Teaching Upper-Division Electromagnetism in the Paradigms Program

http://physics.oregonstate.edu/portfolioswiki

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& the whole Paradigms Team
Support

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• Oregon State University
• Oregon Collaborative for Excellence in the Preparation of Teachers
• Grinnell College
• Mount Holyoke College
• Utah State University
Workshop on the Status of the Upper-Division Physics Curriculum

• Conference site, presentations, and report: http://compadre.org/SUPC/
• Many thanks to Ernest Behringer for an insightful conference summary.
• Upper-division collections on comPADRE are in process.
Workshop on the Status of the Upper-Division Physics Curriculum

• Griffiths E&M text is considered by most to be canonical.
• There is no common agreement about quantum textbooks or goals.
• 1/3 of participant faculty do not feel comfortable enough with thermal content to want to teach the course.
Colorado Goals

• Math/physics connection
• Visualize the problem
• Organized knowledge
• Communication
• Problem-solving strategy
• Expect & check solution
• Intellectual maturity
• Maxwell’s Equations
• Build on Earlier Material

• Problem-solving techniques
  – Approximations
  – Series expansions
  – Symmetries
  – Integration
  – Superposition

http://www.colorado.edu/sei/departments/physics_learning.htm
• Raise your hand if you believe that Green Functions are an advanced topic.
Griffiths 1: Vector Analysis

- Derivatives: Gradient, Divergence, Curl
- Product Rules, Second Derivatives
- Theorems
- Curvilinear Coordinates
- Dirac Delta Function
- Theory of Vector Fields
Griffiths 1: Vector Analysis

- Derivatives: Gradient, Divergence, Curl (in rectangular coordinates)
- Product Rules, Second Derivatives
- Theorems (maybe)
- Curvilinear Coordinates (not $\hat{r}, \hat{\theta}, \hat{\phi}$).
- Dirac Delta Function
- Theory of Vector Fields
Griffiths 2 & 5: Electrostatics (Magnetostatics)

- Introduce $V \quad \vec{E} \quad \rho$
- Direct integration (chop up the source and accumulate) Use explicit coordinates.
- What is script r? Note that Coulomb’s law is a Green function.
- Use the theorems to derive differential versions of Maxwell’s eqns.
Griffiths 2 & 5: Electrostatics (Magnetostatics)

\[ \vec{E} = -\nabla V \]

\[ V = -\int \vec{E} \cdot d\vec{r} \]

\[ \frac{\rho}{\varepsilon_0} = \nabla \cdot \vec{E} \]

\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \, d\tau' \]
Griffiths 4 & 6: Electric (Magnetic) Fields in Matter

- Polarization
- The Field of a Polarized Object
- The Electric Displacement
- Linear Dielectrics
Derivation of Bound Charges

\[
V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{(\vec{r} - \vec{r}') \cdot \vec{P}(r')}{|\vec{r} - \vec{r}'|^3} \, d\tau'
\]

\[
= \frac{1}{4\pi\varepsilon_0} \int_V \vec{P}(r') \cdot \vec{\nabla}' \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) \, d\tau'
\]

\[
= \frac{1}{4\pi\varepsilon_0} \left[ \int_V \vec{\nabla}' \cdot \left( \frac{\vec{P}}{|\vec{r} - \vec{r}'|} \right) \, d\tau' - \int_V \frac{1}{|\vec{r} - \vec{r}'|} \left( \vec{\nabla}' \cdot \vec{P} \right) \, d\tau' \right]
\]

\[
= \frac{1}{4\pi\varepsilon_0} \left[ \int_S \frac{1}{|\vec{r} - \vec{r}'|} \vec{P} \cdot d\vec{a}' - \int_V \frac{1}{|\vec{r} - \vec{r}'|} \left( \vec{\nabla}' \cdot \vec{P} \right) \, d\tau' \right]
\]

\[
= \frac{1}{4\pi\varepsilon_0} \left[ \int_S \frac{\sigma_b}{|\vec{r} - \vec{r}'|} \, da' - \int_V \frac{\rho_b}{|\vec{r} - \vec{r}'|} \, d\tau' \right]
\]
Griffiths 3: Special Techniques

• Uniqueness Theorems
• Method of Images
• Separation of Variables
  – Rectangular Coordinates
  – Spherical Coordinates w/ Azimuthal Symmetry
• Multipole Expansions
Real Title of This Talk

It is impossible to teach upper-division E&M as defined by Griffiths.

(or—you can teach it but your students can’t be expected to follow!)

YOU HAVE TO MAKE CHOICES.
Our Choices

• In the equivalent of 12 weeks:
  – All of Chapter 1, but mixed in with the physics.
  – All of Chapters 2 & 4, emphasizing geometry.
  – An intro version of multipole expansions.
  – An intro version of vector algebra manipulations.
  – An intro version of the uniqueness theorems.

• In quantum mechanics:
  – Separation of variables

• In 10 weeks:
  – Everything else!!! (including Chaps 4 & 6)
CUE Data

What do you notice about this data?

[Bar chart showing average scores for OSU (N=37) and CU (N=103) across questions Q1 to Q17.]
What I notice

• E&M I is hard!
• The Paradigms curriculum is not as successful as Colorado’s (maybe).
• Paradigms students can’t answer questions 1 & 15.
• Paradigms students tail off at the end.
• Otherwise the performance of Paradigms students tends to parallel the performance of Colorado students.
CUE Data

- OSU Normalized Gain 33%
- CU Normalized Gain 34%
CUE Separation of Variables

Q1. An insulating sphere with radius \( R \), with a voltage on its surface \( V(\theta) = k\cos(3\theta) \). Find \( E \) (or \( V \)) inside the sphere at point \( P \).

Q15. Circle all of the following boundary conditions that are suitable for solving Laplace’s equation for finding \( V(r,\theta) \) everywhere due to a charge density \( \sigma \) on a spherical surface of radius \( R \).

(I) \( V_{in} = V_{out} \) at \( r = R \)
(II) \( \vec{E}_{in} = \vec{E}_{out} \) at \( r = R \)
(III) \( E_{in} - E_{out} = -\sigma/\varepsilon_0 \) at \( r = R \)
(IV) \( E_{in} - E_{out} = -\sigma/\varepsilon_0 \) at \( r = R \)
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  - Superposition

http://www.colorado.edu/sei/departments/physics_learning.htm
Our materials

- “Just in time” math.
\[ V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \sum_i \frac{q_i}{|\vec{r} - \vec{r}'|} \]

\[ V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau' \]

\[ \vec{E}(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho(\vec{r}')(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^2} d\tau' \]

\[ \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau' \]

\[ \vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^2} d\tau' \]
Our materials

• Spiral approach with reusable content (distance between points, charge/current density)
\[ V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{|\vec{r} - \vec{r}'|} \]

\[ V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau' \]

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Our materials

• Communication from students to faculty
• Respect for student difficulties.
Prompt

• Find the magnetic vector potential everywhere in space from a spinning ring of charge with radius R, charge Q, and period T. Continue until you get an expression that Maple or Mathematica could evaluate.
\[
V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{|\vec{r} - \vec{r}'|}
\]

\[
V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau'
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\[
\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}')(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^2} d\tau'
\]

\[
\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau'
\]

\[
\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')(\vec{r} - \vec{r}') \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^2} d\tau'
\]
Our materials

- Students do hard examples in groups during class with faculty support but no prior lecture/reading. (back flip).
\[ V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{|\vec{r} - \vec{r}'|} \]

\[ V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau' \]

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The Purpose of Education — A challenging vision

“...the purpose of your education is your growth as an individual and the development of your capacity to contribute to the transformation of society.”

—FUNDAEC

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