How Students' Understanding of Normalization Changes After Taking Quantum Mechanics

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Introduction

- Normalization of kets, wave functions, and vectors that mathematically represent quantum states is particularly important due to the probabilistic nature of quantum mechanics.
- The goal of my research is to examine students' understanding of normalization as they enter and leave a quantum mechanics course.

Research Questions

- What understandings of normalization do students have at the beginning of a quantum mechanics course?

- How do these initial understandings of normalization differ from the understandings of students at the end of studying Quantum Mechanical Spin?

Theoretical Framework

- A *conceptual analysis* (von Glasersfeld, 1995) or “a detailed description of what is involved in knowing a particular (mathematical) concept” (Lockwood, 2013, p. 252) was used in developing the framework.
- See Watson (2017) for more details on the development

		Students’ Understanding
Vectors	Vector Space What is a vector space? Examples of vector spaces	
	Vector Representations	
Norm	Norms: What is a norm? Examples of norms	
	Procedure(s) to Find Norm	
Normalizing	Metaphors for Normalizing	
	Procedure(s) to Normalize	
Normalized Vectors	Properties of Normalized Vectors	
	Reasons for Normalized Vectors	

Methods

- Hour-long, video-recorded, semi-structured interviews with physics students from a university in the northwestern United States
  - Nine were interviewed at the beginning of a junior-level quantum mechanics course
  - Eight were interviewed at the end of a three-week unit on Spin (of which six had participated in the earlier interview)
- Asked questions about several linear algebra concepts relevant to quantum mechanics. This research only focuses on questions where students were asked to normalize vectors.
- Each students' understanding was summarized by filling out the framework. Patterns were found by looking across students.

Results – Familiarity with Complex Vectors

- In the pre-quantum interviews, only two of the nine students were able to normalize a complex vector, and several explicitly mentioned never having seen complex vectors before.

- After studying quantum spin, six of the eight students were able to correctly normalize a complex vector.

Doug Pre-Quantum

$$|\vec{w}| = \sqrt{(3+2i)^2 + (4-i)^2}$$

$$\sqrt{9-4+12i+16-1+8i}$$

$$|w| = \sqrt{20+20i}$$

Doug Post-Quantum Spin

$$\vec{v} = \begin{pmatrix} 3 \\ 2i \end{pmatrix} \qquad \vec{v}\cdot\vec{v} = |3|^2 + |2i|^2$$

$$|\vec{v}| = \mathbb{R}$$

$$= 9 + 4 = 13$$

$$\vec{v}_n = \frac{1}{\sqrt{13}} \begin{pmatrix} 3 \\ 2i \end{pmatrix} \qquad \vec{v}\cdot\vec{v}^* = 3\cdot 3 + 2i(-2i)$$

$$|\vec{v}_n| = 1 \qquad 9 + 4$$

Results – Vector Representations

- In the pre-quantum interviews, students mainly chose to represent vectors in three ways: matrix notation, algebraic vector notation (i.e. $\vec{v}$), and graphically as a directed arrow from the origin.

- In the post-quantum spin interviews, some students explicitly chose Dirac Notation for their calculations, and even expressed why they prefer it.

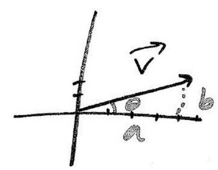
Danielle Pre-Quantum

$\vec{V}\cdot\vec{V}=$

$\sqrt{5^2+2^2}$

$\sqrt{25+4}$

$|\vec{V}|=\sqrt{29}$



$|\vec{V}|=\sqrt{a^2+b^2}$

Danielle Post-Quantum Spin

$\vec{V}=\begin{pmatrix}3\\2i\end{pmatrix}=|V_0\rangle$

$\sqrt{\langle v|v\rangle}$

$\sqrt{(3\langle 1| - 2i\langle 2|)(3|1\rangle + 2i|2\rangle)}$

$\sqrt{9+(-2i)(2i)}$

$\sqrt{9+4}=\sqrt{13}$

$|V_1\rangle=\frac{1}{\sqrt{13}}\begin{pmatrix}3\\2i\end{pmatrix}$

Results – Norm and Inner Product Confusion

- In the pre-quantum interviews, all nine students knew at least one way to find the norm of a real vector
- In the post-quantum spin interviews, four of the eight students showed some possible conflation between the norm of a vector and the inner product of a vector with itself.

David Pre-Quantum

Normalize the following vector: $\boldsymbol{v} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

$$\frac{v}{|v|} = \frac{1}{\sqrt{25+4}}\, v = \frac{1}{\sqrt{29}}\vec{v}$$

"And, I just squared both components, and then took the square root because that's how you get the magnitude of something. <u>Really, it's v over v-dot-v</u>."

David Post-Spin

$$\langle \psi | \psi \rangle = \left(\langle + | 3 + -2i \langle - | \right) \cdot \frac{1}{\sqrt{13}}$$

$$\left(3|+\rangle + 2i|-\rangle \right) \frac{1}{\sqrt{13}}$$

$$\langle + | + \rangle = 1 \quad \langle - | - \rangle = 1$$

$$= \left(9\langle + | + \rangle + -4i^2 \langle - | - \rangle \right) \frac{1}{13}$$

$$\frac{1}{13}\left(9 + 4 \right) = \frac{13}{13}$$

"<u>Which is 13. And that's the magnitude of the vector</u>. So, if we want to get rid of that, then, we want to divide by root 13. Because, we have two vectors in here."

Results – Reasons for Normalization

Pre-Quantum Interviews	Post-Quantum Spin Interviews
Doug: … So, you're shrinking it down to that to get just the direction. … Damian: … I guess, basically what we're doing is we're just cutting down the length to a unit of one. Um, so all we're worrying about here is the direction. … Drake: … Which is useful to, like, for, like, you, if you only want the direction of the vector, not the magnitude. …	David: And that's the magnitude of the vector. So, if we want to get rid of that, then, we want to divide by root thirteen. Doug: … you need to normalize so that way when you're summing up other things, you're getting a probability of 1, otherwise the probability is not going to make any sense. Doran: OH YEAH! I think we are reserving [quantum] states to be unit vectors.

Discussion

- While these physics students had little to no experience with complex vectors before the course, most seemed comfortable working with these vectors after studying quantum mechanics
  - This could partially be a result of becoming familiar with Dirac Notation (Gire & Price, 2015)
- Future Work: How might students' conceptions of normalization (e.g., "getting rid of the magnitude") impact their understanding of normalizing wave functions?

References

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