the Moon; the other, from the Sun to Earth.) Use these two triangles to prove the relationship below. (*Note:* The line connecting the center of the Earth and the center of the Moon (which helps define θ_c) is not part of these triangles. However, you should recognize one of the dashed lines from your work in part (a)!)

$$\frac{r_S - r_E}{R_E} = \frac{r_S + r_M - R_M \sin\theta_c}{R_E - R_M \cos\theta_c}$$

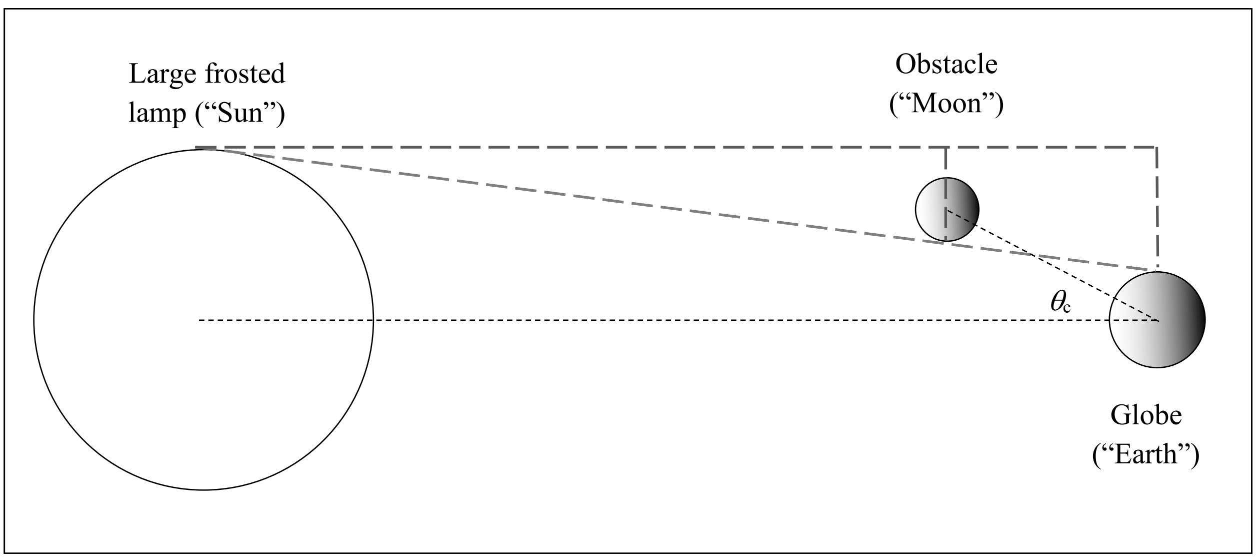

Fig. 4a: Side view diagram for Problem 1.4.

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(c) Explain why θ_c can be regarded as a *small angle,* and then use the small-angle approximation for $\sin\theta_c$ and $\cos\theta_c$ along with the accepted values for the various astronomical parameters to solve for θ_c. (*Note:* You should obtain a value between 1° and 2°.)

(d) Your results in part (c) have important ramifications for solar eclipses. Ideally, the Moon would be found exactly at the location of one of the nodes in its orbit (that is, at an angle $\theta = 0.00°$ from the ecliptic) when it enters a new moon phase. However, at the time of new moon, your result in part (c) suggests that the Moon can be slightly above or below the ecliptic—hence, making θ nonzero in value—and still cause a solar eclipse (at least a partial one, if not a total or annular eclipse). A nonzero value for θ means that the node in the Moon's orbit is located some small azimuthal angle φ away from the Sun-Earth line (where φ is measured within the ecliptic plane).

Given that the Moon's orbit makes an angle of 5.145° relative to the ecliptic, use your answer from part (c) to estimate the maximum value for the aforementioned angle φ that would allow a solar eclipse to occur. You should find that your value for φ corresponds to the angle swept by a node (as the Earth moves around the Sun) over the course of more than 16 days, and hence that the duration of an "eclipse season" at a nodal crossing is twice that much, or more than 32 days. (In other words, two consecutive solar eclipses could occur within the same synodic month!)

(*Note:* The results obtained here are approximate; for example, our treatment here assumes that the Moon is always the same distance from the Earth.)

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