The Triplets Paradox

About acceleration in Special Relativity

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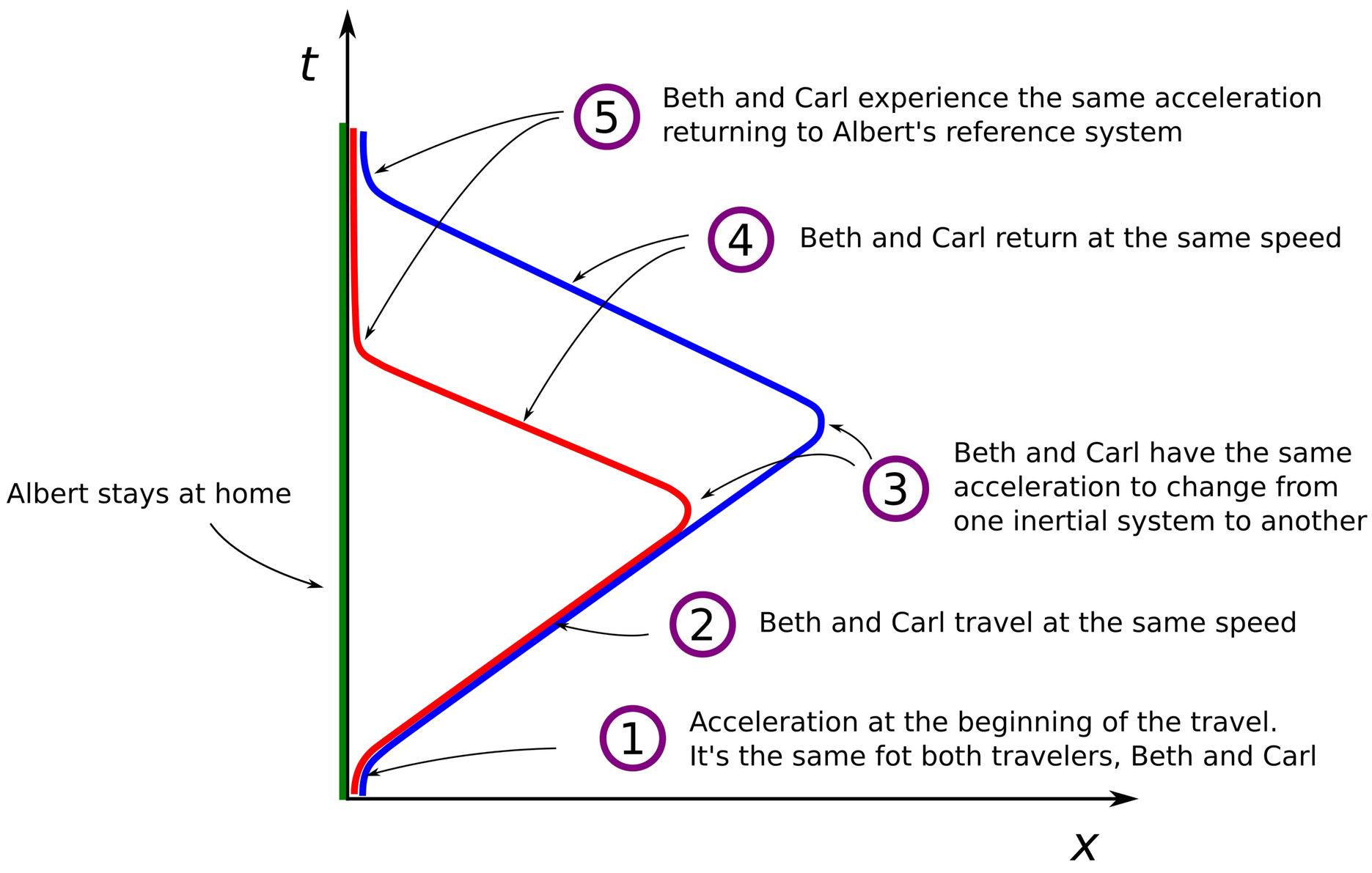
In this work a didactical approach about the well known twin paradox is discussed. In a simple and visual way it is shown that two systems could go through the same accelerations and yet end up having different times at the end of their trips, pointing out that the amount of acceleration is not the key of the problem. The change of inertial reference frame is highlighted to understand the issue together with the role of acceleration in special relativity and the twin paradox in particular.

Twin paradox: one twin travels at high speed and returns to Earth while his brother was waiting.
Why the stationary brother is older?

1) Asymmetry arises from the change in the reference frame of the traveler.
2) Acceleration is key to that change.
3) But, is the acceleration directly related to the the amount of the time shift?



Space-time diagram showing the worldlines of the triplets: Albert, Beth and Carl.

Albert (green line) stays at home in Earth, while Beth (red line) travels back and forth and also Carl (blue line), but on a longer trip than Beth.

When the three meet again, Albert is older than Beth, who in turn is older than Carl.

The time shift between Beth and Carl is NOT due to the accelerations, since both have suffered the same accelerations.

The time shift is due to different *proper times* in their worldlines.

Minkowski metric
Line element:

$$dt_{proper} = \sqrt{c^2 dt^2 - dx^2}$$

A straight line (like Albert) has the longest proper time.
A nonstraight line has a shorter proper time.

Then: Albert's proper time is longer than Beth's and Beth's proper time is longer than Carl's.