QUAD III
KINEMATICS AND SCIENTIFIC METHODS
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THE MECHANICAL UNIVERSE

High School Adaptation

A co-production of the
California Institute of Technology
University of Dallas
and
Southern California Consortium

QUAD III
KINEMATICS AND SCIENTIFIC METHODS

The Law of Falling Bodies
Inertia
Moving in Circles
The Millikan Experiment

An Annenberg/CPB Project
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Materials Development Council

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FOREWORD

Today, scientific and educational leaders are seriously concerned about the quality of science and mathematics education in the United States. It is as though the problems have been rediscovered, 25 years after Sputnik! In addition to those problems which have repeated themselves, today many qualified science and mathematics teachers at the pre-college, college, and university levels are being lured from the classroom by higher-paying jobs in business and industry. Many classrooms, therefore, have become the responsibility of instructors with limited preparation in the subject matter they are called upon to teach. And yet, more than ever the nation's current economic, social, and political needs call for a technologically literate population.

The Mechanical Universe, which served as the basis for the high school materials, addresses one critical need in science education by providing video and print materials that can serve as the basis of a solid, introductory college-level physics course. The video offers an exciting array of audiovisual resources for classroom instruction: close-ups of complicated experiments; extensive computer animation sequences that make abstract concepts and mathematical processes understandable; historical reenactments that provide a philosophical fabric for the development of ideas of physics.

The Mechanical Universe, part of the Annenberg/CPB collection, has as its primary purpose the provision of a quality learning experience for those whose lives cannot fit into the traditional campus schedule. This 52-program introduction to physics also offers a partial answer to some of the current problems of science education, for it can be used to upgrade skills of secondary science teachers and to provide supplementary support in the college and university classes.

Through the sponsorship of the National Science Foundation, selected programs of The Mechanical Universe have been adapted for use in high school. These materials represent the same quality and innovation as the college series, but they are presented in shorter and less mathematically oriented tapes that can be used in a wide variety of high school curricula. Teachers who find themselves teaching high school physics in spite of limited preparation will discover that, by enrolling in The Mechanical Universe course and using the adaptations in their classes, they will enjoy the confident feeling that they are presenting their students with quality instruction.
INTRODUCING
THE MECHANICAL UNIVERSE
High School Adaptation

The adaptations of The Mechanical Universe were created by twelve outstanding high school physics teachers (the Materials Development Council) through the generous support of the National Science Foundation. The clear purpose of the Council and the entire staff was to produce quality materials that would be used to improve instruction in physics. No one was satisfied with the goal of producing materials that would simply motivate or fascinate students, or would provide a change of pace. From the start, the challenge was to create materials which could make wise use of the power of television in developing a sound and solid understanding of physics.

Here with the fruit of these labors: sixteen modules each consisting of a video adaptation from The Mechanical Universe with written support materials. Each module stresses conceptual understanding of underlying physical principles. The written materials support the video dimension of the modules. These support materials provide the teacher with additional background information and mathematical derivations, pre-video and post-video questions, applications, demonstrations, and evaluation questions.

The Mechanical Universe was originally developed for lower-division college courses in physics. The materials from The Mechanical Universe that have been adapted for use in high schools were field tested in 1984-86 by over 100 high school physics teachers located in schools widely scattered across the county in both urban and rural communities that serve various socio-economic populations. As a result of the assessment of the field testing, the videos were re-edited and the written materials were focused more directly on the videos to provide the best support possible for teachers.
PREFACE

These materials are intended for all teachers of high school physics. Teachers new to the arena of physics will discover rigorous, conceptual video presentations of traditional and not-so-traditional topics in classical physics. We hope that each word of the written materials will be savored. They are your resources and we hope that you tap them to capture the excitement of The Mechanical Universe. Experienced teachers will find a different slant to classical physics in the space age: a humanizing, compelling, integrated approach to the greatest revolution in the history of Western civilization. These teachers, too, we hope, will find the written materials continually refreshing resources.

Although The Mechanical Universe is a calculus-based course, the excerpts for high school use were selected to focus on concepts. That is not to say that the videos for high school use are not rigorous; they present sound logic at every stage in the development. Mathematics is occasionally used in the high school materials as a language to relate ideas concisely. In many cases the original mathematical derivations have been modified to be appropriate to the high school level. Nonetheless, mathematical derivations go by quickly in the video and we hope that teachers will replay these sections for their students. The mathematical background sections of the modules, we expect, will be read by all teachers even though they may not necessarily present to their classes the same level of mathematics provided in the print materials. We hope that teachers as well as students will gain a better appreciation of the vital role of mathematics in physics.

No laboratory component is currently suggested. The reason is not because we judge a physics laboratory component to be unimportant or uninteresting. On the contrary, we believe that demonstrations and laboratories lie at the heart of a sound education in high school physics. Instead we concentrated on what we could offer best: instruction through television. There are dozens of laboratory manuals which can be appended easily to these materials and we expect that each teacher will decide how best to handle the laboratories. On the other hand, since many demonstrations and applications to everyday life are presented in the video, we identified simple, short, and effective demonstrations that tie into concepts in the video. We hope that all physics teachers will enjoy performing them.

Not all the topics covered in the modules are conventional to high school physics curricula. Angular Momentum and Harmonic Motion, effectively covered in the videos, are two topics which are not necessarily a part of every curriculum. Navigating in Space, on the other hand, represents an exciting application of Kepler's ellipses and Newton's gravity that is not covered in typical curriculum. Other topics, such as The Fundamental Forces and Curved Space and Black Holes, provide tantalizing looks at twentieth century physics from the perspective of classical physics.

The Mechanical Universe is the story of the Copernican revolution, why it was necessary, and how it unfolded in the work of Galileo, Kepler, and Newton. It is the story of the eventual wedding of the heavens with the earth through the synthesis of mechanics and astronomy. History is presented in the series, not for the sake of historical detail, but for a fuller sense of how scientific thought proceeded through the intellectual searches and triumphs of men who reshaped the society of their times. We hope the infectious spirit of The Mechanical Universe will inspire teachers and students and will contribute to a lifelong scientific interest in the workings of the universe.
ACKNOWLEDGEMENTS

The adaptations of these instructional materials for high school use would not have been possible without the assistance of a long list of people who aided through the dedicated use of their diverse and specialized skills.

Heading the list is Professor David L. Goodstein, of Caltech, whose inspiration and guiding force in the creation of The Mechanical Universe led to the development of these materials.

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STRUCTURE OF THE MATERIALS

The written materials are designed to support and extend the VIDEO presentation of each module. The format and content of the materials are designed to help the user (1) to integrate the concept(s) presented in the VIDEO with traditional high school materials, (2) to supplement and promote conceptual understanding of the phenomena presented in the VIDEO, and (3) to infuse the students with a new spirit of inquiry concerning the mechanics of physics.

Each module is composed of components of written materials. Each component is intended as a resource to promote active engagement of the learner in developing conceptual understanding of the physical phenomena. The five components of the print materials are:

**TEACHER'S GUIDE**

- **Content and Use of the Video** — describes what the VIDEO does and does not cover.
- **Terms Essential for Understanding the Video** — includes the definitions of terms listed in the STUDENT'S GUIDE, discussion of critical elements or relationships.
- **What to Emphasize and How to do It** — includes the objectives of the module, references to demonstrations, possible applications, and suggestions for correcting common misconceptions.
- **Points to Look for in the Video** — includes common misconceptions when relevant; characteristics and questions concerning critical elements presented in the VIDEO. Answers to questions in the STUDENT'S GUIDE are included.
- **Everyday Connections and Other Things to Discuss** — suggests additional questions to promote student participation and discussion. An essential purpose of the questions is to engage students in review and clarification of the concepts.
- **Summary** — reviews the key concepts that have been presented.

**STUDENT'S GUIDE**

(Designed for duplication and distribution)

- **Introduction** — a brief statement about the content or purpose of the VIDEO.
- **Terms Essential to Understanding the Video** — includes terms or critical elements of the VIDEO, with definitions and explanations provided in the TEACHER'S GUIDE.
- **Points to Look for in the Video** — includes common misconceptions when relevant; characteristics and questions concerning critical elements presented in the VIDEO along with figures representative of key points in the VIDEO.

**TEACHER RESOURCES**

- **Supportive Background Information** — summarizes additional historical, physical, and mathematical information that relate to the topics and content presented in the VIDEO.
- **Additional Resources** — includes demonstrations and applications the teacher may use to extend and enrich the treatment of the topic.
- **Evaluation Questions** — provides ten multiple-choice questions dealing with the objectives of the module and two essay questions that require student's explanations of certain concepts related to the topics.

*The repeated showing of the video (in full and part) is essential to student understanding. The division of activities into prevideo and postvideo activities, therefore, is somewhat artificial. It is likely that most, if not all, prevideo activities will precede the initial showing of the video. Sections of the video will undoubtedly be sprinkled throughout the postvideo activities, with a full showing being used for closure where time permits.*
QUAD III

THE LAW OF FALLING BODIES

DO HEAVIER BODIES FALL FASTER THAN LIGHTER ONES? The way that Galileo arrived at an answer to this question was as revolutionary as the answer itself. Although Galileo could not create a vacuum, he could imagine one, and so he realized that in a vacuum all bodies fall with the same constant acceleration. In this video, Galileo's investigation of the law of falling bodies is presented with an emphasis on scientific methods. Galileo's pioneering work in quantifying empirical data earned him the title of "The Father of Modern Science."


THE LAW OF INERTIA

HOW CAN THE MOTION OF A FALLING OBJECT APPEAR THE SAME ON A MOVING EARTH AS ON A STATIONARY EARTH? In defending the Copernican system, Galileo discovered that the description of motion of objects on earth demanded an understanding of inertia quite different from the prevailing Aristotelian conception. He realized that the tendency of an object was not, as Aristotle thought, to reach a state of rest, but rather to continue moving without any propelling force. In this video, the methods Galileo used to arrive at the law of inertia and its immediate consequence, the relativity of motion, are examined. Through his persevering work in mechanics and astronomy, Galileo drove the last nail in the coffin of the Aristotelian world view and firmly established the validity of the Copernican system.

*Running Time: 18:11.*

MOVING IN CIRCLES

HOW DO WE DESCRIBE THE CIRCULAR MOTION OF HEAVENLY BODIES? The stars, said Plato, represent eternal, divine, unchanging beings that move at uniform speed around the earth in the most regular and perfect of all paths—an endless circle. Plato accepted motion in a circle with constant speed—uniform circular motion—as so simple and natural an idea that it needed no explanation. In this video, the kinematics of circular motion is explored; the relationships between radius, velocity, and acceleration are developed. When Newton's universal law of gravitation is incorporated as the driving force, the circular motion of planets, and much more, is revealed.

*Running Time: 7:00.*

THE MILLIKAN EXPERIMENT

HOW DOES SCIENCE PROGRESS? The experimental work of Robert A. Millikan indicates that science progresses through painstaking care in measurements, extraordinary diligence in eliminating errors, and creativity in improving existing methods. In this video, the spirit of scientific discovery is conveyed through Millikan's work in determining the charge of the electron. The jottings in Millikan's laboratory notebooks reveal an interesting relationship between scientific judgment and experimental apparatus.

*Running Time: 15:06.*
TEACHER'S GUIDE TO THE LAW OF FALLING BODIES

CONTENT AND USE OF THE VIDEO - Galileo's law of falling bodies is traditionally introduced early in a high school physics course during the study of kinematics. The introduction of free fall should be preceded by considerable work with both uniform and accelerated motion. The historical development of the thinking about freely falling bodies is not usually stressed at the high school level. However, Galileo's contributions to scientific methodology are considered so important that they constitute a major theme throughout the video.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - Terms are introduced in the video which the student might have heard but whose technical use is still unclear. It would be helpful to review these terms prior to viewing the video. All terms are defined according to their use within the context of this video:

- **time rate of change** of a quantity--the difference between the final and initial value of a quantity divided by the time for the change.

- **speed**--the time rate of change of distance.

- **velocity**--the time rate of change of distance in a particular direction.

- **acceleration**--the time rate of change of velocity.

- **terminal velocity**--a constant velocity achieved by some bodies falling through air. This velocity is the result of air resistance becoming equal to downward force of gravity.

- **direct proportion**--relation between two quantities such that if one increases, then the other increases proportionately; if one decreases, then the other decreases proportionately.

- **free fall**--the vertically downward motion of a body accelerated by a gravitational force.

WHAT TO EMPHASIZE AND HOW TO DO IT - At a time when all the world believed that heavy bodies fall faster than lighter ones, Galileo realized that in a vacuum all bodies fall with the same constant acceleration. This concept, together with the experimental procedures in which Galileo engaged while verifying it, constitutes the focus of the video.

Since students see objects falling at different rates in their environment, the effect of air resistance on falling bodies should be discussed. Although the term "terminal velocity" is not introduced in the video, the concept is important for students to understand. The video should make students more aware of the motion of falling bodies in their everyday world and in the world of the vacuum that Galileo imagined.
Objective 1: Recognize that the distance all bodies fall in a vacuum is proportional to the square of the time \((s \propto t^2)\).

Galileo rolled balls down inclined planes to demonstrate the relationship between distance and the time squared. He gradually increased the angle of incline and from these data he was able to extrapolate how a body would fall if the incline were raised to 90\(^\circ\), thus illustrating free fall. DEMONSTRATION #5 should be performed as a part of the discussion of the connection between distance and the square of time as related to falling bodies.

Objective 2: Recognize that the instantaneous velocity (speed) of any falling body in a vacuum is proportional to time, \(t\); that is \(v \propto t\).

When air resistance is neglected, the velocity of a falling body increases in direct proportion to the time of fall. DEMONSTRATION #3 is helpful in developing this concept. You might also discuss with your students why springboard divers and gymnasts on trampolines try to attain a maximum height. Since distance is proportional to the square of the time, maximum height allows maximum time for performing routines.

Discuss amusement park rides like the free fall ride at Magic Mountain, which is shown in the video. Ask students to describe the sensation that they feel as they take such a ride. Since the acceleration of all objects in free fall is the same, everything feels and looks like it is falling with you; the seat exerts no upward force on you nor do the restraints exert a downward force.

Some students may be familiar with objects falling with constant speed. You might discuss skydiving as an illustration of a free fall with air resistance. Skydivers assume a position that will maximize the effect of air resistance on the diver, thereby retarding downward motion. The difference in terminal velocity between streamlining and a spread-eagle fall can amount to as much as 100 miles an hour. Point out that if a skydiver were on the airless surface of the moon, he would fall with a speed directly proportional to time.

Objective 3: Recognize that in a vacuum all bodies fall with the same constant acceleration.

Students often notice that in the everyday world some bodies fall faster than others. In a vacuum, and only in a vacuum, do bodies fall with the same acceleration. The difference in the rates observed is due to air resistance opposing the motion of bodies not in a vacuum.

DEMONSTRATIONS #1, #2, and #4 will give students an awareness of constant acceleration. A discussion of the acceleration due to gravity, commonly given the notation \(g\), should follow these demonstrations. Point out that the value of \(g\) depends on the geographical location of an object, but that the average usually used is 9.8 m/s\(^2\) or 32 ft/s\(^2\).
Objective 4: Identify the significant aspects of the historical environment which gave rise to the development of the law of falling bodies.

The significance of Galileo's descriptions of motion can be fully understood only when contrasted with Aristotelian physics which Galileo's ideas eventually superseded. During Galileo's life, the Copernican (sun-centered) system was still a revolutionary theory and the most disputed scientific question of the time. Through his fertile experimentation and keen insight into natural phenomena, Galileo helped solidify Copernican theory and hastened the replacement of Aristotelian phenomenology with the science of mechanics.

In order to encourage students to think about the dynamics of falling bodies, you might perform the following activity: Take two identical heavy objects such as shoes. Tie a lighter object to one of the heavy objects with a string. Hold the single object and the compound object side by side as if to drop them. Ask the question, Which one will hit the floor first?

1. If the students answer "the heavier object tied to the lighter one," respond as an Aristotelian by reminding them that the lighter object would hinder the speed of the heavier object.

2. If the students answer "the heavy object alone," respond as an Aristotelian by reminding them that the lighter object tied to the heavier object forms a still heavier object.

3. If the students answer that the two would hit at the same time, respond as an Aristotelian by reminding them that the two systems are not equally heavy.

The described activity provides a good introduction to the video. The historical sketch in ADDITIONAL RESOURCES can also help students to appreciate Galileo's role in examining free fall.
POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with suggested responses and frames from the video.

Will the compound body fall faster or slower than the heavy body alone?

All bodies, regardless of their weight, fall at exactly the same acceleration once the effect of air resistance is removed.

How do the data accompanying the free fall ride relate to Galileo's theory of free fall?

Instead of increasing as the integers, Galileo's theory was that, in successive intervals of time, the distances traveled by a freely falling body should follow the odd numbers, falling one unit of distance in the first time interval, three units of distance in the second, etc.

Galileo reached his conclusions after a brilliant series of experiments in which he timed balls as they rolled down steeper and steeper inclines, moving closer and closer to the vertical path of a free falling body.

After three seconds, how far has a falling body traveled? What is its velocity at this time? (Use \( g = 9.80 \text{ m/s}^2 \))

The body has traveled 44.1 meters. Its velocity is 29.4 meters per second.

\[
\text{acceleration} = g \\
v = gt \\
s = \frac{1}{2} gt^2
\]
EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the ideas contained in the video, you might pose the following questions to your students.

1. Why does an acorn fall to the ground more quickly than an oak leaf?

*The effects of air resistance must be considered. Observations indicate that air resistance is proportional to the surface area of an object. The surface area of the leaf is larger than that of the acorn; therefore, the upward push of the air on the leaf has a greater effect than on the acorn. Consequently, the leaf falls more slowly.*

2. When astronaut David R. Scott of the Apollo 15 Mission found himself on the surface of the moon, he simultaneously dropped a hammer and a feather. This event is dramatized in the video. What were the results and why?

*The hammer and feather hit the surface of the moon simultaneously; however, both fell in what appears to be slow motion. Since the surface of the moon is airless, there is no air resistance to any falling object. Because the gravitational force on the moon is 1/6 that on the earth, objects fall more slowly.*

3. Given Galileo’s values for distance traveled, the distance is:

<table>
<thead>
<tr>
<th>Distance</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>25</td>
<td>5</td>
</tr>
</tbody>
</table>

*Answer: (c)*

4. Given the following data, the velocity is:

<table>
<thead>
<tr>
<th>Velocity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

*Answer: (b)*
5. Given the following data, the acceleration is:

<table>
<thead>
<tr>
<th>Acceleration</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0 to 1</td>
</tr>
<tr>
<td>2</td>
<td>1 to 2</td>
</tr>
<tr>
<td>2</td>
<td>2 to 3</td>
</tr>
<tr>
<td>2</td>
<td>3 to 4</td>
</tr>
<tr>
<td>2</td>
<td>4 to 5</td>
</tr>
</tbody>
</table>

(a) constant.
(b) proportional to time.
(c) proportional to time squared.
(d) proportional to time cubed.

Answer: (a)

6. How would today's public react to a major new scientific discovery as compared to the public of four hundred years ago?

The public tends to react according to the paradigm of the day. Today's public, ranging from the wealthy to the poor, is more open to discovery of the unknown and the unconventional. A legal approach would likely be used to solve the controversy that might arise because of the discovery. The society of four hundred years ago was quite closed to new ideas, except from the known, wealthy, and noble. Theological decree was employed to resolve any controversy that arose due to a discovery.

NOTE: Other explanations are equally valid as long as their development is logically supported.

7. Two golf balls are dropped from the fourth-story window of a physics building one second apart. As they fall, what happens to the distance between the balls?

(a) It increases.
(b) It decreases.
(c) It remains the same.

Explain your answer.

The distance is proportional to the square of the time falling. The longer an object is in free fall, the more distance it has fallen. Since the first ball is in the air longer, the distance between the two balls increases.

8. If the cable broke while you were riding in an elevator, what would happen if you tried to sit down?

If the cable broke, both you and the elevator would accelerate in free fall. You would be falling at the same rate as the walls around you. Since everything is falling at the same rate, you would be unable to sit down unless you grabbed a railing inside the elevator.

9. A baseball thrown straight up rises for two seconds before it begins to fall back to earth. A second baseball falling from rest takes two seconds to reach the earth. What do the motions of the two baseballs have in common?

The two balls fall from the same height since their time of fall is the same. During the fall, both have the same constant acceleration g. Therefore, both balls strike the ground with the same velocity.
SUMMARY - The video describes Galileo's ingenious experiments which determined the relationship between distance and time for a body in free fall. He concluded that the distance an object falls is directly proportional to the square of the time of free fall. The speed or velocity at which bodies move varies directly with the time. Galileo formulated the law of falling bodies which says that all bodies fall with the same acceleration. For bodies in free fall this is called gravity and is a constant with the special notation g. The average value of g is 9.8 m/s² or 32 ft/s².

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed prior to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that follow the viewing(s) of the VIDEO and give closure to the lesson.
STUDENT'S GUIDE TO THE LAW OF FALLING BODIES

INTRODUCTION - Although Galileo could not create a vacuum, he could imagine one, and so he realized that in a vacuum all bodies fall with the same constant acceleration. This video investigates the relationship between time, distance, and acceleration for bodies in free fall.

Terms Essential for Understanding the Video

time rate of change
velocity
terminal velocity
free fall

speed
acceleration
direct proportion

*** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ***

Points to Look for in the Video

Will the compound body fall faster or slower than the heavy body alone?

How do the data accompanying the free fall ride relate to Galileo's theory of free fall?
After three seconds, how far has a falling body traveled? What is its velocity at this time? (Use $g = 9.80 \text{ m/s}^2$)

\begin{align*}
\text{acceleration} &= g \\
v &= gt \\
s &= \frac{1}{2} gt^2
\end{align*}
TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - Prior to Galileo, much of the thinking about freely falling bodies was qualitative and did not involve experimentation. For centuries, only words were used to describe the motion of objects – words based on the writings of the Greek philosopher Aristotle (about 350 B.C.). These descriptions presupposed that everything had a "natural place" and, when removed from its natural place, possessed a tendency to return. Hence, fire (or smoke) rose through air to return to its natural place, whereas rocks fell through both air and water to return to their natural place, the earth. In explaining the behavior of falling bodies, Aristotle reasoned that mass is a factor governing the speed of the fall. He believed that the heavier an object, the greater its content of earth and, therefore, the stronger its tendency to return to the earth. Thus, heavier bodies would fall faster than lighter bodies.

Since every falling body encounters air resistance during its fall through the air, Aristotle's description of motion was well supported by everyday observations. When asked how bodies would fall through a vacuum, Aristotle argued that, without the resistance of air, all bodies would fall with the same infinite speed. However, such motion contradicted his description of falling bodies, and since direct observation and logic were the prevailing methods of proof in Aristotle's time, motion through a void was dismissed as inconceivable. For Aristotle, a vacuum was impossible.

Almost 2,000 years passed before there emerged a figure whose brilliant insight could break the stranglehold of Aristotelian influence on science. Galileo Galilei could not produce a vacuum, but he could imagine one. To understand the motion of falling bodies, Galileo knew that air resistance had to be ignored. In so doing, Galileo paved the way for restoring the connection between the heavens and the earth. Aristotle had postulated that the terrestrial matter of fire, air, water, and earth differed entirely from celestial matter, or quintessence. Consequently, the heavens were thought to be governed by a different physics than was the earth. Whereas the natural motion of bodies on earth consisted of rising and falling, natural motion of celestial bodies was seen as endless revolution in circles. Galileo realized that free-fall motion was at the heart of understanding the observable motions of all bodies, both on the earth and in the heavens. Although the explanations of those motions were not heard during Galileo's lifetime, his work on free fall served as their foundation.

Before Galileo, a number of medieval scholars attempted to describe the motion of falling bodies. In the fourteenth century, Albert of Saxony argued that the speed of a body is proportional to the distance it has fallen: compared to its speed after having fallen one foot, a body is traveling twice as fast after falling two feet, three times as fast after three feet, and so on. Without any means to measure this motion precisely, Albert's argument seemed in accord with common sense. Nicole Oresme, a French mathematician, shifted his focus to the relationship between time and speed. He suggested that the speed is directly proportional to the time spent falling. In the fifteenth century, Leonardo da Vinci formulated a law in terms of quantities that might be easier to measure: intervals of distance and time. He proposed that a body would fall greater distances in successive intervals of time and that these distances would follow the consecutive integers, i.e., one unit of distance in the first time interval, two more units of distance in the second time interval, three more units of distance in the third time interval, and so on.
The formulations of Albert, Nicole, and Leonardo were based on qualitative observations. It remained for Galileo to grasp the usefulness of experimentation and the consequent quantification of observation. Although his medieval predecessors had examined the same falling motion he had, they asked different questions. Galileo’s genius lay in his ability to ask the right questions and to distinguish between that which demanded attention and that which could be ignored. He was also amazingly perceptive in performing experiments to select the best description of a phenomenon from a number of possibilities.

Galileo experimented using balls rolling down an inclined plane. He observed that in successive time intervals the total distance traveled by a ball was proportional to the consecutive odd numbers. For example, in the first time unit, the ball traveled one unit of distance, in the second time unit, it traveled three units of distance, in the third time unit, five units of distance; and so on.

Galileo’s law of odd numbers can also be stated in terms of total distance, i.e., after the first time interval, one unit of distance; after the second time interval, a total of four units of distance; after the third time interval, a total of nine units of distance; and so on. The total distance that the ball rolled down the inclined plane was proportional to the square of the time of roll. Interestingly enough, Nicole had formulated a time-squared law which provides an equivalent description of falling bodies. It is not known whether Galileo, who came three centuries later, had ever heard of Nicole, but he did use strikingly similar arguments.

Galileo arrived at his law experimentally, without the benefit of sophisticated timing devices. Instead he used a dripping water clock to time balls rolling down inclined planes. As he made the plane steeper, the ball would roll faster, but the motion always had the same property: the total distance was proportional to the square of the elapsed time. He concluded that the law would also hold if the plane were vertical, in other words, in free fall.

Galileo’s experimental methods fathered modern scientific research. Galileo himself must have been aware of their importance. In his work *Dialogues Concerning Two New Sciences*, he concludes his discussion of accelerated motion with the following:

We may say the door is now opened, for the first time, to a new method fraught with numerous and wonderful results which in future years will command the attention of other minds.
ADDITIONAL RESOURCES

Demonstration #1: Paper and Book Drop

Purpose: To demonstrate the effect of air on falling objects.

Materials: A piece of paper and a book with equal areas.

Procedure and Notes: First, place the paper and the book side by side and ask what will happen as they are dropped. Most students will answer correctly that the book will fall faster. Encourage the Aristotelian explanation, for example, “Of course the book will fall faster, it is heavier.” Now place the paper under the book. Again, ask for predictions. Drop the paper and the book and notice they fall with the same acceleration. When asked why, students will probably guess correctly, but possibly give the wrong reasons. Some will say that the book is pushing the paper ahead of it, which will lead you to your next demonstration. Place the paper on top of the book and ask what will happen when they are dropped. The answers and reasons may vary widely. Drop the paper and book. Notice that they fall at the same acceleration, since you have limited air resistance on the paper. One final demonstration supporting this concept is to wad the paper and show that it falls with roughly the same acceleration as the book.

Explanation: Although mass at first appears to influence the rate of fall of an object, surface area is the important criterion.
**Demonstration #2: Water Drop**

<table>
<thead>
<tr>
<th>Purpose:</th>
<th>To demonstrate that in free fall objects fall with the same acceleration.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials:</td>
<td>One aluminum can with the lid cut off and a hole near the bottom on the side.</td>
</tr>
<tr>
<td>Procedure and Notes:</td>
<td>First fill the can with water and let some water run out of the hole into a wastebasket. Place your thumb over the hole. Allow students to guess what will happen to the system when it is placed in free fall. Release the can into the wastebasket from a sufficient height to allow the students to see the effect.</td>
</tr>
<tr>
<td>Explanation:</td>
<td>Since the can and the water are both in free fall, no water can leave the can.</td>
</tr>
</tbody>
</table>
Demonstration #3: Velocity vs. Time

Purpose: To show that velocity is proportional to time for freely falling bodies having the same shape, regardless of their masses.

Materials: Ticker-tape timer apparatus; four pieces of ticker tape 2 m long; various masses (250 g, 500 g, 1 kg, 2 kg); sturdy ring stand; four meter sticks.

Procedure and Notes: Tape a mass to the end of the ticker tape. Place the mass about 1½ m above the floor just below the timer with the ticker tape in position to pass through the timer when the mass is released. Make ticker tapes for each of the masses. Then ask small groups at their desks to analyze the tapes, using measurements with the meter sticks. Have the groups provide the following information: Beginning with the slow-speed end of the ticker tape, measure the distance the mass falls between the fifth and sixth ticks. Call this interval \( t = 1 \). Since the speed is distance/time, the tape speed at that point (in cm/tick) is just the interval measured (in cm). Find the speeds for the successive time intervals between the tenth and eleventh ticks, the fifteenth and sixteenth ticks, and the twentieth and twenty-first ticks. Tabulate the class results on the board as follows:

<table>
<thead>
<tr>
<th>mass</th>
<th>1 tick</th>
<th>2 ticks</th>
<th>3 ticks</th>
<th>4 ticks</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 g</td>
<td>1.2</td>
<td>2.3</td>
<td>3.5</td>
<td>4.6</td>
</tr>
<tr>
<td>500 g</td>
<td>1.3</td>
<td>2.4</td>
<td>3.6</td>
<td>4.7</td>
</tr>
<tr>
<td>1 kg</td>
<td>1.3</td>
<td>etc.</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>2 kg</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

For one or more of the masses plot \( v \) vs. \( t \).

1. Was the velocity of the falling mass proportional to \( t \)?
2. Did the difference in the masses affect the results? Why?

Explanation: As indicated in the graph below, \( v \propto t \).

Since these objects fall with the same acceleration, the mass should not affect the result.
Demonstration #4: Terminal Velocity

Purpose: To demonstrate an object falling at terminal velocity.

Materials: Styrofoam packing material which has the appearance of a watch glass and the diameter of approximately 1 in.

Procedure and Notes: First it may be helpful to define terminal velocity by asking the students what is meant by the word “terminal”. For example, what is a terminally ill patient? Most students would know that the final condition of the patient is unchanging. After discussion, simply drop the packing material from a sufficient height to see the effect.

Explanation: If the room is draft-free, the packing material quickly attains terminal velocity and then continues to fall at that constant velocity.

Note: This demonstration can also be done using balloons. Drop a balloon which has not been inflated. Then drop balloons which have been inflated various amounts and observe the fall of each. (Do not use helium to inflate balloons.)
Demonstration #5: Beads on a String

Purpose: To demonstrate that for objects in free fall the time of fall and the distance fallen are related by the expression

\[ s \propto t^2 \]

and that these objects obey Galileo's law of odd numbers.

Materials: Two pieces of string 8 ft. in length and ten beads which can be strung or taped to the string.

Procedure and Notes: Five beads are strung on the first string, one at the bottom, the others at intervals of 2 ft. up the string. Stand on a chair and drop the string. Listen to the irregular sound of the bead clicks as they strike a trash can or metal plate. Successive clicks are clearly not multiples of the first interval heard. On the second string, place a bead at the bottom and the others up the string at intervals as shown in the table below. When this string is dropped, a regular pattern of clicks is heard with an interval of about 0.18 s.

<table>
<thead>
<tr>
<th>Number</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squared</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>Distance(in.)</td>
<td>0</td>
<td>6</td>
<td>24</td>
<td>30</td>
<td>42</td>
</tr>
<tr>
<td>Difference(in.)</td>
<td>6</td>
<td>18</td>
<td>30</td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>

Galileo's law of odd numbers:

- First bead: 6 in.
- Second bead: 18 in.
- Third bead: 30 in.
- Fourth bead: 42 in.

Note: Use the law of odd numbers to determine the spacing between beads, i.e., if the 1st bead is 6 in. from the end of the string, then the second must be 3 times (or 18 in.) farther, the third must be 5 times (or 30 in.) farther, etc.

Explanation: The top beads are accelerated longer, travel faster, and cover larger distances in a shorter time. The beads on the second string are strung so that the successive distances are proportional to odd numbers. Galileo's law of odd numbers can be seen to follow.
Historical Sketch

Purpose: To imagine the historical development of the treatment of free fall.

Materials: Inclined plane and two different-sized steel balls; water clock with powder funnel; one-hole cork with plastic tube and graduated cylinder; gowns and/or signs for historical characters.

Voice (offstage): The year is 350 B.C. The place is Athens, Greece. The great teacher of the day, Aristotle, is about to deliver a discourse on the topic of falling bodies.

(Arrow, Aristotle enters and moves to center.)

Aristotle: Today we are going to discuss falling bodies. It is obvious to the observer that heavy objects fall faster than light objects. This is because objects are made of earth and want to return to their own kind. The more you have of an object, the more need it will have to seek out its own kind and to do so more quickly. If an object is ten times heavier than another object, the heavier object will fall ten times faster. This is obvious. You ask: "Have you ever done an experiment to confirm this stated fact?" The answer is, of course, "No!" Philosophers like myself are thinkers and do not dirty our hands in the same way that a tent maker would do in his construction.

(Aristotle leaves stage.)

Voice (offstage): The year is 1630 A.D. The place is Pisa, Italy. Galileo Galilei, known as Galileo because we like to address our friends on a first-name basis, is at work. A representative of Aristotelian thought, Simplicio, engages him in conversation.

(Arrow, Galileo enters and begins tinkering with an inclined plane. Another gowned figure, Simplicio, a representative of Aristotelian thought, enters behind him.)

Simplicio: What are you doing, Galileo?

Galileo: By allowing balls to roll down the incline, I am experimentally attempting to find the relationship between the distance traveled and the time it takes for the ball to get to the bottom of the plane.

(Simplicio operates the water clock while Galileo takes data. He determines the time for s = 30 cm and s = 120 cm. He also takes data for two balls of different sizes rolled the same distance, e.g., 60 cm. He tries to establish the same time for both balls.)

Galileo: What do you think would be the result if we were to try this at a steeper incline, Simplicio?

Simplicio: There is nothing special about the angle you choose, Galileo, so I imagine the result would be the same.

Galileo: Let's try it at the vertical angle, since we would expect the result to be the same as on the incline.
(An attempt is made to obtain similar data, but with little success because the ball accelerates too rapidly to allow measurements.)

Simplicio: Well, it looks as if there is no way that anyone will ever find out if these relationships hold at steeper and steeper angles.

Galileo: Nevertheless, I believe that I have discovered an important relationship: \( s \propto t^2 \), which should work even for bodies in free fall.

Note: All roles could be played by the teacher except that of Simplicio, which could be played by a student.
EVALUATION QUESTIONS

A lady is riding along in a free fall amusement park ride as shown in the video and in the drawing at the right. Questions 1-5 refer to her motion:

1. Suppose the free fall ride were in a vacuum, the lady's speed at the finish of the fall, Point B, would be

   A. greater than her speed in air.
   B. less than her speed in air.
   C. the same speed as her speed in air.
   D. zero.

2. If the free fall ride were in a vacuum and the lady dropped a small ball at Point A, the speed of the ball at Point B would be

   A. greater than the lady's speed at Point B.
   B. less than the lady's speed at Point B.
   C. the same as the lady's speed at Point B.
   D. zero.

3. Galileo's law of odd numbers is an equivalent way of stating that the total distance fallen is proportional to

   A. the square of the time spent falling.
   B. the time spent falling.
   C. the average speed of the fall.
   D. the instantaneous speed.

4. The speed of the lady after falling for two seconds is

   A. eight times greater than her speed after falling for one second.
   B. four times greater than her speed after falling for one second.
   C. two times greater than her speed after falling for one second.
   D. the same as her speed after falling for one second.

5. The acceleration of the lady after falling for two seconds is

   A. four times greater than the acceleration at the end of the first second of fall.
   B. three times greater than the acceleration at the end of the first second of fall.
   C. two times greater than the acceleration at the end of the first second of fall.
   D. the same as the acceleration at the end of the first second of fall.
6. The total distance that a tight-rope walker might fall is proportional to

A. the time of the fall.
B. the square root of the time of fall.
C. the average speed of fall.
D. the square of the time of fall.

7. After Galileo did experiments using a ball rolling down an incline, he concluded that if you raise the incline to 90° causing free fall

A. a heavy object would fall faster than a lighter one.
B. a lighter object would fall faster than a heavy one.
C. the same light object tied to the same heavy object would fall faster than the heavy object alone.
D. the light object and the heavy object would fall at the same rate.

8. Which of the following speed vs. time graphs represents a falling body in a vacuum?

A.  
   ![Graph A]

B.  
   ![Graph B]

C.  
   ![Graph C]

D.  
   ![Graph D]

9. Given Galileo’s values for distance traveled in meters, the distance traveled after 3 seconds would be

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>?</td>
<td>3</td>
</tr>
</tbody>
</table>

10. Given the following data, the velocity in meters per second at the end of the third second would be

<table>
<thead>
<tr>
<th>Velocity (m/s)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>?</td>
<td>3</td>
</tr>
</tbody>
</table>
ESSAY QUESTIONS

11. Suppose a freely falling body is equipped with a gauge that gives readings for height above ground. Discuss how the reading for height would change with each second of fall.

12. A bathroom scale is an acceptable piece of equipment for measuring the weight of a body. Suppose you stand on the scale and the reading is 160 lb. Why would the reading become zero if both you and the scale were dropped during the weighing procedure?

KEY:

1. A
2. C
3. A
4. C
5. D
6. D
7. D
8. A
9. B
10. A

SUGGESTED ESSAY RESPONSES

11. Since the distance traveled by a falling object is directly proportional to the square of time falling, the height gauge reading will decrease. A freely falling body falls \( s = \frac{1}{2} gt^2 \) 4.9 m during the first second of flight; 19.6 m the second; 44.1 m the third, etc. If the height is \( L \) at \( t = 0 \), then at the end of the succeeding seconds, the gauge will read \( h - 4.9 \) m; \( h - 19.6 \) m; \( h - 44.1 \) m, etc., until \( h \) becomes zero \( (h = \frac{1}{2} gt^2) \).

12. The reading on the bathroom scales becomes zero because both bodies are accelerating at the same constant rate.
TEACHER'S GUIDE TO INERTIA

CONTENT AND USE OF THE VIDEO - Inertia and the law of inertia are typically introduced in high school physics classes in conjunction with Newton's first law of motion. In the video, Galileo's contribution to the development of this concept is emphasized. In addition to its traditional place immediately preceding Newton's laws, inertia also has a bearing on the following topics: (1) an historical introduction to mechanics, (2) a contrast of kinematics and dynamics, (3) a consideration of projectile motion, and (4) a discussion of relative motions and frames of reference.

Prior to viewing the video, students should have studied the kinematics of linear motion. Some background in projectile motion is also advisable.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - Since the following terms are introduced in the video, it might be helpful if students are familiar with them prior to viewing. All terms are defined as they are used within the context of this video.

natural motion--the motion of objects, not subject to any outside influences. According to Aristotle, natural motion meant that a moving object would eventually come to rest in its natural place; according to Galileo, natural motion meant that a moving object would continue to move with constant speed in a straight line.

inertia--the tendency of an object to resist changes in its state of motion.

mass--a measure of the inertia of an object; simply stated, the amount of material contained in an object.

friction--a force generated by a surface on an object, and parallel to the surface, that opposes the motion of the object on the surface.

projectile--an object thrown at any angle into the air.

frame of reference--the point of view of an observer when looking at and describing an object's state of motion.

relative motion--a description of an object's motion as seen from a particular frame of reference.

WHAT TO EMPHASIZE AND HOW TO DO IT - Newton's first law is based on Galileo's investigations of the idea fifty years earlier. The combined genius of these two great scientists drove the last nail in the coffin of the Aristotelian world view. The law of inertia challenged Aristotle's idea that rest is "natural." Galileo realized that it was equally "natural" for an object in motion to continue in motion. He also understood that any reference frame is suitable for describing motion. Galileo's penetrating insights into inertia and frames of reference constituted powerful arguments in support of the Copernican system because they gave credence to the idea of a moving earth. His experiments and reasoning demonstrated that the earth could move and that its movement would not change the appearance of motions of objects on it.
The video develops two important concepts: (1) the law of inertia: a body will remain at rest or continue to move with a constant speed in a straight line, unless acted upon by an outside force, and (2) an observer will perceive motion differently when viewed from various frames of reference.

Objective 1: Distinguish between Aristotle's and Galileo's descriptions of natural motion.

You may want to discuss with your students some of the historical data found in the SUPPORTIVE BACKGROUND INFORMATION. In particular, students should be aware of Aristotle's concepts of natural place and natural motion. Natural place, according to Aristotle, was the place in which an object was usually found and to which an object always tried to return. For a stone, that was the earth. Natural motion was the motion of an object toward its natural place; once in its natural place, an object remains at rest.

To reinforce Galileo's description of natural motion, perform DEMONSTRATION #5 on Galileo's inclined planes. Discuss what happens as the angle of the plane is gradually lowered toward the horizontal.

Objective 2: Describe the law of inertia and explain situations where it applies.

Prior to viewing the video, students should understand the terms inertia and mass. Inertia is the tendency of an object to resist change in its motion; mass is a measure of inertia. In this context, mass is sometimes specifically called inertial mass because its magnitude is determined by how difficult it is to change its state of motion. As an introductory activity, the following two situations might be posed to students:

1. A young boy is quietly sitting in the back of his red wagon. His older brother decides to give him a fast ride. He quickly pulls forward on the wagon handle. What happens to the younger boy?
   
   *He tumbles off the rear of the wagon. His inertia tends to keep him at rest while the wagon is pulled out from under him.*

2. A lady goes to the farm to purchase a carton of eggs. She places the eggs on the seat of her car. While driving home she brakes suddenly to avoid another car. What happens to the eggs?
   
   *The eggs tend to keep moving in a straight line while the car stops; the result is that the eggs hit the dashboard.*

These situations can be demonstrated. Exert a sudden force on a small cart which contains a block of wood standing on end, and observe the result. Allow the moving cart and block to crash into a barrier. Discuss the result with students.

This is also an excellent opportunity to perform a demonstration of pulling a flat cart with a block on top around a curve. Point out to the students that the block slides off along the path in which it was moving.
After viewing the video, DEMONSTRATION #1 on the penny stack and #2 on the hammer and card can be used to illustrate the aspect of the law of inertia which states that objects tend to remain at rest unless acted upon by an outside agent. DEMONSTRATION #5 on Galileo’s inclined planes and #6 on the ball on a string show the aspect of the law where an object tends to remain in motion along a straight line unless an unbalanced force acts upon it.

You might ask students to identify examples of inertia with which they may be familiar. Responses could include:

1. Using seat belts to restrain a moving passenger.
2. Beating or shaking rugs to remove dirt.
3. Flicking a paint brush to remove thinner or water when cleaning up.
4. Shaking by a dog to remove water from its coat.
5. Coasting along a level stretch of road on a bicycle.
6. Pounding the bottom of a hammer handle or rake to secure the head.

Several of the questions from the section on EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS relate to the above responses and help students in linking the law of inertia with the world around them.

Objective 3: Recognize that the descriptions of motion are not the same when viewed from different frames of reference.

The concepts of frames of reference and relative motion are difficult ones for students to comprehend. The video has a number of sequences which illustrate these concepts in detail. Since the initial viewing may be somewhat confusing, it is important that these sequences be replayed several times.

The animated sequence of a ball dropped from the top of a tower is first shown following Aristotelian physics. It would be helpful to stop the video at this point and emphasize that this perspective fails to take into account the law of inertia.

Aristotelians argued that on a spinning earth the tower would move with the earth and the ball would fall away from the tower. Since this did not actually happen, they concluded that the earth did not move.

Later in the animated sequence, a ball dropped from the top of a tower is shown from a Galilean perspective that takes into account the law of inertia as seen from two different frames of reference. The first frame of reference is that of a ground observer who sees the ball fall straight down to the foot of the building.
The second view, shown here, is that of an observer outside of the reference frame of the earth. This observer sees the path of the ball as parabolic. The ball has the same horizontal speed as the tower before being released. It continues to move horizontally with that same constant speed as it falls; therefore, it lands at the base.

Since the different frames of reference can be confusing to students, several replays of this video segment may be necessary to insure full understanding.

Similarly, the animated sequence of the ball dropping and hitting the sailor's head illustrates the Aristotelian viewpoint which does not describe accurately what actually happens. The Galilean perspective is shown with real footage of a man dropping a ball from the mast of a ship. The ball falls along the mast. To an observer in the moving boat, it falls straight down. It may be helpful to trace the path of the ball on the screen with your finger as the students watch. This will emphasize the parabolic path as seen by an outside observer.

The classic PSSC film entitled "Frames of Reference" is an excellent resource and can be easily incorporated into this module. You might ask students for other examples of frames of reference. Responses could include:

1. The apparent motion experienced when a person in a car stopped at a light sees another slowly moving car out of the corner of his eye. The person feels he is the one moving.
2. The illusion that the moon is following your car when you are moving.
3. The moving sensation experienced when viewing fast-moving clouds from the ground.
4. The illusion of motion experienced in an automated car wash when the rollers sweep past the car.

Objective 4: Recognize the historical significance of Galileo's scientific accomplishments.

The video and SUPPORTIVE BACKGROUND INFORMATION summarize Galileo's scientific achievements. In mechanics, his descriptions of inertia and motion were attempts to reconcile observed phenomena with the idea of a moving earth. In astronomy, the discoveries which he made with his telescope affirmed the Copernican heliocentric universe. Galileo's ideas so threatened the traditional thought of his time that authorities tried to suppress his work. You may wish to discuss with your students the revolutionary nature of Galileo's ideas. In a parallel discussion you may want to contrast Galileo's use of experimentation with Aristotle's method of arriving at a scientific theory solely through thought.
POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions and some suggested responses along with illustrations from the video.

Galileo realized that a ball, after rolling down one inclined plane, would roll up another until it regained its original height, no matter what the steepness of the second plane was. What would happen if the second plane were horizontal?

*If the second plane were horizontal, the ball could not regain the height at which it had started. On a horizontal plane with a perfectly smooth surface, a ball would never come to rest, it would keep moving forever.*

The video frame shown here is part of a sequence in which the spaceship changes direction but the asteroid does not. Explain why.

*The inertia of the asteroid keeps it moving in the same direction. The force of the rocket engine produces an acceleration which causes a change in velocity. In this case the change in velocity is manifested in a directional change.*

What would an Aristotelian observer on deck think should happen to the ball if dropped from the mast of a moving ship?

*Aristotle's view was that all things by their nature have rightful homes to which they want to return. An Aristotelian observer would have no doubt at all about what should happen if a ball is dropped from the mast of a moving ship. The ball should fall directly toward the center of the earth, while the ship sails out from under it. That sailor on deck would see the ball hurtle toward the rear of the ship, rather than fall to the foot of the mast.*
What would a Galilean observer standing on the shore think should happen?

The Galilean observer would expect the ball to trace out a parabolic path as the ship moved out from under it.

The video frame shown illustrates the path of a ball dropped from the top of a tower as viewed from someone in space. Discuss why the path appears as it does in this frame of reference.

Galileo realized that the principle of inertia would cause the motion of any projectile to follow a parabolic path, whether at sea or on land.

A ball dropped from a tower anywhere on the surface of the spinning earth behaves the same. Sharing the horizontal motion of the planet, a ball falls straight downward, and lands at the base of the tower. The video frame shown here represents the frame of reference of someone in space. In this frame the ball moves in a parabola.
EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in the video, you might pose the following questions to your students.

1. A car stopped at a red light is rear-ended by another car. Why might a passenger in the stopped car develop the condition known as whiplash?

   The head tends to remain at rest while the rest of the body, supported by the seat, is moved forward. This puts great stress on the neck vertebrae. That's why high back headrests are standard in cars.

2. Why is it more difficult to pedal a bike when starting from rest than when moving along at a constant speed?

   In maintaining motion, you have only to overcome friction, but to change the state of motion, you have to overcome inertia as well as friction.

3. While stopped at a red light, out of the corner of your eye you notice the car in the lane next to yours. Suddenly you see your car rolling backwards; you slam on the brakes, but you don't feel any change in motion. Why?

   Your car was not moving with respect to the earth. The other car moved forward and, since you were looking at it, it became your frame of reference. Therefore, your car appeared to be moving backwards.

4. In order to maintain a constant speed on a skateboard, the rider continually pushes against the sidewalk with his foot. How would Aristotle have explained this situation? Galileo?

   Aristotle would say the natural state of motion is rest and there must be an outside agent preventing it from coming to rest. Galileo would say the natural state is motion and the pushing is necessary to counteract the outside agent (friction).

5. How does the law of inertia explain the fact that, when a sharp jerk is given to a dusty throw rug, it removes some of the dust?

   When the rug is rapidly moved, the inertia of the dust allows it to remain behind and hence "fall off."

6. Explain, in terms of the law of inertia, the value of the use of safety belts in automobiles.

   The person is moving forward at the same speed as the car. If the car suddenly brakes, or swerves, the person tends to remain in the same straight line of motion unless restrained by an outside influence, the seat belt.

7. Why is it difficult to stop a running back on the opposing football team?

   The player has a large amount of inertia. An object (player) in motion tends to remain in motion unless an outside influence (you) acts on it.
8. What would Galileo and Aristotle have predicted about the motion of a hockey puck after it has been struck?

Galileo would have predicted that the puck would continue moving along the ice in a straight line at the speed with which it left the hockey stick. Aristotle would have predicted that the puck would slow down and eventually stop. (Actually the puck does slow down, due to friction between the ice and the puck.)

9. Why is the law of inertia considered such a giant step forward in the development of physics?

Unlike his predecessors, Galileo used actual experimentation to substantiate his concepts. The law of inertia not only afforded a better understanding of observed phenomena, but also allowed Galileo to reconcile these phenomena with the idea of a moving earth. This lent credence to the Copernican system and set the stage for the true Renaissance in natural philosophy (scientific thought), culminating in the discoveries of Newton.

10. Can an object continue to move if no unbalanced force acts on it? If so, give an example.

Yes, an object continues to move with a constant speed in a straight line unless acted upon by a net outside force. For example, Voyager I is now so far from the sun and other planets that their gravitational forces are small enough to be ignored. Voyager is traveling through deep space essentially at a constant speed in a straight line.

11. Can an object at rest begin moving if no unbalanced force acts on it?

No, an object at rest will remain at rest unless acted upon by an outside force.

12. How do Aristotle’s and Galileo’s descriptions of “natural motion” differ?

Aristotle viewed rest as the “natural” state. Natural motion, therefore, was an object’s attempt to return to its natural place. Violent motion was the result of some mover or force causing the object to move away from its natural place. To Galileo it was “natural” for an object in motion to stay in motion with a constant speed in a horizontal direction.

13. Do motions look the same to observers in different reference frames? Cite an example.

No. For example, a ball dropped from the mast of a ship appears to fall straight down if viewed from the position of the moving ship. Someone on shore would see the ball trace out a parabolic path in space, because it continues to move horizontally along with the ship as it falls.
SUMMARY - Galileo's powerful insights into the study of motion of objects, together with his use of experimentation, led him to a formulation of the principle of inertia. This principle, shown to hold for any frame of reference, embodies the idea that the natural motion of a moving object is to remain in motion in a straight line at a constant speed, and that the natural motion of a resting object is to remain at rest. This formulation led to the demise of the Aristotelian world view by demonstrating that the earth could move and that its motion did not change the appearance of motions of objects on it.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed prior to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that follow the viewing(s) of the VIDEO and give closure to the lesson.
STUDENT'S GUIDE TO INERTIA

INTRODUCTION - The video introduces Galileo's accomplishments and contrasts his view of motion with that of Aristotle. The law of inertia is described and motion is viewed from different reference frames.

Terms Essential for Understanding the Video

natural motion  inertia
mass           projectile
friction       relative motion
frame of reference

*** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ***

Points to Look for in the Video

Galileo realized that a ball, after rolling down one inclined plane, would roll up another until it regained its original height, no matter what the steepness of the second plane was. What would happen if the second plane were horizontal?

The video frame shown here is part of a sequence in which the spaceship changes direction but the asteroid does not. Explain why.
What would an Aristotelian observer on deck think should happen to the ball if dropped from the mast of a moving ship?

What would a Galilean observer standing on the shore think should happen?

The video frame shown illustrates the path of a ball dropped from the top of a tower as viewed from someone in space. Discuss why the path appears as it does in this frame of reference.
TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - In the 150 years from about 1550 to 1700 A.D., the science of physics emerged from its dark ages into an age of enlightenment. Around 350 B.C., the ancient Greek philosopher Aristotle, a student of Plato, formulated theories of motion. To Aristotle all motion was the consequence of either natural motion or violent motion. Natural motion was an object’s attempt to return to its “natural place”; violent motion was the result of some mover or force causing the object to move away from its natural place. Aristotle’s theories were based on logic and common sense rather than experimentation. Nonetheless, they were sufficient to explain many observed phenomena. Although lost for many centuries, the writings of Aristotle were discovered and popularized by the philosopher and theologian Thomas Aquinas about 1200 A.D. These ideas gained general acceptance and belief, until challenged by the seventeenth-century Italian scientist Galileo Galilei.

Part of Galileo’s brilliance was in his use of experimentation. Through experiments, Galileo correctly described the motion of freely falling objects and formulated a principle of inertia. His key experiment involved balls allowed to roll down and then up inclined planes, as shown in the illustration below.

![Diagram of Galileo's experiment](image)

Galileo purposely chose balls because he wanted to minimize friction and focus purely on the motion. As he changed the angle of the second inclined plane, he noticed that the ball rolled farther along the plane, but always returned to the same height from which it was released. From this observation, Galileo imagined what would happen if the second plane were level (horizontal). If that were the case, the ball could never reach its original height; it would keep on going forever at the same speed. This was the crucial idea: bodies do not tend to come to rest; they tend to stay in motion. Galileo thought this was true only for horizontal motion, but the law turned out to be more general.

Inertia is the property of an object to resist changes from rest or from motion with a constant speed in a straight line. Galileo understood that the natural state of motion was motion with a constant speed in a horizontal direction. The tendency of an object was not, as Aristotle believed, to reach a state of rest, or “natural place,” but to continue moving in a straight line without any propelling force.

But Galileo’s concept of motion was not quite accurate. To him, a horizontal surface was one which was everywhere perpendicular to the direction pointing toward the center of the earth. That described a sphere with its center at the center of the earth; horizontal motion over such a spherical surface would actually be in a circle, rather than a straight line. An object moving in uniform circular motion is not an example of the principle of inertia, since a force is required to keep it moving in that circle. The French philosopher Rene Descartes straightened out the law of inertia.
Isaac Newton built upon Galileo's principle of inertia and formulated the first law of mechanics:

A body will remain at rest or continue to move with a constant speed in a straight line, unless acted upon by an outside force.

It is important to note that this law seeks only to describe how things move (kinematics) without attempting to explain why (dynamics). (The module Newton's Laws provides additional information on his three laws of mechanics.)

Galileo understood that there is no absolute motion and that the only meaningful description of motion of an object is with respect to another which is used as a frame of reference. All motion is relative. Perception of motion depends upon the point of view, or frame of reference, of the observer. In formulating his principle of inertia, Galileo realized that a reference frame which is moving at a constant speed in a straight line cannot be distinguished from one at rest by an observer within that frame of reference. The earth approximates such a frame of reference, which we now call an inertial reference frame.

In addition to his work in mechanics, Galileo is also famous for his contributions to astronomy. He popularized the telescope, using it to measure the height of mountains on the moon, observe the phases of Venus, and to discover the Jovian satellites. These discoveries led him to affirm the heliocentric universe of Copernicus in a manner which caused him to spend the last eight years of life under house arrest.

At that time, the official theory was the Ptolemaic geocentric universe, a complicated system of equants, deferents, epicycles, and circles within circles. Despite its complexity, Ptolemy's system offered the comforting notion that the earth, and therefore man himself, was the central focus of the universe, a concept embraced by the powerful Catholic Church. Embroiled in the unpopular Inquisition, and being torn apart by the Protestant Reformation, the Church viewed the theories of Copernicus as an additional threat to its strength and authority. Galileo's descriptions of motion and inertia were an attempt to reconcile observed phenomena with the idea of a moving earth. For his belief in and attempt to popularize the Copernican sun-centered system, Galileo was silenced.
ADDITIONAL RESOURCES

Demonstration #1: Penny Stack

Purpose: To demonstrate the concept of inertia as a property of an object's state of rest.

Materials: Stack of fifty pennies; spatula or a wide flat knife.

Procedure and Notes: 1. Stack the pennies on a table top.
2. Quickly slide the spatula or knife along the surface, knocking the bottom penny out of the stack.
3. Repeat with quick, but steady, back-and-forth motion.

Explanation: The law of inertia says that objects at rest (the stack of pennies) tend to remain at rest if the outside influences (i.e., the friction between the bottom penny and the next one up in the stack) are small enough to ignore.
Demonstration #2: Hammer and Card

Purpose: To demonstrate the concept of inertia as a property of an object's state of rest.

Materials: Cardboard shipping label, or piece of manila folder cut to 3 × 5 in. (or a notecard); piece of string; heavy object such as a hammer.

Procedure and Notes: 1. Tie the string onto the end of the cardboard so that it may be pulled.
   2. Place the hammer or other heavy object upright on top of the card.
   3. Pull the card out from under with a quick jerk. Note that the object remains motionless.

Explanation: The law of inertia says that objects at rest (the hammer) tend to remain at rest if outside forces are small enough to ignore (the friction between the card and the hammer). Here you should remind the students that they have undoubtedly seen the trick of pulling the tablecloth out from under the dishes on the table.
Demonstration #3: Hanging Mass on a String

Purpose: To demonstrate the concept of inertia as a property of an object’s state of rest.

Materials: 2 pieces of string; 1-kg or other sufficiently heavy object; ring stand (optional).

Procedure and Notes: 1. Tie one piece of string to the top of the mass, the other to the bottom.
2. Attach to the ceiling or ring stand as shown.
3. Ask students to predict whether the top or bottom string will break first if the bottom is pulled. Depending on their response, either pull uniformly or with a quick jerk to reveal an opposite effect.

Explanation: If the students predict the top string will break first, you can give a quick jerk to cause the bottom one to break. The mass at rest tends to remain at rest causing the bottom string to break. If the students predict the bottom string will break first, give a long, slow continuous pull. In this case your pull causes the mass to move and an object set in motion will continue in motion. (In this case it accelerates down due to both gravity and the force of the pull.) This moving inertia causes the top string to break since the combined pull you exert and the weight of the mass exceeds the forces holding the string together.
Demonstration #4: Flying Chalk Dust

Purpose: To demonstrate the concept of inertia as a property of an object’s state of motion.

Materials: Chalk eraser with a lot of dust in it.

Procedure and Notes:
1. Throw the eraser at the chalkboard or wall.
2. Ask students to explain their observation of the chalkdust imprint on the blackboard.

Explanation: The law of inertia says that objects in a current state of motion (the dust particles traveling in the eraser) tend to remain in the state of motion if the outside influences are small enough to ignore. When the eraser hits the board, the dust particles continue forward until the board stops them. Mention how the use of seat belts in cars as a method of stopping a moving passenger in a sudden braking situation is analogous to the board stopping the dust as it flies in its car (the eraser).
Demonstration #5: Galileo's Inclined Planes

Purpose: To demonstrate Galileo's experiment on inclined planes which led to his statement of the law of inertia.

Materials: Masonite sheets (or similar smooth, flat sheets); level smooth table top; marble or other small ball.

Procedure and Notes:
1. Arrange the planes as shown.
2. Release the marble or ball and note its motion. It should reach nearly the same height on the other side. Mention the effect of friction on keeping it from reaching the same height. Explain how Galileo imagined the ideal case in the absence of friction.
3. Vary the angle of the receiving incline to lower and lower values. In the case of zero angle (remove the second incline), note how the ball moves at essentially constant speed.

![Diagram of inclined planes with a ball being released at various angles.]

Explanation: The ball should reach the same height, but friction consumes some of the motion (energy). As the second incline is lowered, the ball travels longer and at a more nearly uniform rate in its "effort" to reach the original height. As discussed by Galileo, when the second incline is horizontal, the ball travels at a constant speed since it never reaches the level from which it was released.
Demonstration #6: Ball on String

Purpose: To demonstrate the motion in a straight line, due to inertia, of an object released from circular motion.

Materials: 1-m length of string; ball, rock, nut, etc.

Procedure and Notes: 1. Attach the ball to the string.
2. Swing the ball slowly in a vertical plane.
3. Release the string at a certain time and ask students to note that the ball flies off in a line tangent to the circle at the point of release.

Explanation: At the moment of release (at top in diagram above) the mass was instantaneously traveling in the same direction as the velocity vector. When the external influence (the string tension pulling centripetally on the mass) is absent (release of string), the object in motion remains in motion in a straight line at a constant speed. Students may notice and correctly point out that the object actually makes a parabolic trajectory to the ground. Discuss what outside agent is acting to change the motion of the mass.
EVALUATION QUESTIONS

Question 1 refers to the diagram above. Ignore friction.

1. If the ball were released at position x on a frictionless surface, one would expect the ball to reach position ________?

A. position a  
B. position b  
C. position c  
D. position d

Questions 2-3 refer to the following diagram. Ignore friction.

2. The incline on the right has been removed from the first diagram. The ball is again released. According to Galileo, one would expect the ball to

A. roll a distance equal to the height of the original position.  
B. eventually come to rest.  
C. roll forever.  
D. roll a distance equal to 9.8 times the height of the original position.

3. According to Aristotle, one would expect the ball released in the sketch in question to

A. roll a distance equal to the height of the original position.  
B. eventually come to rest.  
C. roll forever.  
D. roll a distance equal to 9.8 times the height of the original position.
4. A ball rolls down an inclined plane and then along a horizontal surface in an actual experiment. After it leaves the plane, it slowly comes to rest. Why is this not a violation of Galileo’s law of inertia?

A. Because the natural state is that of rest.
B. Because the object cannot regain its original height.
C. Because there is an outside agent.
D. Because it should keep moving in a straight line.

A train moves along a horizontal track with a constant speed of 4 m/s. A physics student drops a coin from the ceiling of one of the passenger cars. The coin takes one-half second to hit the floor. Questions 5 and 6 refer to this event.

5. According to the student in the train, where does the coin strike the floor?

A. At a point directly beneath the coin’s original position.
B. At a point somewhere ahead of the coin’s original position.
C. At a point 2m ahead of the coin’s original position.
D. At a point 2m behind the coin’s original position.

6. If the falling coin could be seen by a person standing outside, beside the tracks, how would that person describe the path of the falling coin?

A. As a straight vertical line.
B. As a straight diagonal line in the opposite direction of the moving train.
C. As a parabolic curve in the direction of the moving train.
D. As a parabolic curve in the opposite direction of the moving train.

7. Imagine a freighter in deep space cruising along with a constant velocity. A mechanical arm on the freighter reaches into a garbage bay, pulls out a load of garbage, and releases it. According to a passenger on the freighter, the garbage

A. is left behind as the freighter speeds away from it.
B. moves ahead of the freighter.
C. moves away perpendicular to the freighter.
D. stays right outside the freighter where it was released.

8. It is more difficult to push a stalled car from rest than it is to keep it in motion. Which of the following best explains why this is true?

A. An object in motion tends to slow down and an object initially at rest speeds up.
B. Objects at rest tend to remain at rest, whereas those in motion tend to stay in motion.
C. There is more air friction to overcome for a car at rest.
D. The natural condition is that of rest and all objects wish to return to that condition.

9. Which of the following is not a consequence of Galileo’s scientific accomplishments?

A. He established experimentation as the basis for scientific investigation.
B. He was able to reconcile observations to the concept of a moving earth.
C. He ultimately replaced Aristotelian concepts of natural motion.
D. He established the concept of absolute motion which is independent of the frame of reference.
10. A ship is traveling in a straight line at a constant speed toward the left. Which of the following best simulates the path of a ball released from the mast of the moving ship, as seen by an observer on the shore?

![Diagram of paths A, B, C, D]

ESSAY QUESTIONS

11. When tearing a sheet from a roll of paper towels, would it be better to use a quick jerk or a steady pull? Why?

12. How did Galileo reconcile observed phenomena, such as a projectile falling from a tower, with the idea of a moving earth?

KEY

1. C
2. C
3. B
4. C
5. A
6. C
7. D
8. B
9. D
10. A

SUGGESTED ESSAY RESPONSES

11. If a quick jerk is used, the first sheet moves very quickly because it has little inertia. The rest remains behind and produces a tear at the perforation. If a small steady pull is used, the whole system begins to move, thereby producing little stress at the perforation.

12. Galileo's experiments led him to the principle of inertia and the recognition that there is no absolute motion. All motion is relative, depending on the frame of reference of the observer. Galileo realized that a reference frame moving at constant speed in a straight line cannot be distinguished from one at rest, and that the moving earth approximated this type of reference frame. Galileo realized that at the moment of release, the ball would have the same horizontal speed as the tower. His principle of inertia recognized that the natural tendency of the ball would be to continue in this horizontal motion. The ball would continue to move horizontally and land at the base of the tower.
TEACHER'S GUIDE TO MOVING IN CIRCLES

CONTENT AND USE OF THE VIDEO - In the video it is assumed that students have studied the kinematics of linear motion, projectile motion, dynamics, Newton's laws and orbital motion. Although not the typical order in high school textbooks, it is strongly recommended that a viewing of Moving In Circles be preceded by a viewing of The Apple and the Moon, since this latter video is a treatment of both projectile and orbital motion.

The kinematical derivation of centripetal acceleration which is presented differs from the one given in many high school texts. Consequently, the mathematical explanations which parallel those in the video are provided in the SUPPORTIVE BACKGROUND INFORMATION.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - The video introduces terms which the student might have heard but whose technical use is unclear. Therefore, it would be helpful to list these terms and brief definitions on the board:

- **scalar**—a quantity that has only magnitude or size. (Note: Since the word magnitude is often new to students, this is a good time to re-emphasize its meaning.)

- **vector**—a quantity that has *both* direction and magnitude. A change in a vector occurs if the magnitude or the direction changes.

- **distance**—a measure of how far an object moves without regard to direction; thus distance is a scalar. A car odometer, for example, measures distance.

- **displacement**—a vector that represents a measurement of an object's position from an agreed point of origin, e.g., the radius vector represents the displacement from the center of a circle to a particular point on the circle.

- **speed**—the time rate of change of distance, or how fast something is going. Since distance is a scalar, speed is also a scalar. A speedometer indicates the speed of a car.

- **velocity**—the time rate of change of displacement, or how fast something is going in a specified direction. Since displacement is a vector, velocity is also a vector.

- **acceleration**—the time rate of change of velocity. (Note: In previous units where discussion was confined to motion in a straight line, it was sufficient to say that acceleration was "how fast something is getting faster." However, in circular motion it must be stressed that acceleration deals with any change in velocity. This applies both to changes in direction and changes in magnitude.)

WHAT TO EMPHASIZE AND HOW TO DO IT - Through an analysis of uniform circular motion—motion in a circle with constant speed—the video develops the concept that objects moving in a circle at constant speed accelerate toward the center of the circle. The dynamics of this motion are described by Newton's laws.
Objective 1: Recognize that uniform circular motion exists when an object has constant speed and constant magnitude of acceleration directed to the center of the circle.

Students should already be familiar with projectile motion, where the object has constant velocity in the x-direction and constant acceleration in the y-direction. The result is a parabolic path. But if the object has constant speed and the acceleration is always constant in magnitude and perpendicular to the velocity, the resulting path is a circle (constant tangential speed, constant radial acceleration). At every point in its path, the perpendicular acceleration re-directs the velocity into the circular path. The relations among the directions of the speed and acceleration are illustrated below.

![Diagram of circular motion](image)

DEMONSTRATION #6 deals with the common misconception held by many students. If an object is initially set into a circular path, they tend to think that it will continue in this circular path even in the absence of a centripetal force. Since circular motion is a rather sophisticated concept, it is strongly recommended that the video be shown twice and the pause capabilities of the video recorder be used to advantage. The ADDITIONAL RESOURCES section offers a wide range of discussion questions which help to clarify and reinforce the central ideas in the video.

Objective 2: Recognize that circular motion results when a force of constant magnitude acts on a body in a direction always at right angles to its motion.

DEMONSTRATION #1 and DEMONSTRATION #2 may be used here as an introduction to the forces involved in circular motion. After reviewing the video, you might ask students how the swinging bucket and the coin on a coat hanger are related to the motion of the orbiting moon? The coin is held in its circular path by the centripetal force provided by the hanger, just as the rope provides the force to keep the bucket accelerating toward the water. The gravitational force supplies the centripetal force which holds the moon in orbit.

Objective 3: Describe the relationships between the directions of the radius, velocity, and acceleration vectors in uniform circular motion.

The graphics in the video demonstrate these relationships with precision and power. However, it will probably be necessary to show the sequence several times and to pause frequently if students are to benefit fully from the graphics. The SUPPORTIVE BACKGROUND INFORMATION can be especially helpful in explaining those relationships. DEMONSTRATION #3 may be used to establish the direction of centripetal acceleration toward the center of the circle, which is illustrated below.
Objective 4: Use the expression \( a = \frac{v^2}{r} \) in describing circular motion and in solving problems.

Most texts have a number of problems which will illustrate the use of this equation. The module *The Apple and the Moon* describes Newton's use of this relation to determine the acceleration of the moon toward the earth.

Objective 5: Use Newton's laws to describe the dynamics of circular motion and to interpret problems involving objects moving in circular paths.

Students should recognize that \( F = ma \) states that whenever there is an acceleration, there must be an unbalanced force acting in the same direction. Conversely, if there is an unbalanced force, there must be an acceleration. In the case of the moon orbiting the earth, the gravitational force of the earth on the moon causes its centripetal acceleration.

Several demonstrations in ADDITIONAL RESOURCES help in a deeper analysis of circular motion. DEMONSTRATION #3 establishes that the direction of centripetal acceleration is toward the center of the circle. DEMONSTRATION #4 stresses the vector nature of centripetal force and thus clarifies why the moving object is always accelerated toward the center of the circle. DEMONSTRATION #5 emphasizes that centripetal acceleration is a function of the radius.

The section on EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS offers a number of applications of these concepts. Discussion will help students to a fuller understanding and appreciation of ideas presented in the video.
POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT'S GUIDE. Here are those questions along with some suggested responses and sketches of the related frames from the video.

The moon is always falling, yet stays in the sky. The gravitational force from the earth makes the moon accelerate, according to $F = ma$. If the moon continues to accelerate all of the time, why does it move at nearly constant speed?

Velocity is a vector which can change magnitude and/or direction. The magnitude would change if there is an acceleration parallel to $v$. The direction would change if there is an acceleration perpendicular to $v$. Since the force is directed always toward the center (centripetal force), there is no force tangent to the circle; therefore, there is no acceleration along the path or no change in the magnitude of the velocity. The acceleration continually redirects the velocity vector, which continually redirects the radius vector. They all keep in step with each other—constant in magnitude, but changing continually in direction.

When an object moves in a circle, what are the directions of the position, velocity, and acceleration vectors at a particular instant?

The position is indicated by a vector from the center to the object. The velocity is directed along the tangent to the circle and the acceleration is directed toward the center of the circle opposite to the radius vector; the velocity and acceleration vectors are perpendicular. Notice that the velocity vectors in the picture are always parallel.

A car rounds a curve at a constant speed. How does its acceleration compare to that of another car which rounds the same curve at twice the speed?

The acceleration toward the center, or the centripetal acceleration is equal to the tangential speed squared divided by the radius. According to $a = \frac{v^2}{r}$, the acceleration of the faster car is four times that of the slower.
A car rounds a curve at a certain speed. Then it rounds a curve with half the radius at the same speed. How do the accelerations compare?

Since the speed is constant but the radius is half as great, the acceleration in the smaller radius is twice the acceleration in the larger radius, as can be seen from $a = \frac{v^2}{r}$.

According to Newton's second law, any body that executes uniform circular motion is being accelerated by some external force. What is that force for a body in orbit?

The force is the gravitational force, i.e., $F = \frac{GMm}{r^2}$.

If you know the centripetal acceleration of an object, how do you find the force acting on it?

Use Newton's law, $F = ma$, where the acceleration is given by $a = \frac{v^2}{r}$. 
EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the ideas contained in the video, you might pose the following questions to your students.

1. Identify some applications of centripetal acceleration.
   
   A. The spin-dry cycle in a washing machine. The water tends to continue in a straight line since the clothes do not exert a large enough force on the water droplets to keep them moving in a circular path and the clothes are pulled away from the water.
   
   B. A cream separator, blood separator, or centrifuge. A separation of more dense from less dense substances occurs.
   
   C. The “spinout” or “barrel” in an amusement park. Riders are held in place by the centripetal force of the wall on the rider.

2. A car moves around a flat circular track at a constant speed of 12 m/s. Discuss the following:

   (a) Is the car accelerating?
   (b) What is the velocity vector of this car?
   (c) Where is the velocity vector directed?
   (d) What is the direction of the acceleration vector?
   (e) Compare the directions of the velocity and acceleration vectors in relation to the radius of the circular track.

   (a) Yes, the car has centripetal acceleration due to its change of direction.
   (b) The velocity vector has a magnitude of 12 m/s and is tangent to the circular path.
   (c) The velocity vector is along the tangent, in the same direction as the motion of the car.
   (d) The acceleration vector is directed toward the center of the circle.
   (e) The velocity vector is at right angles to the radius vector. The acceleration vector is directed inward along the radius. (The velocity vector is also at right angles to the acceleration vector.)

3. Consider the motion of the tip of a second hand on a clock or watch.

   (a) Discuss its speed.
   (b) Discuss its velocity.
   (c) Discuss its acceleration.
   (d) Discuss its period.

   (a) The speed is constant and can be calculated using \( v = \frac{2\pi r}{T} \).
   (b) The direction of the velocity is always at right angles to the second hand.
   (c) It is accelerating. There is a centripetal acceleration directed toward the center.
   (d) The period is one minute or 60 seconds.

4. Discuss the forces acting on a car that rounds a corner on a level surface.

   A car rounding a curve on a level surface is subject to the full weight of the car downward and the normal force of the road on the car upward. The frictional force between the road and the tires must be sufficient to provide the centripetal force necessary to keep the car on the desired path. If the speed of the car is too great, the frictional force may not be able to keep the car moving in a circle, and thus the car would skid.
10. What happens to the motion of an object that experiences a force that is always at some angle other than a right angle to the direction of the object's motion?

The object will change in direction and change in speed. For example, when a planet moves in an elliptical path around the sun, except at maximum distance (aphelion) or minimum distance (perihelion), the planet will be changing speed and direction. In the illustration, the gravitational force can be resolved into tangential force which speeds up the object and a normal force which changes the planet's direction. Only at the aphelion and perihelion is the force at right angles to the path.

11. Draw a diagram for an object moving in a circle indicating the direction of the radius vector, velocity vector, and acceleration vector.

\( v \) is tangent to the circle, \( r \) is directed outward, and \( a \) is directed inward.

12. A student may be surprised to learn that a car might accelerate at a much greater rate when it turns a corner than when it speeds up on a straightaway. Test this statement using the following typical situations:

(a) A car accelerates from rest to a speed of 30 m/s in 10 s on a straightaway.

(b) A car moving at a constant speed of 30 m/s can turn in a circle of radius 100 m without skidding.

\[(a) \quad a = \frac{\Delta v}{\Delta t} = \frac{30 - 0}{10} = 3 \text{ m/s}^2.\]

\[(b) \quad a_c = \frac{v^2}{r} = \frac{30^2}{100} = 9 \text{ m/s}^2.\]

Both of these suggested values are typical maximum values for a car. Encourage your students to think of these situations as being real. If it helps, 30 m/s is about 60 mi/hr. Cars and passengers often experience greater accelerations when they turn corners than when they stop or speed up.
5. Discuss why curves are banked.

Sometimes the frictional forces are not sufficient to provide the desired centripetal force. Therefore, the road is banked to create an inward horizontal component of the normal force of the road against the car so that the frictional force with which the road pushes the car toward the center of the turn need not be so large. Therefore, banked curves can be negotiated at faster speeds than unbanked ones.

6. Suppose a moving object that has two forces acting on it is accelerating.

(a) Is it possible for the object to be moving at a constant speed?  
(b) Can the velocity of the object be zero?  
(c) Can the sum of the forces be zero?  
(d) Must the forces act along a single line?

(a) Yes. The object can be moving in a circular path at constant speed, and the acceleration would then be centripetal acceleration.  
(b) No. As long as the object is moving, it must have a velocity.  
(c) No. The sum of the forces can't be zero. There must be an unbalanced force in order to produce an acceleration according to \( F = ma \).  
(d) No. The forces, being vectors, may still produce a resultant force that causes an acceleration.

7. Describe the motion of an object which moves in the absence of any external force.

The object will move at constant speed in a straight line.

8. An object is moving along a horizontal frictionless surface. Suddenly a horizontal force is exerted on the object. This force remains constant in size but is always directed at right angles to the object's motion. Describe the motion of the object.

The object will move with constant speed in a circle of radius determined by \( F = \frac{mv^2}{r} \).

9. We have the same situation as in the above question, except the force increases in size while continuing to act at right angles to the object's motion. Describe the motion of the object.

The object will move at a constant speed in a spiral with an ever decreasing radius of curvature.

The speed remains constant because the force is always at right angles to the object's motion. From \( a = \frac{v^2}{r} \) and \( F = ma = \frac{mv^2}{r} \) we see that, since \( F \) increases and \( m \) and \( v \) remain constant, \( r \) must decrease. (It's interesting to discuss the limiting case of this situation.)
13. A car moves clockwise at constant speed around the track illustrated below:

Each curved section is approximately a semicircle connected by the straight sections AB and CD.

(a) Draw a vector representing the car's velocity at Point y, and mark it \( \mathbf{v} \).
(b) Draw a vector representing the car's acceleration at Point y, and mark it \( \mathbf{a} \).
(c) Indicate where the car's acceleration might be zero if there is such a position.
(d) If the radius of the curved section BD is twice the radius of the curved section AC, compare
the acceleration at Point x with the acceleration at Point y.
(e) How does the horizontal force acting on the car when it is at Point x compare with the
force on it when it is at Point y?

\[ a_x = \frac{1}{2} a_y \]
\[ F_x = \frac{1}{2} F_y \]

14. Discuss why the grooves in the tread on a bicycle tire often run around the circumference of
the tire (see illustration below) rather than transverse to the edge of the tire.

This illustrates that centripetal accelerations are almost always greater than starting or stopping
accelerations, particularly with bicycles (see question 12). The tread is designed to produce the
greatest frictional force at right angles to the wheel's direction of travel. (Also, compare the front
and rear tires of a motorcycle.)

Note: This question can be used to initiate much discussion about what tread on a tire actually
does.
SUMMARY - Acceleration is the rate of change of velocity and velocity is a vector. Initial studies of motion are usually confined to a straight line. However, when objects are allowed to move in curved paths, their velocity can change even though their speed might remain constant. Any change in velocity means that acceleration has taken place.

An object moving in a circle at constant speed (uniform circular motion) is an example of constant speed with a constantly changing velocity. Here the velocity changes in direction even though its magnitude remains constant. Since the change in the direction of the velocity vector is always toward the center of the circle, the acceleration is centrally directed or, "centripetal."

Newton's second law requires that force and acceleration always be in the same direction. Therefore, an object moving in a circle must always be experiencing a net force directed toward the center of the circle. As planets move around the sun, this centripetal force is supplied by the force of gravity.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed prior to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that follow the viewing(s) of the VIDEO and give closure to the lesson.
STUDENT'S GUIDE TO MOVING IN CIRCLES

INTRODUCTION - This video develops the concept that objects moving in a circle at constant speed accelerate toward the center of the circle. As indicated by Newton's laws, the force is in the same direction as the acceleration. The velocity is at right angles to the force and the acceleration. As we shall see, the acceleration does not produce a change in speed.

Terms Essential for Understanding the Video

- scalar
- vector
- distance
- displacement
- speed
- velocity
- acceleration

*** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ***

Points to Look for in the Video

The moon is always falling, yet stays in the sky. The gravitational force from the earth makes the moon accelerate, according to \( F = ma \). If the moon continues to accelerate all of the time, why does it move at nearly constant speed?

When an object moves in a circle, what are the directions of the position, velocity, and acceleration vectors at a particular instant?
A car rounds a curve at a constant speed. How does its acceleration compare to that of another car which rounds the same curve at twice the speed?

A car rounds a curve at a certain speed. Then it rounds a curve with half the radius at the same speed. How do the accelerations compare?

According to Newton's second law, any body that executes uniform circular motion is being accelerated by some external force. What is that force for a body in orbit?

If you know the centripetal acceleration of an object, how do you find the force acting on it?
TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - The study of circular motion has its origins in the fascination which celestial bodies held for the Greek philosophers. One such philosopher was Plato (427–347 B.C.). Although he had little interest in science, he nevertheless had a profound influence upon it because of the significance attributed to everything that he said and wrote. As a consequence, his *Timaeus*, which discussed a model of the universe, dominated theoretical astronomy for twenty centuries.

The stars, Plato said, represent eternal, divine, unchanging beings. They move at uniform speed around the earth in the most regular and perfect of all paths - an endless circle. Plato accepted motion in a circle with constant speed – *uniform circular motion* – as so simple and natural an idea that it needed no explanation. Any heavenly body, he proclaimed, must move, if not in a single perfect circle, then in some combination of circles.

For the Greeks, astronomy was an important profession. Its job was to predict where each heavenly body could be found at any time on any given night. The need for prediction was practical: agriculture, navigation, and – most important in those days – casting horoscopes. For the fixed stars, simple models were enough. But for the planets, there were problems, since they were observed to stop and reverse their motion with respect to the fixed stars. It was hard to see how such retrograde motion could be due to motion in a circle at a uniform rate about the earth.

Numerous astronomers undertook the task of reconciling the irregular motions of the planets with the ideal mathematical system of uniform circular motion. This effort culminated in the publication of Ptolemy’s *Almagest*, a compilation of all the astronomical knowledge of antiquity. This work was the standard for 1,400 years, up to the time of Copernicus in the sixteenth century. Believing that the Ptolemaic system of the *Almagest* was not sufficiently Platonic, Copernicus thought that perhaps he could restore the uniform circular motion and perfection of the heavens by placing the sun at the center of the universe. It remained for Johannes Kepler, a century later, to abandon circles, and for Isaac Newton to explain in what dynamical sense the sun was the center of the system.

When objects move in circles at constant speed, they accelerate. The acceleration is directed toward the center of the circle along the radius vector and is called centripetal acceleration. The velocity vector is at all times tangent to the circle and at right angles to the radius vector. The force which causes the acceleration is called centripetal force. *Centripetal force is not a new kind of force.* It is simply the name given to any force directed at right angles to the path of the moving body. A centripetal force is thus a force which produces circular motion.

Since acceleration is usually introduced in a discussion of straight-line kinematics, students often have the impression that it requires a change in the magnitude of the velocity. They are surprised to discover that an object moving at constant speed can have an acceleration. This is the key idea brought out in the video: an object will be accelerating if its velocity changes in *direction*, despite the fact that its speed remains the same.

The following mathematical explanations parallel the video. Since they may differ from those used in some texts, it is important to view these segments several times.
As an object moves with constant speed in a circular path, the locus of the tip of the radius vector describes a circle. (See Figure 1.) In a given interval of time, the chord between the tips of the two radius vectors represents a change in position (Δr). This change is itself a vector. If smaller and smaller intervals of time (Δt) are taken, the vector which represents the change in position per unit time (Δr/Δt) approaches the tangent. Since Δr/Δt defines the velocity, the velocity vector is tangent to the circle and perpendicular to the radius vector.

![Figure 1](image)

In Figure 2, \(v_1\), \(v_2\), \(v_3\), and \(v_4\) are vectors representing tangential velocity at four different times. By definition a vector remains the same even if moved, as long as its magnitude and direction are unchanged. Figure 3 shows velocity vectors redrawn so that their tails are concurrent. Imagine the tangential velocities at every point of the circular path in Figure 2. If all these velocity vectors were plotted as in Figure 3, the locus of their heads would be a circle of radius \(v\).

![Figure 2](image)
![Figure 3](image)

Figure 4 shows this locus for a radius vector, while Figure 5 shows it for the corresponding velocity vector. Notice that velocity vectors are always parallel to one another and perpendicular to the radius vector.

![Figure 4](image)
![Figure 5](image)
The procedure used to derive the velocity vector can also be used to derive the acceleration vector. As smaller and smaller time intervals are taken (see Figure 6), the vector which represents the change in velocity per unit time \( \Delta v/\Delta t \) is the acceleration and is at right angles to the velocity vector.

![Figure 6](image)

Figure 6

Figure 7 shows the locus of the radius vector, and Figure 8 shows the locus of the velocity vector. Just as the velocity vector is always perpendicular to the radius vector, the acceleration vector is always perpendicular to the velocity vector.

![Figure 7](image)  

Figure 7  

![Figure 8](image)  

Figure 8

The acceleration vector is always seen to be parallel to the radius vector and in the opposite direction. Therefore, the acceleration is directed toward the center of the circle. This centripetal acceleration is shown in the composite vector diagram in Figure 9.

![Figure 9](image)

Figure 9

As an object moves in a circular path, its velocity vector is always perpendicular to the radius vector and the acceleration vector always points toward the center of the circle.
The centripetal acceleration is described by Newton's laws. His second law indicates that every acceleration is caused by an unbalanced force. Since the acceleration is directed toward the center of the circle, there must be a corresponding force in that direction. This is the centripetal force. It is always acting on the moving object, pulling it out of its normal linear motion into a circular path.

Unit vectors are used on occasion in this video. A unit vector can be defined as any vector divided by the magnitude of its length \((\mathbf{r} / |\mathbf{r}|)\). The unit vector, then, has a magnitude of 1, yet retains the direction of the original vector. In the video, the chosen method of indicating the unit vector is a circumflex placed over the vector symbol \((\hat{\mathbf{r}} = \mathbf{r} / |\mathbf{r}|)\). The purpose of introducing this special vector is to provide for consistency in vector expressions. For example, in the equation for the law of universal gravitation, \(\mathbf{F}\) is a vector. Therefore \(-G\frac{mM}{r^2}\) must also be made a vector. The numerator is a scalar, and although \(r\) is a vector, \(r^2\) is a scalar. The right side of the equation would seem to be a scalar. The unit vector \(\hat{\mathbf{r}}\) is then introduced to maintain the vector nature of the force equation:

\[
\mathbf{F} = -\frac{GmM}{r^2} \hat{\mathbf{r}}.
\]

The video also refers to the "rate of change of a vector," which describes how fast the vector is changing. A vector can change by changing its size, or its direction, or both. The change in a vector is itself a vector. The vector, \(\mathbf{v}\), is itself undergoing uniform circular motion. Its rate of change is the acceleration.
ADDITIONAL RESOURCES

Demonstration #1: Swinging Water Bucket

Purpose: To demonstrate and discuss how water will remain in a bucket when swung in a vertical circle.

Materials: Pail or coffee can with a metal handle attached to a short piece of rope.

Procedure and Notes:
1. Before doing the demonstration, hold the pail with a small amount of water in it and ask your class how you could turn the pail upside down and not have the water fall out. Establish in the discussion that the water would remain in the pail if the bottom were accelerated downward at a rate equal to, or greater than, the acceleration of gravity.
2. (Practice this in advance.) Whirl the pail in a vertical circle at a sufficient speed to keep the water from falling out. Stress that the water remains in the pail because the centripetal acceleration of the bottom of the pail is equal to, or greater than, the acceleration of gravity when the pail is upside down.
3. Rough quantitative estimates of the radius of the pail’s path can be made, and then the maximum possible period of revolution which will keep you dry can be computed.
4. As an amusing alternative, have an identical pail on hand filled with small bits of paper which you can replace unknown to the students, and then manage to have this “water” fall on your head.

Explanation: It is not “centrifugal” force which keeps the water in the pail. The bottom of the pail is accelerating toward the center of the circle at a rate greater than the acceleration due to gravity; hence the water cannot fall out.
Demonstration #2: Coin on a Wire Coat Hanger

Purpose: To show how a centripetal force can act to whirl a coin in a vertical circle.

Materials: A specially bent coat hanger and a coin. It may be necessary to file the end of the hanger flat to provide a level platform upon which to rest the coin.

![Diagram of coin on wire coat hanger]

Procedure and Notes: Have fun with this one. With a little practice you will be able to whirl the hanger in a vertical circle without losing the coin. Ask the students to indicate the direction of the force that the hanger exerts on the coin while it moves in the circular path.

Explanation: The hanger provides the centripetal force which holds the coin in the circular path.
Demonstration #3: The Submerged-Cork Accelerometer for Centripetal Acceleration

Purpose: To establish that the direction of centripetal acceleration is toward the center of the circle.

Materials: A submerged-cork accelerometer. In its usual form this consists of a large jar with a cork attached by a string to the lid. The jar is filled with water and is inverted as illustrated below:

An alternative is to use an Erlenmeyer flask. It is possible to thread the string through the stopper with a strong needle.

Procedure and Notes: 1. By accelerating the accelerometer in a straight line, establish that the cork leans in the direction of acceleration:

   \[ \begin{align*}
   & \text{Velocity to the right} \\
   & v \rightarrow \\
   & \quad \text{and speeding up.} \\
   \end{align*} \]

   \[ \begin{align*}
   & \text{Velocity to the right} \\
   & v \rightarrow \\
   & \quad \text{and slowing down.} \\
   \end{align*} \]

2. Hold the accelerometer at arm’s length and ask the students to describe the movement of the cork if you rotate in a circle. (Be sure you ask before doing the following.) Now swing the accelerometer in a horizontal circle and observe that the cork leans inward, and hence the acceleration is toward the center of the circle.

Explanation: The explanation of how the accelerometer works can be quite involved and perhaps should be avoided when it is first introduced. The basic idea is that the water has more inertia than an equivalent volume of cork; hence the total inertia of cork and water can be invoked to explain the observed behavior of the cork. (See *The Physics Teacher*, vol. 2, no. 4, April 1964, p. 176.) In this demonstration, the important idea to stress is that the cork always leans in the direction of acceleration. Hence, when the accelerometer is revolved in a circle, the cork shows the acceleration to be inward.
Demonstration #4: Rubber Stopper on a String

Purpose: To illustrate the basics of centripetal force in a way that can easily evoke a great deal of class discussion.

Materials: A strong piece of string attached to a rubber stopper.

Procedure and Notes: 1. Show the students that if you pull the string in a straight line with the stopper trailing behind and try to keep the string taut (this can be done only for short distances), the stopper accelerates. Keep working with the stopper moving in a straight line path, and stress that, as long as the string is taut, the stopper accelerates in the direction of the string.

2. Whirl the stopper in a horizontal circle, and discuss how the string pulls on the stopper toward the center of the circle; therefore, the stopper must be accelerating toward the center of the circle.

3. To review the principle of inertia and to dispel the myth of centrifugal force, ask the students to predict where the stopper will go if released. (The stopper always moves off tangent to the circle.)

4. More elaborate discussions can be introduced by whirling the stopper in a vertical circle and including the force of gravity on the stopper as well as the tension in the string.

Explanation: This is a perfect time to stress the vector nature of Newton's second law. The unbalanced force and the acceleration are always in the same direction; therefore, the tension in the string provides the unbalanced force which accelerates the stopper toward the center of the circle. (We call this centrally directed acceleration "centripetal acceleration" and the force which causes it "centripetal force.")
Demonstration #5: The Liquid Accelerometer

Purpose: To use a commercial liquid accelerometer to demonstrate the centripetal acceleration as a function of radius.

Materials: Phonograph turntable with a Masonite disk covering the platter. A commercially available liquid accelerometer. [Such devices are usually called "accelerometers." Suppliers, including catalogue numbers are: Sargent Welch (3327C), Fisher Scientific (S52168), Cenco (72717-020).] The liquid accelerometer consists of a thin, transparent rectangular container half filled with some liquid:

Anyone who has ever attempted to carry a pan filled with water has some idea how the accelerometer works. As a less satisfactory option, a large beaker or battery jar can be used in place of the liquid accelerometer.

Procedure and Notes:

1. The accelerometer can be placed on a small rolling cart and accelerated across a table top to establish its behavior under linear acceleration. It can be shown that $a = g \tan \theta$, where $\theta$ is the angle the surface of the liquid makes with the horizontal.

2. Attach the accelerometer to the Masonite disk with its center over the center of the turntable. Ask the class to predict its behavior when the turntable rotates, then demonstrate. Experiment with different angular speeds. Also, displace the accelerometer to the right or left of center of the turntable. (Caution: Make sure the accelerometer is firmly attached to the turntable in all of these demonstrations.) Challenge your more capable students to develop the mathematical expression for the shape of the curve which results.
3. As an optional experiment, work out a way to attach a large beaker or battery jar to the turntable. With the jar half full of water and rotating at constant angular speed, the surface of the water will form a paraboloid. This demonstration has an interesting application in the construction of parabolic mirrors using casting resin. In order to get a smooth surface, the table must be able to rotate without vibration at a constant angular speed for the duration of time required for the resin to set.

Explanation:

The demonstrations with the liquid accelerometer are quite delightful and never fail to enchant the class. The liquid inside the accelerometer indicates acceleration because, unlike solids, liquids cannot exert (or support) a shearing force tangent to the surface.
Demonstration #6: Does "Curvilinear Impetus" exist?

Purpose: Often students believe that if an object is given an initial circular motion it will, when released, continue moving in a circular path. This demonstration should help to dispel this misconception.

Materials: Overhead projector, level, transparency marking pen, steel ball bearing.

Procedure and Notes:
1. Carefully mark a section of a curved "track" on the surface of the leveled overhead projector. Make the curved section small enough to allow plenty of room for launching the steel ball into this section.

2. Have a student attempt to roll the steel ball on the surface of the projector in such a way that it will pass through the marked "track" on its own without touching the sides. Many students will try to set the ball rolling in a circular path and will be surprised to discover that when released the ball will not conform to its original circular motion.

Explanation: Curvilinear impetus does not exist and the students' frequently failed attempts to set the ball into a circular path should underscore this fact. This is a good time to make a full restatement of Newton's first law.
EVALUATION QUESTIONS

Two pennies, A and B, are on a record turntable that is rotating at 33 revolutions per minute. Penny A is farther from the center than Penny B. The following questions refer to the motion of the pennies.

1. The net force on Penny A is
   A. constant and directed radially outward.
   B. greater than that on Penny B.
   C. zero.
   D. in the same direction as the velocity of the penny.

2. At each instant the acceleration of Penny A is
   A. less than the acceleration of Penny B.
   B. zero.
   C. directed radially inward.
   D. directed radially outward.

3. Which penny has the greater speed?
   A. Penny A.
   B. Penny B.
   C. The speed is the same for both.
   D. More information is needed for a comparison.

4. If Penny A should slide off the turntable, what would be its motion just as it leaves the table?
   A. Radially outward.
   B. Radially inward.
   C. Along a tangent and in the direction of rotation of the table.
   D. Along a tangent and opposite the direction of rotation of the table.

5. Which of the following correctly indicates the relationship between the position, velocity, and acceleration vectors for Penny B at a particular instant?

   (A) ![Diagram A](image)
   (B) ![Diagram B](image)
   (C) ![Diagram C](image)
   (D) ![Diagram D](image)
6. Which of the following must be true about the net force acting on an object which moves at constant speed around a circle?

A. The force must be constantly increasing in magnitude.
B. The force must always be directed toward the center of the circle.
C. The force must always be in the same direction.
D. The force must be in the direction of motion (tangent to the circle).

7. An object moving over a frictionless horizontal surface experiences a force. The force is always at right angles to the object’s velocity and increasing in magnitude. Which of the following is the best description of the motion of the object?

A. The object moves in a circle but its speed changes.
B. The velocity of the object changes but its speed remains constant.
C. The object moves in a spiral with a decreasing radius.
D. The acceleration of the object is constant and directed toward the center of its circular path.

8. A car rounds a curve at 20 mph. The same car now rounds the same curve at 30 mph. How does the acceleration acting on the car in the second situation compare with the acceleration acting on the car in the first situation?

A. The acceleration is $\frac{2}{3}$ as much.
B. The acceleration is $\frac{3}{2}$ as much.
C. The acceleration is $(\frac{2}{3})^2$ as much.
D. The acceleration is $(\frac{3}{2})^2$ as much.

9. If an object is accelerating, which of the following must always be true?

A. It must be increasing its speed.
B. It must be increasing or decreasing its speed.
C. It must be changing its direction.
D. It must have a net force acting on it.

10. A car traveling at velocity $v$ enters a curve of radius $r$. As the car enters a second curve of radius $\frac{1}{4} r$, it is slowed to $\frac{1}{4} v$. What is the centripetal force on the car as it enters the second curve compared to the first?

A. 1:1
B. 1/2:1
C. 1/4:1
D. 1/16:1
ESSAY QUESTIONS

11. A racing car rounds a curve at high speed. While in the curve, the driver hits the brakes. In the diagram, draw the acceleration vectors and the resultant acceleration.

12. A child whirls a bucket of water over his head, yet the water doesn't fall out. Discuss why the water remains in the bucket even though it is upside down.

KEY

1. B
2. C
3. A
4. C
5. D
6. B
7. C
8. D
9. D
10. C

SUGGESTED ESSAY RESPONSES

11. As indicated on the diagram below, the centripetal acceleration is directed radially inward (toward the center of the circular track) since the car is moving in a circle. In addition, there is a tangential acceleration (a deceleration) that slows down the car and is a result of the braking. This tangential acceleration is directed opposite to the velocity. The result acceleration is the vector sum of the centripetal acceleration and the tangential acceleration.
12. When the bucket is overhead, the centripetal acceleration on the bottom of the bucket is downward. The water remains in the bucket as long as the downward acceleration of the bucket is greater than the acceleration of gravity.
TEACHER'S GUIDE TO THE MILLIKAN EXPERIMENT

CONTENT AND USE OF THE VIDEO - In the detailed examination of the Millikan experiment presented in this video it is assumed that students have studied gravitational fields and electric fields. Since the experiment involves an analysis of the forces acting on an electrically charged falling droplet of oil, students are expected to understand the dynamics of Newton's laws. The video might be used most effectively in conjunction with the study of electricity or immediately following that study. Terminal velocity and viscosity are not included in many curricula. The teacher may wish to explain briefly their meanings.

The nature of scientific research is strongly emphasized in the video; however, students will appreciate these processes more fully if they are already well grounded in the topics mentioned previously.

TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO - Before viewing the video, the following terms should be briefly discussed with the students:

cathode ray--rays which come from the negative electrode in an evacuated tube.

electron--the negatively charged elementary particle; the smallest known amount of negative electricity. J.J. Thomson demonstrated that the cathode rays were electrons.

ionize—to add or subtract electrons from a neutral atom; an atom so charged is called an "ion."

electric field—a region of space that has the ability to exert forces on electric charges; electric fields exert forces on charges in the same way that gravitational fields exert forces on masses.

electric field strength—a measure of the intensity of an electric field; if the strength of the electric field is E, then the force on a charge q, placed in the field will be given by: $F = qE$.

viscosity—a measure of how much a fluid resists flowing; in the video this is related to air friction or the force the air exerts on objects moving through it.

terminal velocity—the constant velocity an object attains while falling through a fluid. Terminal velocity of a falling object occurs when the downward force of gravity equals the upward force of fluid friction. In the video the terminal velocity of oil drops moving through air will be used to measure the weight of the drops and the charge on the drops.

fundamental—in the context of this video the word fundamental means "basic" or even "smallest possible value"; often the word "elementary" is used and has the same meaning.
WHAT TO EMPHASIZE AND HOW TO DO IT - Although twentieth-century physicists tend to be specialized as "theoretical" or "experimental" physicists, every scientist recognizes that without experimental verification even the most elegant theoretical constructions remain little more than speculations. At time, an experimentalist will make an unexpected discovery which will suggest problems never imagined before by those whose minds are their only laboratories. At other times he or she will be intrigued by a well-defined problem and will through imagination and diligence, tackle the task of finding the definitive solution. Such was the work of Robert A. Millikan. He devised an ingenious experiment which made it possible to measure the fundamental unit of charge with unprecedented accuracy.

The video emphasizes the idea that Millikan's measurement of the charge of an electron stands as one of the classic experiments in the history of physics.

Objective 1: Explain the basic idea behind the method used by Millikan to determine the elementary charge.

A discussion of the Millikan experiment might seem out of place early in a physics course. However, the video can form a review of the forces involved when objects fall under the force of air friction; it can be the basis of an excellent discussion of the methods of science; and it can provide a good introduction to the later study of electricity.

Millikan's work established the fact that charge is quantized. To help insure that students recognize and appreciate the significance of this discovery, you might perform DEMONSTRATION #2 with students the day preceding the video. Student's work can then be collected and discussed in conjunction with viewing the video.

Objective 2: Be able to describe how air friction affects the motion of a falling object.

Often all of the discussions of falling objects in elementary physics courses are confined to "free fall." The details of how objects move though fluids can be quite complex, nonetheless the basic idea of the forces on an object falling at terminal velocity can be understood by high school students. Where simple sliding friction is independent of velocity, fluid friction is a velocity dependent force and presents a different but frequently encountered situation.

Since viscosity is most often associated with liquids, students may not have thought about air as a fluid. With respect to the Millikan experiment, the effect of viscosity on the falling oil drop is the effect of air resistance.

Objective 3: Recognize that all charge is a multiple of the smallest unit of charge.

Probably long before the results of the Millikan oil drop experiment were published, it was suspected that all atoms of a given substance were exactly alike. Even an estimate of the average charge of the electron, which had become identified as the same particle in cathode rays, had been made previous to Millikan's work. However, students should realize that it took enormous effort and careful attention to detail before it could be firmly established that there was a smallest possible multiple of electric charge. DEMONSTRATION #1 offers a good follow-up to the video since it reinforces the concept that charge is quantized.
Objective 4: Use the Millikan experiment as a model to describe experimental research.

The Millikan oil drop experiment serves as a case history which can be used to give students an opportunity to discuss and consider experimental research. Using the actual data of Millikan the video takes great care to show how the scientist finally decided to publish particular results. When students are first learning about scientific objectivity, it might appear to them that an experimental scientist would be "cheating" if he didn't publish all of his data. The video should help students to understand that a trained scientist can criticize and discard data in order to insure that others will be able to repeat the results.

EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS provides discussion questions which can enhance student understanding of the Millikan experiment and its importance to the scientific community. The section on ADDITIONAL RESOURCES also offers some suggestions for implementing a successful lab and/or simulation of the Millikan experiment, if time permits such activity.
POINTS TO LOOK FOR IN THE VIDEO - Several questions are posed in the STUDENT’S GUIDE. Here are those questions along with suggested responses and frames from the video.

The mass times the acceleration of the drop is equal to the sum of all the forces acting on it. What are the forces acting on a falling drop of oil in air?

The weight of the droplet and the viscosity (air resistance) act on the falling droplet.

What did Millikan do to find the charge on a single droplet?

Millikan applied an electric field, which exerted a force equal to the charge of the droplet times the electric field strength, and measured the time of fall with this additional force acting.

Millikan established that every charge was a whole number multiple of a fundamental charge. Why is this an important discovery?

Prior to Millikan's discovery, charge was spoken of in terms of an "average." Millikan's work proved that a fundamental unit of charge exists.
EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS - To reinforce further the concepts presented in the video, you might pose the following questions to your students.

1. Discuss the significance of the Millikan experiment in terms of its impact on the following scientific developments:
   
   (a) The verification of the fundamental unit of charge, an important first step in the development of modern electronics.
   (b) The establishing of the electron volt as a unit of energy.
   (c) The quantization of charge which helped to establish the basic concepts of quantum mechanics.
   (d) Another accurate method for determining Avogadro's number.

2. Suppose Millikan measured the charge on 12 drops and obtained these values (in coulombs):

   \[ \begin{align*}
   3.2 \times 10^{-19} & & 17.6 \times 10^{-19} & & 8.0 \times 10^{-19} \\
   11.2 \times 10^{-19} & & 1.6 \times 10^{-19} & & 19.2 \times 10^{-19} \\
   16.0 \times 10^{-19} & & 6.4 \times 10^{-19} & & 12.8 \times 10^{-19} \\
   4.8 \times 10^{-19} & & 9.6 \times 10^{-19} & & 14.4 \times 10^{-19} 
   \end{align*} \]

   (a) What is the smallest value in the group?
   (b) Are all the other values consistent with the basic assumption of the Millikan oil-drop experiment? If not, which aren't?
   (c) Do you think it is possible for an oil drop to acquire a charge smaller than \( 1.6 \times 10^{-19} \) C? How might you verify your answer?
   (d) Would the conclusion in Millikan's experiment depend upon whether the drops had a positive or negative charge?

   (a) \( 1.6 \times 10^{-19} \) C.
   (b) Yes, results are consistent.
   (c) The question can be answered yes or no. The important part of the answer is the explanation. If "no," the experiment has been performed many times, always giving units of \( 1.6 \times 10^{-19} \). If "yes," the charge could be \( 1/3 \) the smallest charge shown. However, with this number of charges, the probability is small.
   (d) No.

3. Discuss the method used by Millikan to determine the smallest unit of charge. For each of the following ask students to write a one paragraph discussion:

   (a) The equipment used.
   (b) The care taken.
   (c) The method of data collection and interpretation of the data collected.
   (d) The conclusions Millikan formed from the data.
(a) Answers might include: protective pot, charged metal plates, atomizer, microscope, stopwatch.

(b) Answers might include: protective metal pot, constant temperature bath, filtered air for atomizer, filtered light for viewing moving drop with terminal velocity, carefully organized data collection in notebook.

(c) Answers might include: dated and organized notebooks; all data included (good and bad), expected data highlighted, unexpected data explained.

(d) Answers might include: all charge comes as a multiple of some fundamental charge; slight variations may be explained by looking at the equipment problems.

4. Explain what is meant by a fundamental unit of charge.

*Answers might include: smallest amount of electric charge, a quantum unit of charge, an atom of electricity, all other charge is a multiple of the fundamental.*

5. An experimenter makes eight measurements of the charge on oil drops in a Millikan-type experiment. A graph of the results, using electrostatic units (esu) rather than coulombs as the unit of charge, is shown in the figure below. What interpretation would you give to the data?

\[ \text{CHARGE (10}^{-10}\text{ esu)} \]

\[ \text{TRIALS} \]

*There seem to be about four different levels at } 4.5 \times 10^{-10}, 9 \times 10^{-10}, 14 \times 10^{-10}, 18.5 \times 10^{-10}\text{ esu. Therefore, the charge seems to be about } 4.6 \times 10^{-10}\text{ esu, with trials having charges of from 1 to 4 units.}*

6. If you are given many closed sacks filled with different amounts of identical marbles, how could you determine the weight of a single marble without opening the sacks? (Assume the weight of the sack material can be ignored.)

*Weigh all the sacks and look for a common multiple that represents the mass of one marble. It is valuable to discuss whether the method would work if the masses were different. Note how the number of marbles in the sack may affect the results.*
SUMMARY - The video demonstrates how scientific knowledge increases. Scientists gain ideas from others and then extend and refine them. Experiments are conducted painstakingly with results continually checked. Findings which are inconsistent are double checked. Millikan was the first to establish that charge is quantized. An immediate consequence of his discovery was the realization that the mass of the electron was also quantized. Eventually there was experimental verification of the fact that matter was not infinitely divisible.

NOTE OF EXPLANATION REGARDING THE STUDENT'S GUIDE - The following two pages of the STUDENT'S GUIDE should be duplicated and distributed to the students for use in preparation for viewing the video.

In general, the STUDENT'S GUIDE lists topics, terms, and questions, and the TEACHER'S GUIDE provides definitions, discussion, and answers to the questions. It is very important to have the students receive an appropriate "preparatory set" for viewing the VIDEO and also, following the showing of the VIDEO, to have a systematic discussion, analysis, and summarization of the objectives of the module.

The students should be informed that the INTRODUCTION, TERMS ESSENTIAL FOR UNDERSTANDING THE VIDEO, and POINTS TO LOOK FOR IN THE VIDEO should be read and discussed prior to viewing the VIDEO. These should also be rediscussed following the viewing.

Answers to the questions listed in the STUDENT'S GUIDE have been included under POINTS TO LOOK FOR IN THE VIDEO in the Teacher's Guide. The questions which follow this section of the Teacher's Guide and deal with EVERYDAY CONNECTIONS AND OTHER THINGS TO DISCUSS as well as the SUMMARY should be discussed as a part of the activities that follow the viewing(s) of the VIDEO and give closure to the lesson.
STUDENT'S GUIDE TO THE MILLIKAN EXPERIMENT

INTRODUCTION - The video emphasizes the idea that Millikan's measurement of the charge of an electron stands as one of the classic experiments in the history of physics. Students should recognize that all charge is a multiple of the smallest unit of charge. They should also understand that a trained scientist can criticize and eliminate data in order to insure that others will be able to repeat his results.

Terms Essential to Understanding the Video

cathode rays  electric field intensity
electron     viscosity
ionize       terminal velocity
electric field fundamental

*** NOTE: Parts of the video, especially mathematical equations, may go by quickly on the screen. If you have questions, you should ask your teacher to replay these sections. ***

Points to Look for in the Video

The mass times the acceleration of the drop is equal to the sum of all the forces acting on it. What are the forces acting on a falling drop of oil?

What did Millikan do to find the charge on a single droplet?
Millikan established that every charge was a whole number multiple of a fundamental charge. Why is this an important discovery?
TEACHER RESOURCES

SUPPORTIVE BACKGROUND INFORMATION - Cathode rays were discovered in the middle of the nineteenth century and soon became a research topic of intense interest throughout all of Europe. By 1897, J. J. Thomson at the Cavendish Laboratories in Cambridge, England, had proven that cathode rays were really negatively charged particles. His exhaustive series of experiments gave convincing evidence that these particles were also fundamental parts of atoms. Thomson proposed the name corpuscles for them, but they quickly became known as electrons.

Thomson’s most important cathode ray experiment resulted in the determination of the charge-to-mass ratio (e/m) of electrons. His original value of e/m was a thousand times greater than the corresponding ratio for ions in solution. Since it was unlikely that the charge on the electron was much larger than the charge on an ion, it seemed most probable that the mass was a thousand times smaller. To calculate an accurate value for the mass of an electron, however, required an accurate value for its charge.

The first measurements of the electron’s charge were performed in a series of experiments at the Cavendish Laboratories between 1897 and 1903. These early measurements relied on the cloud method. This consisted of condensing small water droplets on charged particles in a cloud chamber. Then the number of droplets in the cloud was estimated, and the total charge on the cloud was measured. These experiments gave results varying from $0.7 \times 10^{-19}$ to $2.8 \times 10^{-19}$ coulomb. However, the average of these results was confirmed by electrolysis experiments to be of the correct order of magnitude. There were several major problems with these experiments. The two most significant problems were these: interpretations required many assumptions about the behavior of water droplets in clouds, and the result was only a mean value of many imprecise results.

In 1906 at the University of Chicago, R. A. Millikan began to repeat the Cavendish cloud experiments. Millikan’s refinement was to try to balance the cloud between two charged metal plates. This attempt was not totally successful in its original form, but did lead to another important modification. Millikan found that he could observe and measure individual water droplets in the cloud rather than work with the entire cloud and statistical averages. Millikan repeated this experiment several times, balancing individual charged droplets in an electric field between the plates to determine the charge on each droplet. The main difficulty encountered in the experiment was that during the observation of the droplet some of the water would evaporate. Nevertheless, the value of the electron charge obtained from balanced water droplets was within 3% of the modern accepted value. The work was announced in 1909.

To overcome evaporation Millikan made one more modification: he decided to use oil droplets rather than water droplets. The immediate problem was how to produce the oil droplets since they could not be formed in a cloud chamber. Harvey Fletcher, a graduate student, suggested an atomizer, a simple but powerful idea that worked. Now Millikan would redo his experiment with the utmost care. Using oil droplets would minimize evaporation. A heavy metal pot would shield the metal plates from outside influences. The air going through the atomizer was filtered, as was the light used to illuminate the oil droplet. Finally, the entire apparatus was immersed in a constant-temperature oil bath. Millikan found it difficult to tell when the droplet was exactly balanced and motionless, so he used a moving droplet and timed it very accurately. To make these timings in the darkened room, he used a special printing chronograph, accurate to one-hundredth of a second. In 1911 Millikan published his final value for the electron charge: $4.77 \times 10^{-10}$ electrostatic units, or $1.60 \times 10^{-19}$ coulomb.
Although Millikan's experiment is famous for its determination of the electron charge, that is not its greatest significance. There were several determinations of the electron charge before the oil-drop experiment. The most important finding of the new experiment was that electric charge is quantized in units the size of the electron. In Millikan's own words:

Here, then, is direct unimpeachable proof that the electron is not a "statistical mean," but that rather the electrical charges found on ions all have either exactly the same value or else small exact multiples of that value.

Today physicists are searching for fractionally charged particles called quarks. Based upon a study of elementary particles, quarks carry charges of \( \pm \frac{2}{3} \) e and \( \pm \frac{1}{3} \) e and are the building blocks of particles which exist inside nuclei. Modifications of Millikan's historical experiment are used by some of these quark hunters.
ADDITIONAL RESOURCES

A Mathematical Analysis of the Millikan Experiment

In analyzing mathematically the Millikan experiment, most high school texts concentrate on the analysis of oil droplets or latex spheres balanced by gravitational and electric fields. A pair of parallel plates provides a nearly uniform electric field, the magnitude of which is given by \( E = \frac{V}{d} \), where \( V \) is the potential difference across the plates and \( d \) is their separation. Since the droplet (mass \( m \), charge \( q \)) is held motionless, the only forces acting on it are the force of gravity (\( F_g = mg \)) and the electric force (\( F_E = qE \)). Determining the mass of the oil droplet becomes a major problem which Millikan solved by allowing the droplet to move through air, observing its velocity, and calculating its mass using fluid dynamics. This complicated process is simplified in most high school presentations by using calibrated latex spheres of known mass.

When the droplet is held motionless, there is no air friction. Since it is in equilibrium, the net force acting is zero. Therefore,

\[
F_E = F_g,
\]

implies that

\[
qE = mg.
\]

Substituting \( E = \frac{V}{d} \), we get

\[
\frac{Vq}{d} = mg.
\]

Solving for the charge, we find

\[
q = \frac{mgd}{V}.
\]

When Millikan balanced the electric and gravitational forces on a droplet and observed it for some time, he noticed that suddenly it would start to move and a new potential difference would be required to bring it to rest. A careful analysis of the potential difference required to bring the droplet to rest revealed that the amount of charge always came in small integer multiples of a particular value. Charge is quantized!

Because of the experimental difficulties of knowing when the droplet was completely motionless, and because it was necessary to move the droplet in order to determine its mass, the measurements in the actual experiment were made on moving droplets. Here, too, the quantized nature of electric charge could be observed. Frequently while observing a droplet moving at constant velocity against the retarding force of the air, the droplet would suddenly change velocity. An analysis of the forces required to move the droplet at this new velocity always revealed that the charge was some multiple of the smallest value.

Although the mathematical analysis of the case of moving droplets is not usually discussed in high school, nor is it considered in the video, it was such an important part of Millikan's original work that it is included here for reference. Any object falling through air or any fluid will eventually reach a terminal velocity, where the force of gravity is matched by the viscous force. When the sum
of the forces on the droplet equals zero, the droplet will fall at constant speed. The law which describes the viscous force was found in the nineteenth century and is known as Stokes' law:

\[ F_v = 6\pi R \eta v, \]

where \( F_v \) = force of viscosity, \( R \) = radius of droplet, \( \eta \) = viscosity of material, and \( v \) = speed of the droplet. At terminal velocity the force of gravity equals the viscous force, so we have

\[ 6\pi R \eta v = mg, \tag{1} \]

where \( m \) is the mass of the droplet and \( g \) is the acceleration of gravity. Millikan found the value of \( v \) by allowing the droplet to reach terminal velocity and then timing its motion over a known distance. Still the mass of the droplet cannot be found unless the radius is known. We can find another equation in \( m \) and \( R \) using the density of the oil (which can be easily measured):

\[ \rho = \frac{m}{\text{volume}}, \]

where \( \rho \) is the density of the oil. Since the volume of a spherical droplet is \( \frac{4}{3}\pi R^3 \), where \( R \) is its radius, the density of a droplet is given by

\[ \rho = \frac{m}{[\frac{4}{3}\pi R^3]}. \tag{2} \]

Eliminating \( m \) in Eqs. (1) and (2) gives the radius of a droplet in easily measured quantities:

\[ R = \sqrt{\frac{6\eta v}{4/3 \pi \rho g}}. \tag{3} \]

The next steps in the experiment are to move the droplet upward using an electric field and to measure its terminal velocity while moving upward, \( v_{up} \). The force of viscosity is now directed downward, so that equilibrium requires

\[ F_E = F_g + F_v, \tag{4} \]

where

\[ F_v = 6\pi R \eta v_{up}, \quad F_g = mg, \quad \text{and} \quad F_E = qE. \]

Substituting these values in Eq. (4) we get

\[ qE = mg + 6\pi R \eta v_{up}. \]

Using Eq. (1) to replace \( mg \) we find

\[ qE = 6\pi R \eta v + 6\pi R \eta v_{up}, \]

and solving for the charge we obtain

\[ q = 6\pi R \eta (v + v_{up}). \]
When Eq. (3) is used to determine the radius of a droplet, this last result then yields the charge on the droplet in terms of measurable quantities. The great bulk of Millikan's actual experimental time was devoted to moving small droplets upward using the electric field, carefully measuring their velocity, and then allowing them to fall under the influence of gravity alone, and again measuring their velocity. After measuring a single droplet rise and fall several times, Millikan would change the charge on the droplet with x-rays and repeat the experiment. In this manner, many different charges could be measured on the same droplet, often over a period of several hours.

Millikan also took into account the buoyancy of air in analyzing all the forces on a droplet. This correction has been left out in the above discussion because its effects are so small.
Helpful Suggestions in Performing the Millikan Experiment

One of the major problems in the Millikan oil-drop experiment is the determination of the mass of the drop. The theory behind the measurement is usually not discussed in high school, and the measurement itself is time-consuming. However, microscopic latex spheres of known diameter and density can now be produced and are used in the calibration of electron microscopes. Several high school lab manuals have recognized the convenience of these spheres of identical mass in developing a variation of the Millikan experiment.

Although the modified experiment is still difficult to perform, some teachers have had success with this version. If you decide to do it, be sure to try it yourself so that you can be prepared for students' difficulties. Many experienced teachers have abandoned this revised experiment as hopeless; those who have had some success suggest the following:

1. Correct lighting and proper microscope focus are very important. Removing the cell and placing a needle point at the place where the spheres will appear can make careful prefocusing and lighting possible.

2. Sometimes it seems impossible to get any "drops" at all. Make sure all parts of the atomizer-to-cell path are clear of obstructions. Also make sure that the ball valve on the rear of the atomizer bulb is working correctly so that the jet of air passes only outward; otherwise the spheres can be withdrawn from the cell immediately after they enter it. New atomizer bulbs can be purchased at the perfume counter in a drug store.

3. Following the many excellent suggestions given in the Project Physics Handbook is beneficial.

The latex sphere version of the oil-drop experiment can be a frustrating experience, and many teachers use it only as a special "research" exercise for interested students. If you can spare the time, you might try it as a traditional lab exercise, but it might be more realistic to perform DEMONSTRATION #1 and use the class time to discuss the actual Millikan oil-drop experiment in the context of scientific research.

References: Experiment 5-3, "The Measurement of the Elementary Charge," in Project Physics Handbook, Holt, Rinehart and Winston. The Project Physics instructions are excellent and include many important details. The method used is static balance of the spheres only and not the full method of Millikan.

Experiment 17, "The Millikan Experiment," and Experiment 16, "Measuring Small Electric Forces," in Laboratory Manual for PSSC, D.C. Heath. The PSSC instructions are satisfactory but do not include the detail of the Project Physics Handbook. PSSC uses the idea of terminal velocity with a moving sphere rather than the static-balance method of Project Physics.
As alternatives to laboratory exercises, the following are suggested:

1. Film and Discussion

"The Millikan Experiment," 16-mm film, Physical Science Study Committee. This film recreates the Millikan experiment using latex spheres rather than oil drops. The film, along with a follow-up activity such as DEMONSTRATION #2 in this module or suitable problems from available texts, might be considered as an alternative to the laboratory listed above.

2. Computer Simulation of the Millikan Experiment

The Millikan experiment is ideal for computer simulation because of the difficulty encountered in performing this experiment.

"Charge," Minnesota Educational Computer Consortium, vol. 1, version 22. This is a text-only simulation, but is one that is commonly available for Apple computers.

"A Demonstration of the Millikan Experiment," by Don Barron, Physics Teacher, Wheaton High School, 12601 Dalewood Dr., Wheaton, MD 20906. This student-written program includes both a section on theory and a graphic simulation. It runs on an Apple and is available by sending a disk to the above address.

Several other commercial simulations from various companies are available. Experience indicates that graphic simulations are preferred to text-only simulations.
Demonstration #1: Large-scale "Quantum" Effects

Purpose: To perform a large scale demonstration which imitates the basic concept of the Millikan oil-drop experiment.

Materials: Laboratory balance accurate to one gram or better; several large steel ball bearings of the same mass; a paper sack.

Procedure and Notes: In advance of the demonstration, arrange a way to place the ball bearings in the sack out of view of the class. Place a small number of bearings in the sack, determine its mass, and record the mass on the board. Out of sight of the class, remove or add bearings, determine the mass of the sack, and record the result. Continue until several measurements have been made.

Discuss how the data might be used to answer the following two questions:

1. Do all of the items in the sack have the same mass?
2. If so, what is the mass of each item?

Explanation: The demonstration should help to establish, by analogy, how the Millikan oil-drop experiment shows that charge comes in multiples of a fundamental unit.
Demonstration #2: A Millikan - Experiment Analogy

Purpose: To imitate the concept of the Millikan oil-drop experiment and establish the fact that charges occur in multiples of a fundamental charge.

Materials: Ten lettered envelopes containing various numbers of index cards:

<table>
<thead>
<tr>
<th>Envelope letter</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of index cards</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Procedure and Notes: Determine the mass of each envelope. Then draw a histogram (bar graph) with the masses in ascending order.

1. What would lead you to believe that all the index cards have the same mass?

2. What would be your guess as to the mass of each index card?

3. What can you say about the mass of the envelope?

4. What is your guess for the mass of the envelope and why?

It would probably be best to start this activity about 15 minutes before the end of class and have the students draw the histogram and answer the questions overnight as a homework assignment. The discussion could begin the next day’s class period.

Explanation: The mass of the envelopes and index cards might vary from brand to brand. Assuming typical values of about 2.7 grams for envelopes and 1.9 grams for the index cards, the histogram shown below should result.

1. The histogram seems to show all steps about the same height.

2. The mass of the index cards seems to be about 1.9 grams because that corresponds to the size of the steps in the histogram.
3. Since the smallest mass that was measured was about 4.6 grams, that would have to be the largest possible value. Other possible values might be 2.7 grams (4.6 grams minus the mass of one index card) or 0.8 grams (4.6 grams minus the mass of two index cards).

4. There is no one correct answer to this question. Do not expect the students to give only 2.7 grams. The students should be able to give a defense of one of the following: 4.6, 2.7, or 0.8 grams.

Reference: "Milli-can Experiment," by James H. Nelson, in *The Physics Teacher*, vol. 18, no. 1, January, 1980, p. 39. This is a similar experiment to the one described, but uses 35-mm film cans and coins. It contains some excellent extensions and points for discussion.
Demonstration #3: Cloud Chamber

Purpose: To demonstrate the operation of a cloud chamber. Cloud chambers were the first devices used to measure the charge of the electron. The Millikan experiment is a highly refined version of the early cloud chamber experiments. This experiment shows that droplets form on ions.

Materials: Diffusion cloud chamber, power supply, light source, 2 lb. of dry ice, isopropyl alcohol, radioactive source (alpha, or beta).

Procedure and Notes: Since there are many different types of cloud chambers, no general procedure can be given here. Follow the manufacturer's procedures and observe proper precautions with the high-voltage power supply.

Explanations: All cloud chambers are based on the ionization of a gas caused by charged particles moving through a gas at high speeds. A collision between a charged particle and an atom in its path can give an electron of the atom enough energy to escape from the atom. The process forms two charged particles: the electron itself and the ionized atom, which is now positively charged because it has lost a unit of negative charge.

When an alcohol molecule of the vapor inside the cloud chamber passes near an ion, it becomes polarized; the positive and negative centers of charge are slightly displaced. Consequently, the molecule can be attracted to the ion. Other molecules in the vicinity react similarly, and the ion thus acts as a center of condensation. When the temperature of the gas is below the dew point of the vapor, a droplet of alcohol quickly forms. A speeding particle leaves thousands of ions in its wake; the resulting droplets comprise the vapor trail. ("Cloud Chambers for Detecting Nuclear Events," by Gareth D. Shaw, in The Amateur Scientist, edited by C. L. Strong, p. 323, New York, Simon and Schuster, 1960.)

Cloud chambers can be frustrating to work with. If you have more than one, set up several to increase the probability of having one work. Once one is operating, it is usually good for a couple of hours. You are trying to obtain a supersaturated gaseous solution that will condense to form a vapor trail when an ion intercepts a molecule of alcohol.

Suggestions:

1. Use lots of dry ice (a 2-lb. block). You want a large enough temperature change from the top of your apparatus to the bottom so that the saturated air at the top will be supersaturated when it reaches the lower levels.

2. Darken the room and use side lighting such as a slide-projector light. The blower on the projector also will help to dissipate the heat of the lamp. You may need to shift the light, because an angle of about 120° is needed between the lamp and your eyes in order to see the paths.
3. Moisten the base with alcohol. Don't flood it, or the sensitive region for the trails may be obscured.

4. Use an electric field to clear off old trails and spurious ions.

5. **Know what to look for:** Alpha trails are thicker than beta trails, because alpha particles are more massive and can ionize more atoms before losing energy. Forking trails usually involve a collision with a nucleus.

6. Make a trial run without the radioactive material to enable you to differentiate between background trails and the actual trails produced by the sources.
EVALUATION QUESTIONS

1. The Millikan oil-drop experiment
   A. is an important test of the force of gravity on small objects.
   B. gave the first study of the effect of viscosity on falling droplets.
   C. determined the average charge on clouds of oil droplets.
   D. established that charge comes in integer multiples.

2. When a small spherical oil droplet falls through air, it quickly reaches a constant speed. This speed
   A. is independent of the mass of the droplet.
   B. would be smaller if the mass of the object is larger.
   C. would be larger if the mass of the object is larger.
   D. is a universal constant.

3. In the Millikan oil-drop experiment, a droplet of oil moves upward at a constant velocity in the electric field. Which of the following is true?
   A. The charge on the droplet must be positive.
   B. The droplet moves at constant speed because the electric field is upward.
   C. The upward force on the droplet must be larger than the downward force on the droplet.
   D. The forces on the droplet are balanced.

4. A microscopic droplet of oil is moving downward at constant speed in a Millikan type apparatus. The electric field is turned on and the droplet quickly begins moving upward at a constant speed which is twice the falling speed. In this situation, which of the following statements is true?
   A. The unbalanced force on the droplet is upward.
   B. The charge on the droplet must be negative.
   C. The electric force on the droplet is three times the weight of the droplet.
   D. The viscous force on the droplet is three times the weight of the droplet.

5. The diagram below represents a particle of mass, m, and charge, q, located between two parallel plates. The electric field strength between the two plates is E.

```
+ + + + + + + + + +
● q
- - - - - - - - -
```

The particle will remain stationary when
   A. qE is greater than mg.
   B. qE is equal to mg.
   C. qE is less than mg.
   D. E equals g.
6. Which of the following forces on the moving oil drop had to be considered in order to measure the charge on the drop?

   A. The force of gravity.
   B. The electric force.
   C. The force of air resistance.
   D. All of the above.

7. Suppose you use enough batteries to supply an electric force in the upward direction equal to three times the downward force of gravity. After the drop reaches terminal velocity it moves

   A. upward as fast as it fell under gravity alone.
   B. upward twice as fast as it fell under gravity alone.
   C. upward three times as fast as it fell under gravity alone.
   D. upward four times as fast as it fell under gravity alone.

8. Which of the following could be the charge on the oil drop in question 7?

   A. 1/4 the charge of an electron.
   B. 1/2 the charge of an electron.
   C. Three times the charge of an electron.
   D. All of the above.

9. When observing a single drop for extended periods of time, Millikan found that many different values of electric field were necessary to move the drop at a constant speed. The most likely reason that the different values of electric field were required was because

   A. the mass of the drop changed.
   B. the charge on the drop changed.
   C. the radius of the drop changed.
   D. the temperature of the air changed.

10. When air friction is considered, a falling object

    A. falls at the acceleration due to gravity.
    B. falls at a constant acceleration which is less than the acceleration due to gravity.
    C. has the largest acceleration just when released.
    D. can only fall at terminal velocity.

ESSAY QUESTIONS

11. An oil drop is being pulled upward by the electric field in a Millikan oil drop apparatus. The drop is moving at a constant velocity. Carefully illustrate and label all of the forces acting on the drop while it is being pulled upward at constant velocity. Briefly discuss each of the labeled forces.

12. The video discusses how Millikan chose to publish some of his results while rejecting others. It is suggested that this practice might be "unscientific". If scientists exercise this censorship over their data, what prevents someone from making a discovery that isn't real?
KEY

1. D
2. C
3. D
4. D
5. B
6. D
7. B
8. C
9. B
10. C

SUGGESTED ESSAY RESPONSES

11. $F_E$ is the force due to the electric field. $F_g$ is the force due to the gravitational field, or the weight of the drop. $F_v$ is the force of air friction.

12. The scientist firmly believes that all scientific experiments can be repeated. If his data cannot be repeated under the conditions given in his experiment, then some other scientist will "find him out".