07/23/19 TPT FAVORITES #DH08

Provo UT

THE JOY OF SOLVING PHYSICS PROBLEMS

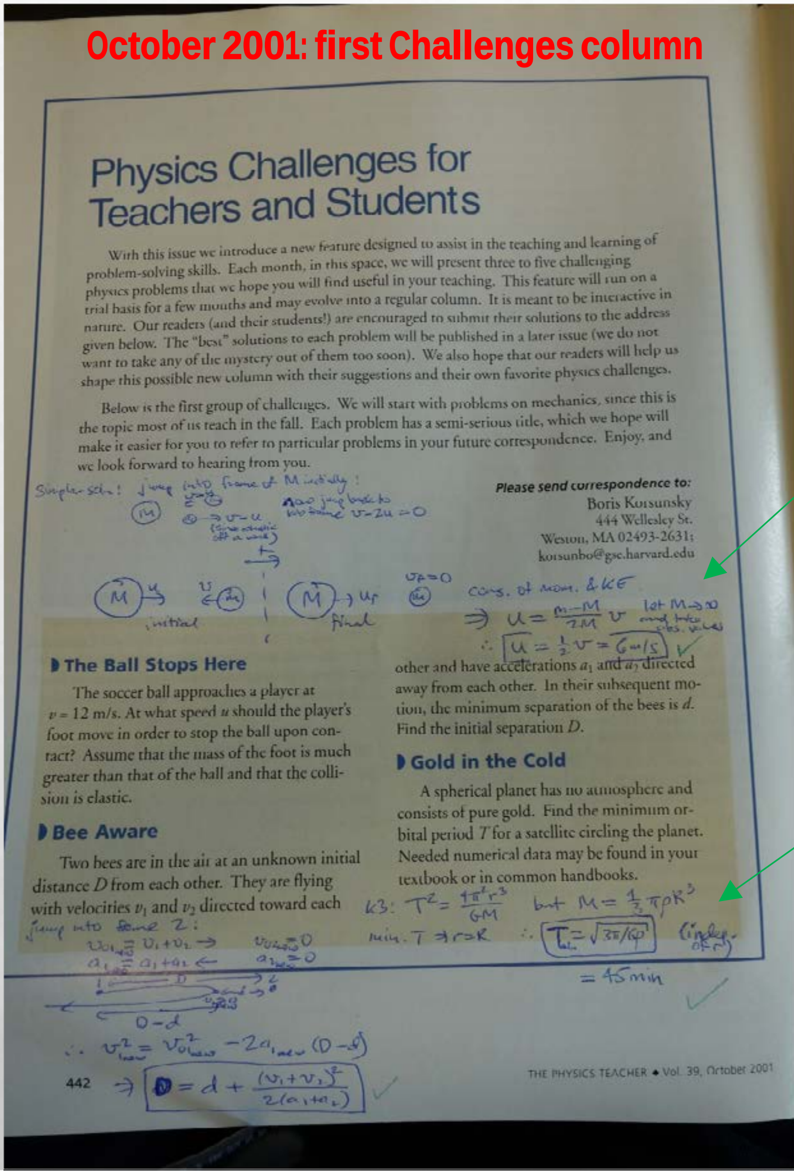
CARL E. MUNGAN, U.S. NAVAL ACADEMY, ANNAPOLIS MD

TPT Column “Physics Challenges for Teachers and Students” by Boris Korsunsky

inaugurated October 2001 and still going strong

white page inserts
in each issue
are my solutions
and notes

October 2001: first Challenges column



Physics Challenges for Teachers and Students

With this issue we introduce a new feature designed to assist in the teaching and learning of problem-solving skills. Each month, in this space, we will present three to five challenging physics problems that we hope you will find useful in your teaching. This feature will run on a trial basis for a few months and may evolve into a regular column. It is meant to be interactive in nature. Our readers (and their students!) are encouraged to submit their solutions to the address given below. The "best" solutions to each problem will be published in a later issue (we do not want to take any of the mystery out of them too soon). We also hope that our readers will help us shape this possible new column with their suggestions and their own favorite physics challenges.

Below is the first group of challenges. We will start with problems on mechanics, since this is the topic most of us teach in the fall. Each problem has a semi-serious title, which we hope will make it easier for you to refer to particular problems in your future correspondence. Enjoy, and we look forward to hearing from you.

Please send correspondence to:
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The Ball Stops Here

The soccer ball approaches a player at $v = 12$ m/s. At what speed u should the player's foot move in order to stop the ball upon contact? Assume that the mass of the foot is much greater than that of the ball and that the collision is elastic.

Bee Aware

Two bees are in the air at an unknown initial distance D from each other. They are flying with velocities v_1 and v_2 directed toward each other and have accelerations a_1 and a_2 directed away from each other. In their subsequent motion, the minimum separation of the bees is d. Find the initial separation D.

Gold in the Cold

A spherical planet has no atmosphere and consists of pure gold. Find the minimum orbital period T for a satellite circling the planet. Needed numerical data may be found in your textbook or in common handbooks.

originally
3 short problems
per month

initially I wrote
solutions by hand

November 2001:
my first article in TPT

Acceleration of a Pulled Spool

Carl E. Mungan, Physics Department, U.S. Naval Academy, Annapolis, MD 21402-5026; mungan@usna.edu

A well-known lecture demonstration[1] consists of pulling a spool by the free end of a string wrapped around its inner diameter. By pulling at different angles relative to the floor, the spool can be made to roll either toward or away from you. This is explained by considering the torque about the point of contact O between the spool and the floor. A number of authors[2] have noted that for a cylinder rolling under the action of a horizontal pulling force, the frictional force can be in the direction of motion of its center of mass C. This often puzzles introductory students. It is therefore helpful to explore the kinematics for the more general case of pulling at an arbitrary angle with respect to the floor.

Pulling the Spool

For a spool whose mass distribution is sufficiently far away from the axis of symmetry, it turns out that there are two special pulling angles. In addition to the familiar angle θ_c in the forward direction at which the spool *cannot* roll without slipping, there is a second characteristic angle θ_m in the backward direction at which the spool *always* rolls without slipping. (Throughout this article, "forward" refers to the direction in which the string would unwind off the bottom of the spool.) In standard homework problems and class demos involving pulled cylinders, θ_m is not investigated.

The critical pulling angle θ_c at which the spool slips in place without rolling occurs when the line of action of the pulling force T is directed through O, as illustrated in Fig. 1. All other forces (the normal force N, the spool's weight mg, and the frictional force f) also have lines of action through O; thus, there is no net torque τ about O and the spool does not rotate about this point. Let the inner radius of the spool (about which the string is wrapped) be R_1 and its outer radius (with which it contacts the floor) be R_2. From the geometry it follows that $\theta_c = R_1/R_2$. For example, if $R_1 = 0.75\ R_2$ then $\theta_c = 41°$.

Now consider pulling at a steeper angle,[3] $\theta > \theta_c$. A free-body diagram for the case in which the spool rolls without slipping (so that

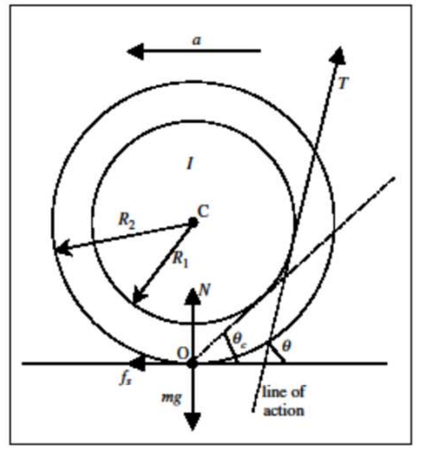
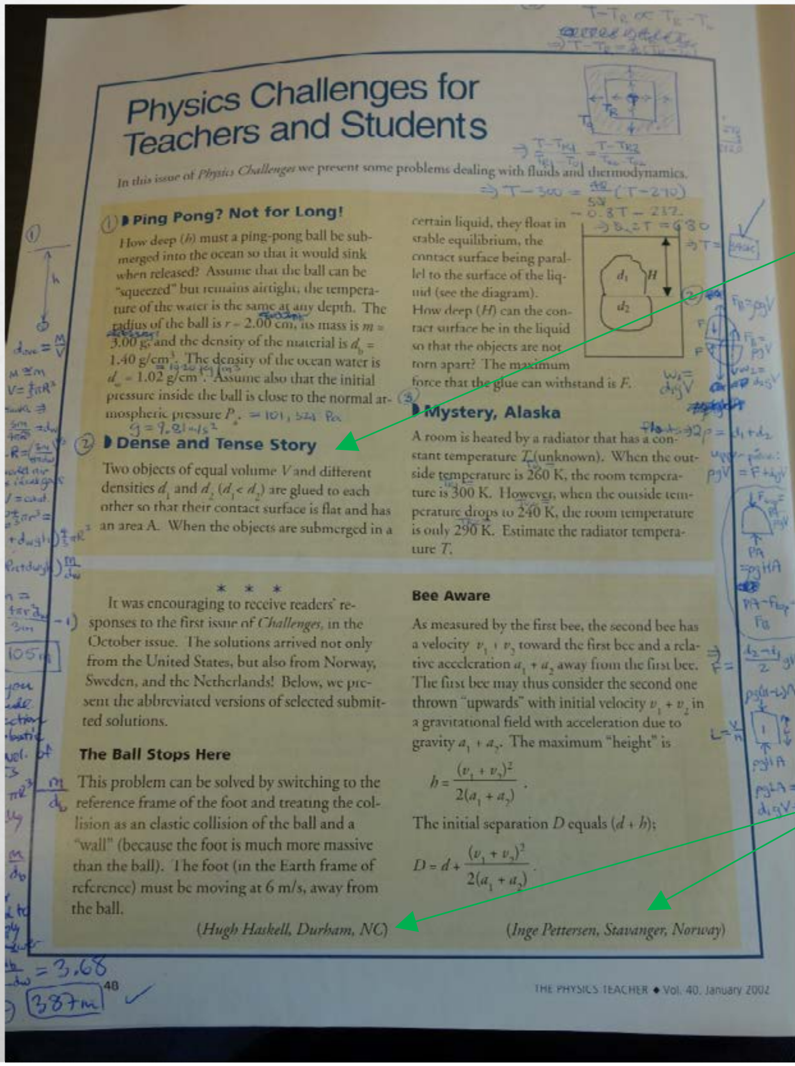


Fig. 1. Free-body diagram of a spool of mass *m* and moment of inertia *I* about its center C. A constant tension force *T* has a line of action making an angle θ with respect to the forward horizontal direction (which is to the right in this sketch). The directions of the static frictional force f_s and of the linear acceleration of the center of mass *a* follow from the equations of motion and depend on θ, as discussed in the text.

January 2002: first solutions appear



Physics Challenges for Teachers and Students

In this issue of *Physics Challenges* we present some problems dealing with fluids and thermodynamics.

Ping Pong? Not for Long!

How deep (h) must a ping-pong ball be submerged into the ocean so that it would sink when released? Assume that the ball can be "squeezed" but remains airtight; the temperature of the water is the same at any depth. The radius of the ball is $r = 2.00$ cm, its mass is $m = 3.00$ g, and the density of the material is $d_b = 1.40$ g/cm^3. The density of the ocean water is $d_w = 1.02$ g/cm^3. Assume also that the initial pressure inside the ball is close to the normal atmospheric pressure P_a.

Dense and Tense Story

Two objects of equal volume V and different densities d_1 and d_2 ($d_1 < d_2$) are glued to each other so that their contact surface is flat and has an area A. When the objects are submerged in a certain liquid, they float in stable equilibrium, the contact surface being parallel to the surface of the liquid (see the diagram). How deep (H) can the contact surface be in the liquid so that the objects are not torn apart? The maximum force that the glue can withstand is F.

Mystery, Alaska

A room is heated by a radiator that has a constant temperature T (unknown). When the outside temperature is 260 K, the room temperature is 300 K. However, when the outside temperature drops to 240 K, the room temperature is only 290 K. Estimate the radiator temperature T.

* * *

It was encouraging to receive readers' responses to the first issue of *Challenges*, in the October issue. The solutions arrived not only from the United States, but also from Norway, Sweden, and the Netherlands! Below, we present the abbreviated versions of selected submitted solutions.

The Ball Stops Here

This problem can be solved by switching to the reference frame of the foot and treating the collision as an elastic collision of the ball and a "wall" (because the foot is much more massive than the ball). The foot (in the Earth frame of reference) must be moving at 6 m/s, away from the ball.

(*Hugh Haskell, Durham, NC*)

Bee Aware

As measured by the first bee, the second bee has a velocity $v_1 + v_2$ toward the first bee and a relative acceleration $a_1 + a_2$ away from the first bee. The first bee may thus consider the second one thrown "upwards" with initial velocity $v_1 + v_2$ in a gravitational field with acceleration due to gravity $a_1 + a_2$. The maximum "height" is

$$h = \frac{(v_1 + v_2)^2}{2(a_1 + a_2)}.$$

The initial separation D equals $(d + h)$:

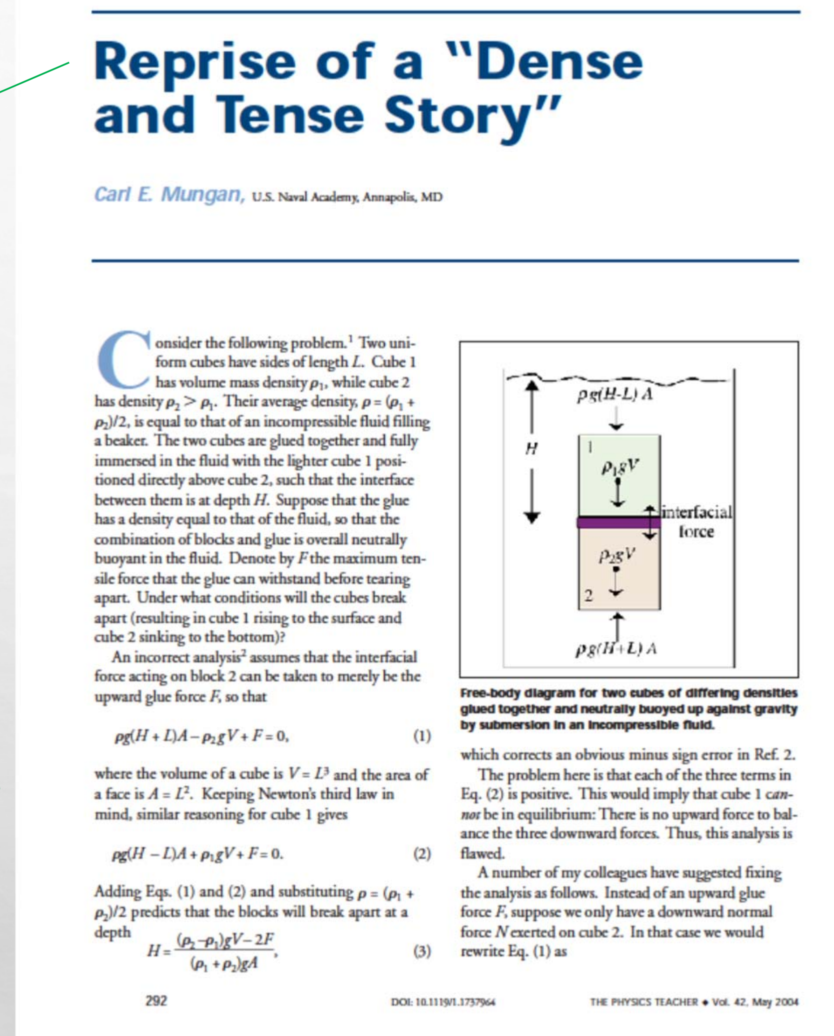
$$D = d + \frac{(v_1 + v_2)^2}{2(a_1 + a_2)}.$$

(*Inge Pettersen, Stavanger, Norway*)

thanks to Ken Ford who traveled to USNA to discuss the issues with me

reader who contributed that solution

May 2004: this Challenge proved especially tricky



Reprise of a "Dense and Tense Story"

Carl E. Mungan, U.S. Naval Academy, Annapolis, MD

Consider the following problem.[1] Two uniform cubes have sides of length L. Cube 1 has volume mass density ρ_1, while cube 2 has density $\rho_2 > \rho_1$. Their average density, $\rho = (\rho_1 + \rho_2)/2$, is equal to that of an incompressible fluid filling a beaker. The two cubes are glued together and fully immersed in the fluid with the lighter cube 1 positioned directly above cube 2, such that the interface between them is at depth H. Suppose that the glue has a density equal to that of the fluid, so that the combination of blocks and glue is overall neutrally buoyant in the fluid. Denote by F the maximum tensile force that the glue can withstand before tearing apart. Under what conditions will the cubes break apart (resulting in cube 1 rising to the surface and cube 2 sinking to the bottom)?

An incorrect analysis[2] assumes that the interfacial force acting on block 2 can be taken to merely be the upward glue force F, so that

$$\rho g(H+L)A - \rho_2 gV + F = 0, \quad (1)$$

where the volume of a cube is $V = L^3$ and the area of a face is $A = L^2$. Keeping Newton's third law in mind, similar reasoning for cube 1 gives

$$\rho g(H-L)A + \rho_1 gV + F = 0. \quad (2)$$

Adding Eqs. (1) and (2) and substituting $\rho = (\rho_1 + \rho_2)/2$ predicts that the blocks will break apart at a depth

$$H = \frac{(\rho_2 - \rho_1)gV - 2F}{(\rho_1 + \rho_2)gA}, \quad (3)$$

which corrects an obvious minus sign error in Ref. 2.

The problem here is that each of the three terms in Eq. (2) is positive. This would imply that cube 1 *cannot* be in equilibrium: There is no upward force to balance the three downward forces. Thus, this analysis is flawed.

A number of my colleagues have suggested fixing the analysis as follows. Instead of an upward glue force F, suppose we only have a downward normal force N exerted on cube 2. In that case we would rewrite Eq. (1) as

Free-body diagram for two cubes of differing densities glued together and neutrally buoyed up against gravity by submersion in an incompressible fluid.

March 2002 solutions: first to be posted online

Physics Challenges for Teachers and Students

* * *

Here we present the solutions to the March 2002 *Challenges*:

A Vacuum Squeezer

From the first law of thermodynamics,

$$dU = dQ - dW,$$

where dU is the change in the internal energy of the gas, dQ is the heat added to the gas, and dW is the work done by the gas.

For a monoatomic ideal gas

$$U = \tfrac{3}{2}nRT,$$

where n is the quantity of gas in moles, R is the universal gas constant, and T is the absolute temperature of the gas. For a fixed quantity of the gas

$$dU = \tfrac{3}{2}nR\,dT. \qquad (1)$$

The work done by the expansion of the gas is

$$dW = P\,dV.$$

If the cross-section area of the piston is A, the spring constant k, and the distance from the left edge of the container to the piston x, then the volume of gas in the cylinder is

$$V = Ax \quad \text{and} \quad dV = A\,dx.$$

The force on the piston exerted by the spring is kx; therefore, the pressure of the gas is

$$P = \frac{kx}{A} \quad \text{and} \quad dP = \frac{k\,dx}{A}.$$

From the ideal gas law:

$$PV = nRT \text{ or } d(PV) = d(nRT) \text{ or}$$
$$P\,dV + V\,dP = nR\,dT.$$

Since $P\,dV = V\,dP = kx\,dx$, the ideal gas law can be written as

$$2P\,dV = nR\,dT$$
or
$$dW = P\,dV = (1/2)nR\,dT. \qquad (2)$$

Substituting for dU and dW in the first law using Eq. (1) and Eq. (2):

$$\tfrac{3}{2}nR\,dT = dQ - \tfrac{1}{2}nR\,dT \quad \text{or} \quad dQ = 2nR\,dT.$$

The heat capacity of the system is then

$$C = \frac{dQ}{dT} \quad \text{or} \quad C = 2nR.$$

With the quantity of gas $n = 1$ mol,

$$C = (2\text{ mol})R \approx 16.6\text{ J/K}.$$

(Contributed by Richard Morra, Pascack Valley Regional HS District (Retired), Hillsdale, N.J.)

Futile resistance

Let V_1 be the voltage across the ammeter in the first circuit. In the second circuit, that voltage becomes $3V_1$ because the current triples. Similarly, the initial voltmeter reading can be denoted V_2, and the final reading $V_2/3$. Since the battery remains in series with the ammeter, the battery current does not depend on the resistance distribution between the battery and the ammeter; for convenience, we can assume that the battery has zero resistance. Then the voltage across the battery remains 6 V, so:

$$V_1 + V_2 = 6$$
$$3V_1 + (V_2/3) = 6,$$
$$\text{and } V_2 = 4.5\text{ V}.$$

(Contributed by Christina Domnisoru, a junior at Maine School of Science and Mathematics, Limestone, Maine)

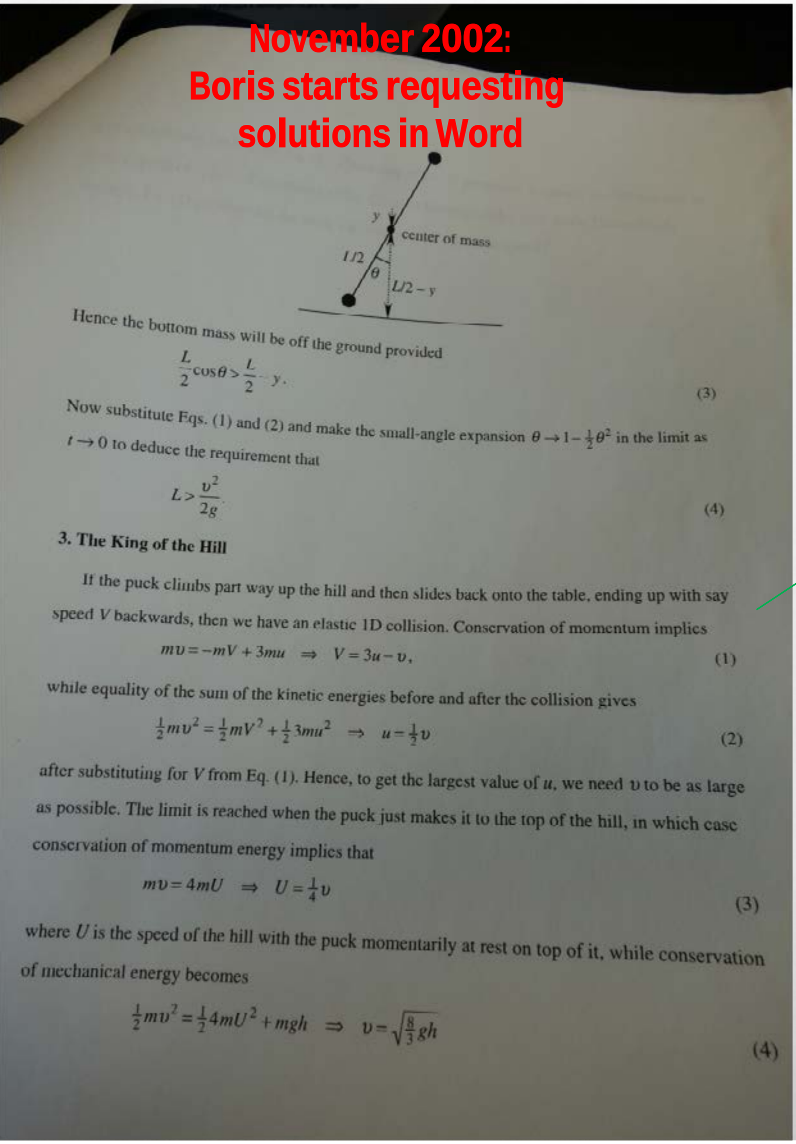
solutions became long enough that they are no longer in the print issue

April 2002 solutions: first time one of mine is published by Boris

V-A-R-iables

First, one can connect the voltage source, the unknown resistor r and the ammeter in series. The voltmeter should be connected in parallel with the ammeter. The ratio of the readings, (V/I) equals the resistance of the ammeter. Then the voltmeter should be reconnected in parallel with both the ammeter and the unknown resistor. The ratio of the new readings (V'/I') now equals the total resistance of the ammeter and the unknown resistor. The unknown resistance r is, therefore, given by $r = (V'/I') - (V/I)$.

(Contributed by Carl Mungan, United States Naval Academy, Annapolis, Md.)



Hence the bottom mass will be off the ground provided

$$\frac{L}{2}\cos\theta > \frac{L}{2} - y. \quad (3)$$

Now substitute Eqs. (1) and (2) and make the small-angle expansion $\theta \to 1 - \frac{1}{2}\theta^2$ in the limit as $t \to 0$ to deduce the requirement that

$$L > \frac{v^2}{2g} \quad (4)$$

3. The King of the Hill

If the puck climbs part way up the hill and then slides back onto the table, ending up with say speed V backwards, then we have an elastic 1D collision. Conservation of momentum implies

$$mv = -mV + 3mu \Rightarrow V = 3u - v, \quad (1)$$

while equality of the sum of the kinetic energies before and after the collision gives

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mV^2 + \tfrac{1}{2}3mu^2 \Rightarrow u = \tfrac{1}{2}v \quad (2)$$

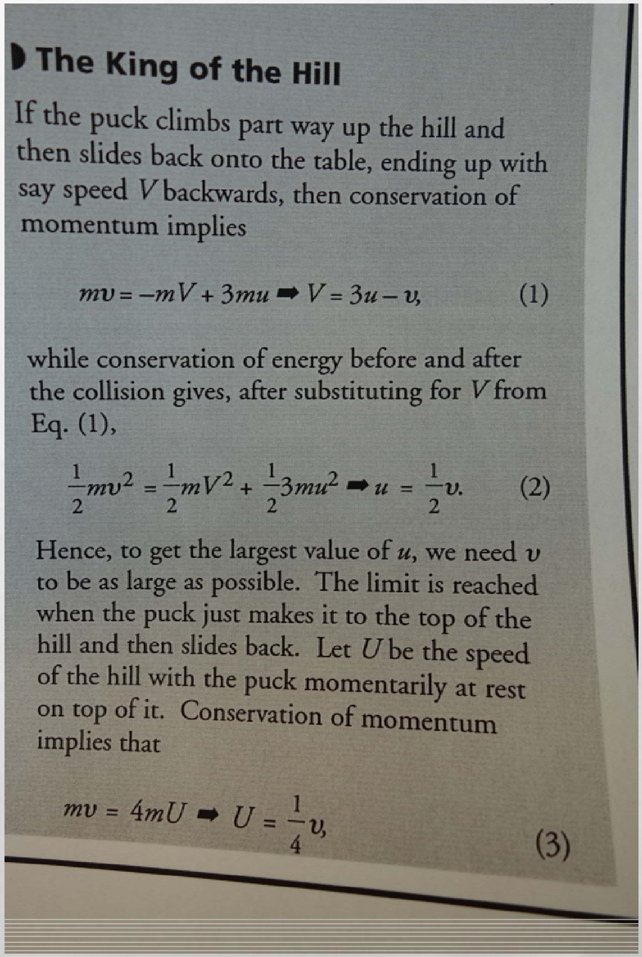
after substituting for V from Eq. (1). Hence, to get the largest value of u, we need v to be as large as possible. The limit is reached when the puck just makes it to the top of the hill, in which case conservation of momentum energy implies that

$$mv = 4mU \Rightarrow U = \tfrac{1}{4}v \quad (3)$$

where U is the speed of the hill with the puck momentarily at rest on top of it, while conservation of mechanical energy becomes

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}4mU^2 + mgh \Rightarrow v = \sqrt{\tfrac{8}{3}gh} \quad (4)$$

for direct conversion into posted PDF



The King of the Hill

If the puck climbs part way up the hill and then slides back onto the table, ending up with say speed V backwards, then conservation of momentum implies

$$mv = -mV + 3mu \Rightarrow V = 3u - v, \quad (1)$$

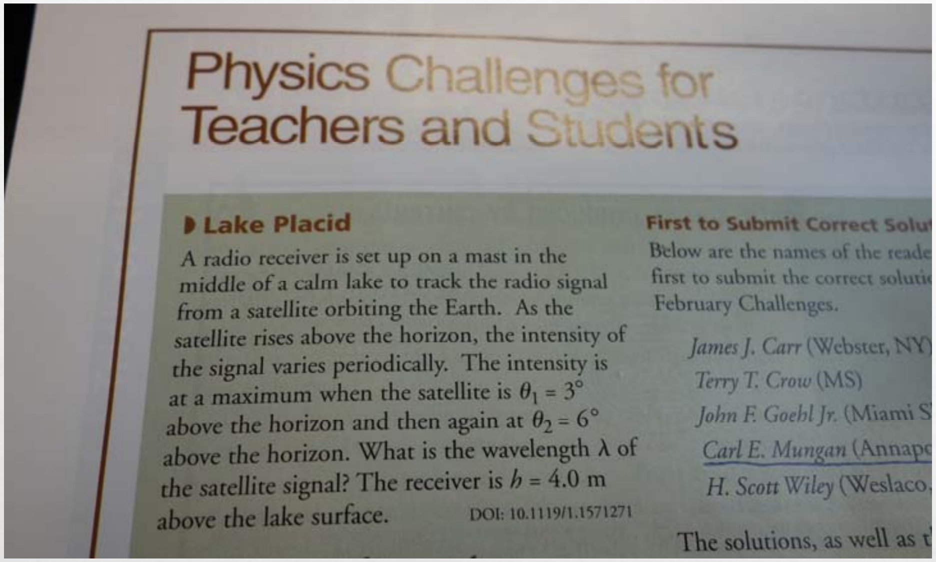
while conservation of energy before and after the collision gives, after substituting for V from Eq. (1),

$$\frac{1}{2}mv^2 = \frac{1}{2}mV^2 + \frac{1}{2}3mu^2 \Rightarrow u = \frac{1}{2}v. \quad (2)$$

Hence, to get the largest value of u, we need v to be as large as possible. The limit is reached when the puck just makes it to the top of the hill and then slides back. Let U be the speed of the hill with the puck momentarily at rest on top of it. Conservation of momentum implies that

$$mv = 4mU \Rightarrow U = \frac{1}{4}v, \quad (3)$$

May 2003 Challenge:
rare instance of an error by Boris



Physics Challenges for Teachers and Students

Lake Placid

A radio receiver is set up on a mast in the middle of a calm lake to track the radio signal from a satellite orbiting the Earth. As the satellite rises above the horizon, the intensity of the signal varies periodically. The intensity is at a maximum when the satellite is $\theta_1 = 3°$ above the horizon and then again at $\theta_2 = 6°$ above the horizon. What is the wavelength λ of the satellite signal? The receiver is $h = 4.0$ m above the lake surface.

DOI: 10.1119/1.1571271

First to Submit Correct Solu

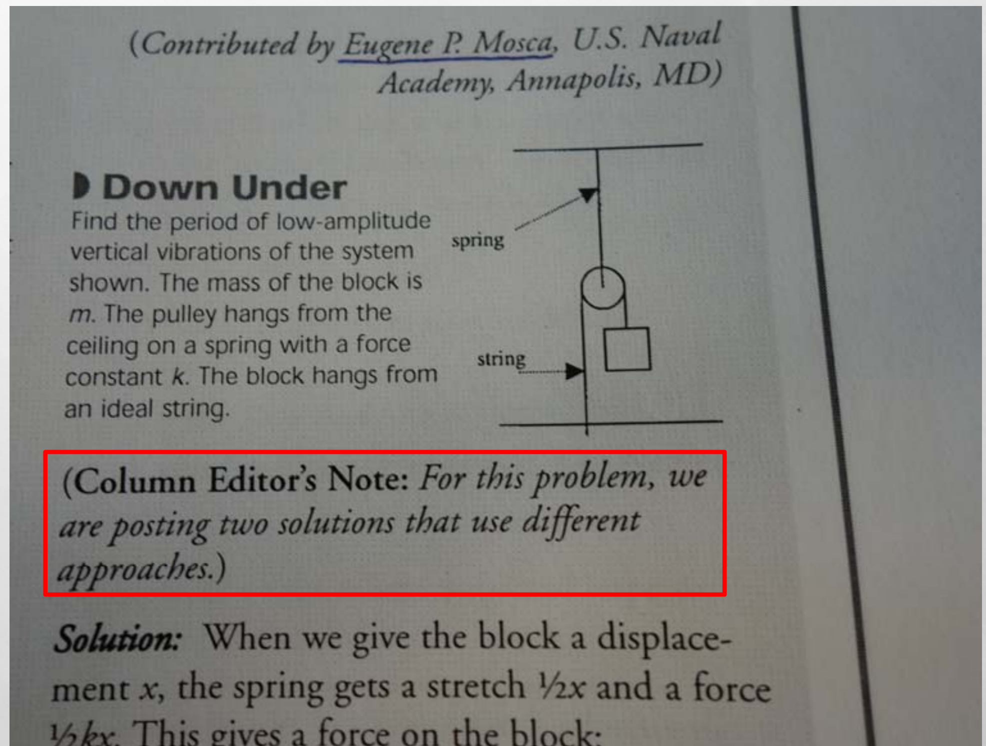
Below are the names of the reade
first to submit the correct soluti
February Challenges.

James J. Carr (Webster, NY)
Terry T. Crow (MS)
John F. Goebl Jr. (Miami S
Carl E. Mungan (Annapo
H. Scott Wiley (Weslaco,

The solutions, as well as t

this Lloyd's mirror problem
has no solution if we account
for the phase change upon reflection

January 2004:
first time that two different
solutions to the same problem
are posted



(*Contributed by Eugene P. Mosca, U.S. Naval Academy, Annapolis, MD*)

Down Under

Find the period of low-amplitude vertical vibrations of the system shown. The mass of the block is *m*. The pulley hangs from the ceiling on a spring with a force constant *k*. The block hangs from an ideal string.

(**Column Editor's Note:** *For this problem, we are posting two solutions that use different approaches.*)

Solution: When we give the block a displacement x, the spring gets a stretch $\frac{1}{2}x$ and a force $\frac{1}{2}kx$. This gives a force on the block:

February 2004 Challenge: rare instance of Boris posing a problem previously solved in TPT

Giving in to Pressure

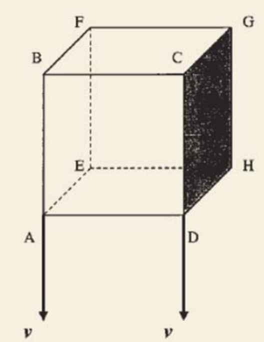
Column Editor's note: A similar situation was discussed in C.E. Mungan's "Irreversible adiabatic compression of an ideal gas," *Phys. Teach.* 41, 450–453 (Nov. 2003).

Challenge: A portion of helium gas in a vertical cylindrical container is in thermodynamic equilibrium with the surroundings. The gas is confined by a movable heavy piston. The piston is slowly elevated a distance *H* from its equilibrium position and then kept in the elevated position long enough for the thermodynamic equilibrium to be reestablished. After that, the container is insulated and then the piston is released. After the piston comes to rest, what is the new equilibrium position of the piston?

September 2005 Challenge: sneaky kind of problem with TWO answers and you had to find both to be counted correct

Back to Square One

A rigid cube ABCDEFGH is in motion. At a certain moment, face ABCD is vertical, and the velocities of vertices A and D are directed vertically downward and equal to v. At the same moment, the speed of point H equals $2v$. What point of the cube has the maximum speed at that moment? What is that speed?

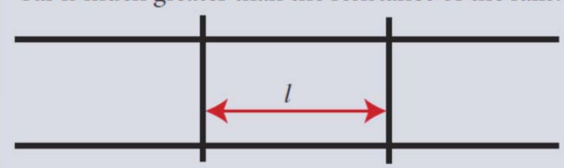


April 2006 Challenge: first recorded instance of Boris drawing a problem from a referenced source

A Faradayan Slip[1]

Two identical conducting bars rest on two horizontal parallel conducting rails. The bars are perpendicular to the rails and parallel to each other as shown. The distance between the bars is l. At a certain moment, a uniform vertical upward magnetic field is turned on. The field quickly reaches its maximum magnitude and then remains constant.

Neglecting friction, find the new distance between the bars. Assume that the resistance of each bar is much greater than the resistance of the rails.



(top view)

1. Adapted from *Physics Olympiads*, by A.I. Slobodyanuk, L.G. Markovich, and A.V. Lavrinenko, published by Aversev, Minsk (2003) (in Russian).

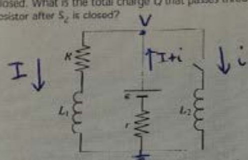
May 2004 Challenge:
erroneous solution even after a revision

Physics Challenges for Teachers and Students

Solutions to May 2004 Challenges

Double Closure

Challenge. An electric circuit contains a battery with emf ε and internal resistance r, two coils with inductances L_1 and L_2, and a resistor R, connected as shown. On the diagram, all shown parameters are given. Initially, both switches are open. Switch S_1 is then closed. After a while, switch S_2 is closed. What is the total charge Q that passes through the resistor after S_2 is closed?



Solution: That charge is finite because the second inductor shorts out the battery so that the final voltage between the top and bottom of the diagram will be zero. It will take some time to reach that condition because the current in L_2 will approach its asymptotic value gradually. Let "I" be the current through L_1, and let "i" be the current through L_2. (Both currents are functions of time.) The voltages across the three vertical parts of the network must be equal:

$$V = RI + L_1\, dI/dt, \quad (1)$$
$$V = \varepsilon - r(I + i), \quad (2)$$
$$V = L_2\, di/dt. \quad (3)$$

Combining Eqs. (1) and (3),

$$RI + L_1\, dI/dt = L_2\, di/dt. \quad (4)$$

It seems likely that V will decay exponentially ($V = V_0 e^{-kt}$) so the terms on the right sides of Eqs. (1) and (3) must decay in similar fashion:

$I = I_0 e^{-kt}$, so $dI/dt = -kI_0 e^{-kt}$,

and

$i = i_f(1 - e^{-kt})$ so $di/dt = ki_f e^{-kt}$.

By plugging those expressions into Eq. (4) and canceling the exponentials we find that $RI_0 = kI_0L_1 - ki_fL_2$. Solving for the unknown constant, $k = RI_0/[I_0L_1 + i_fL_2] = R/[L_1 + (i_f/I_0)L_2]$. But $I_0 = \varepsilon/(R + r)$ and $i_f = \varepsilon/r$, so $i_f/I_0 = (R + r)/r$. Therefore $k = R/[L_1 + L_2(R + r)/r]$.

To check my solution I will see if it is consistent with Eq. (2):

$$V_0e^{-kt} = \varepsilon - r(I + i) = \varepsilon - r[I_0e^{-kt} + i_f(1 - e^{-kt})] = (\varepsilon - r\,i_f) + r(i_f - I_0)e^{-kt}$$

We already know that $(\varepsilon - r\,i_f) = 0$, so we can factor out the exponential:

$$V_0 = r(i_f - I_0) = r[\varepsilon/r - \varepsilon/(R + r)] = \varepsilon[1 - r/(R + r)] = \varepsilon[R + r - r]/(R + r) = \varepsilon R/(R + r).$$

$= I_0 R$ inconsistent with because $\frac{dI(0)}{dt} = -$

Since we know this is correct, the solution seems good. To find the total charge that passes through resistor R we must integrate I with respect to time, from $t = 0$ to ∞:

$$Q = \int I dt = I_0 \int e^{-kt} dt = I_0/k = [\varepsilon/(R + r)][L_1 + L_2(R + r)/r]/R = (\varepsilon/R)[L_1/(R + r) + L_2/r].$$

(Contributed by Art Hovey, Milford, CT)

Column Editor's Note: *We thank Leo H. van den Raadt (Heemstede, The Netherlands) for pointing out a typo in the originally posted solution.*

This revised pdf (posted July 12, 2004) replaces the May Solutions originally posted online.

the actual solution is a double rather than a single exponential

Double-Exponential *LR* Circuit

Carl E. Mungan, U.S. Naval Academy, Annapolis, MD

Simple *LR* and *RC* circuits are familiar to generations of physics students as examples of single-exponential growth and decay in the relevant voltages, currents, and charges. An element of novelty can be introduced by connecting *two* (instead of one) *LR* coils in parallel with a battery. The resulting circuit can still be treated using little more than the basic tools (Kirchhoff's rules plus a trial exponential solution) employed in the standard *LR* analysis. But the solution is now a double exponential, as can be verified by constructing such a circuit.

Consider the circuit shown in Fig. 1, which is adapted from Ref. 1. The resistors include the internal resistances of the coils and battery.[2] (This is the reason for the addition of the resistor R_2, which was absent from the original circuit in Ref. 1.) Assume the currents in the circuit have reached their steady-state values with switch S open. The switch is then closed at $t = 0$. The problem is to find the subsequent currents in the circuit as a function of time.

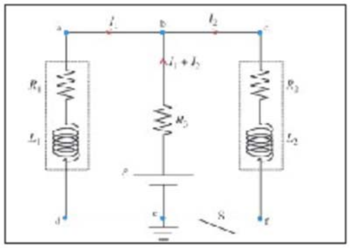


Fig. 1. A circuit consisting of two inductors L_1 and L_2, one battery ε, one switch S, and three resistors R_1 through R_3. The dotted rectangular boxes represent the coils used in the experimental measurements.

General Solution for the Currents After the Switch Is Closed

The inductors prevent the currents from suddenly changing and thus they instantaneously remain at the steady-state values they had at the instant before S was closed,

$$I_1(0) = \frac{\varepsilon}{R_1 + R_3} \quad \text{and} \quad I_2(0) = 0. \quad (1)$$

A long time after the switch is closed the currents attain new steady-state values, which can be derived using the parallel and series rules for resistors,

$$I_1(\infty) = \frac{\varepsilon R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$
and
$$I_2(\infty) = \frac{\varepsilon R_1}{R_1R_2 + R_1R_3 + R_2R_3}. \quad (2)$$

Equations (1) and (2) are obtained by replacing the inductors with ideal wires, since there is no voltage across an inductor in steady state. Provided R_1 and R_3 are nonzero, then $I_1(0) > I_1(\infty)$ and $I_2(0) < I_2(\infty)$.

Now let us find the detailed functional forms of $I_1(t)$ and $I_2(t)$ from $t = 0$ to ∞. Kirchhoff's current junction rule is already built into Fig. 1, since the upward current in the middle branch is the sum of the

December 2004 Challenge

A Home Stretch

A new car model is tested first along a specially designed track. First, the car is driven along a straight stretch of the track of length r. If the car starts from rest at its maximum (constant) acceleration, it takes the amount of time t to cover distance r. The car is then brought to rest and is accelerated again along a circular loop of the track that has a radius r. Assuming that the car is speeding up at the maximum possible constant rate that allows it to remain on the track, how long would it take to cover that loop?

DOI: 10.1119/1.1828731

constant *tangential* or *total* acceleration?

subsequently discussed in C. Edmondson, "Fast Car Physics" JHU Press, 2011

November 2018 Challenge:
Boris essentially repeats a problem by accident

Born to turn

Trying to set a speed record, a motorcyclist is taking off from rest on a circular horizontal track. What fraction of the track will be covered before the maximum possible speed is achieved, if the motorcycle is handled in the optimal way?

Accelerating Around an Unbanked Curve

Carl E. Mungan, U.S. Naval Academy, Annapolis, MD

The December 2004 issue of *TPT* presented a problem concerning how a car should accelerate around an unbanked curve of constant radius r starting from rest if it is to avoid skidding.[1] Interestingly enough, two solutions were proffered by readers.[2] The purpose of this note is to compare and contrast the two approaches. Further experimental investigation of various turning strategies using a remote-controlled car and overhead video analysis could make for an interesting student project.

One approach, exemplified by Scott Wiley's solution, assumes that the tangential acceleration a_t of the car is constant throughout the turn, while the centripetal acceleration $a_c = v^2/r$ continuously increases as the car's speed v builds up. But the magnitude of the total acceleration $a = \sqrt{a_t^2 + a_c^2}$ must never exceed $\mu_s g$, where μ_s is the coefficient of static friction between the tires and road. To minimize the travel time t, the car's acceleration just attains this slipping value as it completes the turn. Using the kinematic relations $v^2 = 2a_t r\phi$ and $r\phi = \frac{1}{2}a_t t^2$ (where ϕ is the angle through which the car has turned), straightforward algebraic manipulations lead to

$$\frac{v}{V} = \sqrt{\frac{2\phi}{k}}, \quad \frac{t}{T} = \sqrt{2k\phi}, \quad \frac{a_c}{A} = \frac{2\phi}{k}, \quad \text{and} \quad \frac{a_t}{A} = \frac{1}{k}, \tag{1}$$

where $k \equiv \sqrt{1 + 4\phi_{max}^2}$ and the variables have been normalized by $V \equiv \sqrt{\mu_s g r}$, $T \equiv \sqrt{r/\mu_s g}$, and $A = \mu_s g$. Equations (1) are plotted in red in Fig. 1 for the case of a 45° turn (i.e., $\phi_{max} = \pi/4$ rad).

The other approach further reduces the driving time by making a equal to $\mu_s g$ during the *entire* turn, rather than merely as the car *completes* it. Following the solution of Eugene Mosca, one can equate $a_t = \sqrt{a^2 - a_c^2} = \sqrt{V^4 - v^4}/r$ to $a_t = dv/dt = (ds/dt) \times (dv/ds) = v\,dv/r\,d\phi$ (where $s = r\phi$ is the distance traveled around the curve) to obtain

$$\phi = \int \frac{v\,dv}{\sqrt{V^4 - v^4}} = \frac{1}{2}\sin^{-1}\left(\frac{v}{V}\right)^2. \tag{2}$$

Inverting this result gives

$$\frac{v}{V} = \sqrt{\sin 2\phi} \Rightarrow \frac{a_c}{A} = \sin 2\phi \text{ and } \frac{a_t}{A} = \cos 2\phi. \tag{3a}$$

(If ϕ_{max} is greater than 45°, then v remains constant with value V, so that $a_c = A$ and $a_t = 0$, at all angles beyond $\pi/4$.) The time is found by substituting $v = r\,d\phi/dt$ into the first equality in Eq. (3a) to get

$$\frac{t}{T} = \int_0^{\phi} \frac{d\phi'}{\sqrt{\sin 2\phi'}} \approx \frac{\phi}{N}\sum_{n=1}^{N} \csc^{1/2}\left[\frac{2\phi}{N}\left(n - \frac{1}{2}\right)\right], \tag{3b}$$

where N is any large integer.[3] This summation can be easily performed in a spreadsheet program.[4] The results are plotted (using $N = 1000$), along with Eq. (3a), in blue in Fig. 1.

 DOI: 10.1119/1.2165440

January 2005

Many readers sent us the correct solutions to the *Challenges* published in October. Below are the names of the readers who were first to submit the correct solutions.

Phil Cahill (Lockheed Martin Corporation, Rosemont, PA)
James J. Carr (Webster, NY)
Michael C. Faleski (Delta College, Midland, MI)
John F. Goehl, Jr. (Barry Univ., Miami Shores, FL)
Art Hovey (Milford, CT)
Eric L. Kemer (St. Andrew's School, Middletown, DE)
Carl E. Mungan (U. S. Naval Academy, Annapolis, MD)
Peter Sadowski, student (Archbishop Murphy H.S., Everett, WA)
Leo H. van den Raadt (Heemstede, The Netherlands)
H. Scott Wiley (Science Academy of South Texas, Weslaco, TX)
Yufei Zhao, student (Don Mills Collegiate Institute, Toronto, Canada)

The solutions, as well as a more complete list of contributors, can be found on our website: http://www.aapt.org/tpt. DOI: 10.1119/1.1845998

Celebrate the World Year of Physics in style! Solve some physics puzzles!

As many of you know, 2005 is the World Year of Physics (WYP). As part of the effort to commemorate this event, AAPT is proud to announce a monthly Problem-Solving Contest open to all physics enthusiasts worldwide.

From January through May 2005, AAPT will publish a new problem weekly on *The Physics Teacher* website (http://www.aapt.org/tpt) in conjunction with the *Physics Challenges* column. You do not have to subscribe to *The Physics Teacher* to participate. The style and the level of the problems will be similar to the ones usually found in the column: tricky and challenging but not requiring high-level mathematics. Submission deadlines and guidelines will accompany each problem.

Each month, a T-shirt and certificate will be awarded to each contest winner. The names of all those who submit at least one correct solution will be recognized on the website. Winning solutions will be published on the website regularly. We hope to be able to recognize winners in the following categories: faculty, college student, and high school student. After the contest ends in May, a special award in each category will be given based on cumulative submissions. Criteria used to determine award recipients may include, but are not limited to: the total number of correct solutions presented, most innovative and elegant solutions, etc. You may only win one T-shirt, but you are encouraged to enter each month's contest to compete for the special awards. Look for the first Problem of the Week in early January. We look forward to your participation! DOI: 10.1119/1.1845999

Note to contributors:
For the duration of the WYP contest, the submission guidelines have been slightly changed. Please read carefully—especially if you are a regular contributor.

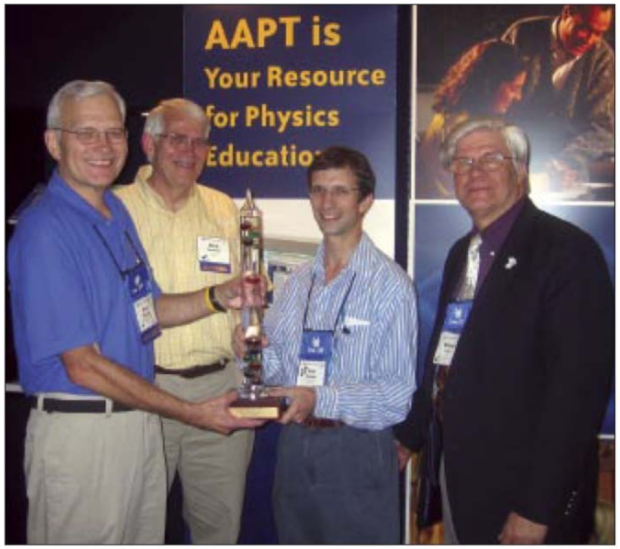
- only email submissions will be considered;
- email your solutions to Boris Korsunsky at korsunbo@post.harvard.edu;
- please email the solutions as Word files;
- please email each solution as a separate file;
- note that each problem, in addition to a very clever title, has a code such as J1. Please name each file as "problem code-first initial-last name." For instance, "J1DVader" if your name is Darth Vader and you are sending the solution to problem J1;
- please state your name, hometown, and professional affiliation in each file.

We look forward to your (and your students') participation.

Please send correspondence to:
Boris Korsunsky
444 Wellesley St.
Weston, MA 02493-2631
korsunbo@post.harvard.edu

to celebrate the World Year of Physics, Boris posted 1 problem a week (4 per month) between Jan and May 2005 as a contest

the number of submissions tripled as a result to over 100 per month



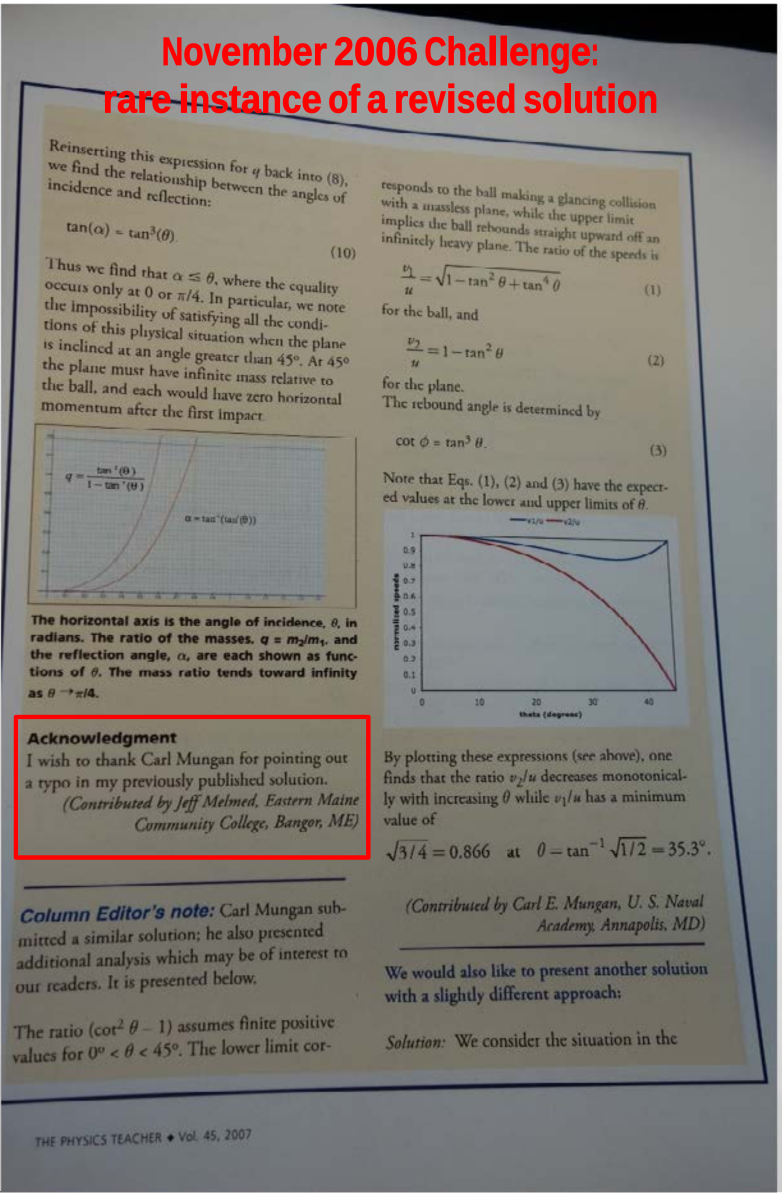
At the AAPT Summer Meeting in Salt Lake City in August, *TPT* Editor Karl Mamola (left) presents a Galileo thermometer to Carl Mungan (U.S. Naval Academy), a grand prize winner of the World Year of Physics Challenges Contest. AAPT Associate Executive Officer Warren Hein (far right) and AAPT President Richard Peterson look on.

(Photo by Melanie Hein)

DOI: 10.1119/1.2060654

starting in Fall 2005 and continuing to the present, there is only 1 problem per month, to make Boris' workload manageable

November 2006 Challenge: rare instance of a revised solution



Reinserting this expression for q back into (8), we find the relationship between the angles of incidence and reflection:

$$\tan(\alpha) = \tan^3(\theta). \quad (10)$$

Thus we find that $\alpha \le \theta$, where the equality occurs only at 0 or $\pi/4$. In particular, we note the impossibility of satisfying all the conditions of this physical situation when the plane is inclined at an angle greater than 45°. At 45° the plane must have infinite mass relative to the ball, and each would have zero horizontal momentum after the first impact.

The horizontal axis is the angle of incidence, θ, in radians. The ratio of the masses, $q = m_2/m_1$, and the reflection angle, α, are each shown as functions of θ. The mass ratio tends toward infinity as $\theta \to \pi/4$.

Acknowledgment
I wish to thank Carl Mungan for pointing out a typo in my previously published solution.

(Contributed by Jeff Melmed, Eastern Maine Community College, Bangor, ME)

Column Editor's note: Carl Mungan submitted a similar solution; he also presented additional analysis which may be of interest to our readers. It is presented below.

The ratio $(\cot^2\theta - 1)$ assumes finite positive values for $0° < \theta < 45°$. The lower limit corresponds to the ball making a glancing collision with a massless plane, while the upper limit implies the ball rebounds straight upward off an infinitely heavy plane. The ratio of the speeds is

$$\frac{v_1}{u} = \sqrt{1-\tan^2\theta + \tan^4\theta} \quad (1)$$

for the ball, and

$$\frac{v_2}{u} = 1 - \tan^2\theta \quad (2)$$

for the plane.

The rebound angle is determined by

$$\cot\phi = \tan^3\theta. \quad (3)$$

Note that Eqs. (1), (2) and (3) have the expected values at the lower and upper limits of θ.

By plotting these expressions (see above), one finds that the ratio v_2/u decreases monotonically with increasing θ while v_1/u has a minimum value of

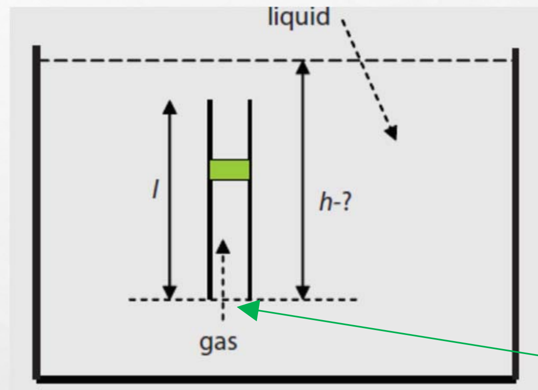
$$\sqrt{3/4} = 0.866 \quad \text{at} \quad \theta = \tan^{-1}\sqrt{1/2} = 35.3°.$$

(Contributed by Carl E. Mungan, U. S. Naval Academy, Annapolis, MD)

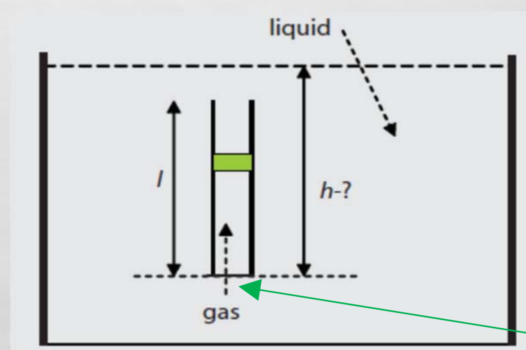
We would also like to present another solution with a slightly different approach:

Solution: We consider the situation in the

January 2009 Challenge: rare instance where the figure in the solution differs from that in the problem



bottom of tube appears to be open



now shown as closed

December 2009 Challenge: first time Boris used a reader-submitted problem

◗ Happy New Year?

Imagine that the mass of the Sun instantly doubles. How long would the Earth's year be?

(Contributed by Carl E. Mungan, U.S. Naval Academy, Annapolis, MD)

the posted solution was from Philip Blanco who subsequently became a frequent collaborator of mine

January 2013 Challenge: my lengthy solution prompts another frequent contributor to write a follow-up article

◗ When It's 2 Cool, It Gets Hot!

An electric freezer is turned on inside a tent for a long time. It is 0 °C outside the tent, +1 °C inside the tent, and -13 °C inside the freezer. What would be the equilibrium temperature inside the tent if another freezer is turned on inside the tent? The outside temperature remains the same. The freezers are identical and follow the Carnot cycle.

Revisiting a Problem of Two Freezers

Don Easton, Lacombe, AB, Canada

The January 2013 *Physics Challenge for Teachers and Students*[1] has some features that are surprising and worth a closer look. The problem concerns a Carnot-cycle refrigeration unit operating inside a tent. It achieves dynamic equilibrium with a freezer ("cold") compartment temperature of $T_C = -13$ °C, tent temperature of $T_H = 1$ °C (the "hot" waste side of the freezer), and temperature "outside" the tent of $T_O = 0$ °C. The problem is to find the equilibrium temperature inside the tent if an identical freezer is brought in and run simultaneously. As explained here, what constitutes an identical freezer is open to interpretation.

Starting from Carl Mungan's solution[2] to the problem, his Eq. (4) can be rewritten in the form

$$T_H^2 - (2+m)T_C T_H + T_C(T_C + mT_O) = 0, \qquad (1)$$

where $m = \sigma_t / \sigma_f$ is the ratio of the thermal conductance (in units of W/K) of the tent walls to that of the freezer walls. Here $\sigma_t = k_t A_t / d_t$, where k_t, A_t, and d_t are the thermal conductivity, surface area, and thickness of the tent walls, respectively, and the corresponding quantities for the freezer give $\sigma_f = k_f A_f / d_f$. When a second freezer is introduced, m is replaced by $m/2$ since A_f is doubled, but Eq. (1) is otherwise unchanged. The temperatures in Eq. (1) and in all equations below are in kelvin, but are converted into degrees Celsius in the text and graphics.

Equation (1) is quadratic in T_H and has two solutions although they may not be real-valued. Checking the discriminant, T_H has real values when

$$T_O \le T_C\left(1 + \frac{m}{4}\right). \qquad (2)$$

In other words, for a given freezer and tent, there is a maximum outdoor temperature for which Eq. (1) has a solution for T_H (or conversely, for a given T_O there is a minimum freezer compartment temperature T_C). The maximum outdoor temperature for which a solution exists depends on the number of freezers inside the tent. Figure 1 shows the solutions to Eq. (1) with T_O variable, $m = 0.7534$ (for one freezer), $m/2 = 0.3767$ (for two freezers), and $T_C = -13$ °C. (The value of m has been chosen to give $T_H = 1$ °C when $T_O = 0$ °C.) The branches of each parabola are labeled either "efficient" or "inefficient" with respect to the operation of the freezers. Although the feedback and cycling of a thermostat forces the efficient solution $T_H = 2.43$ °C for two freezers, one might alternatively envision a refrigeration unit controlled by a dial that fixes the power consumption rather than the cold compartment temperature. There are practical reasons for rejecting the inefficient solution. In Fig. 1, for the values stated in the problem, the inefficient solution for one freezer results in a temperature $T_H = 169$ °C. That is above the boiling point of water and possibly above the melting temperature of the tent.

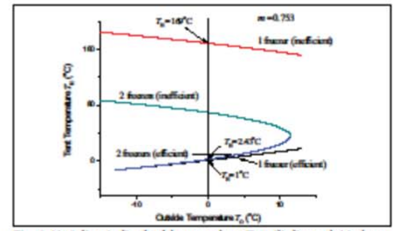


Fig. 1. Variation in the tent temperature T_H with the outside temperature T_O. The vertical line marks the conditions specified in the original statement of the problem.

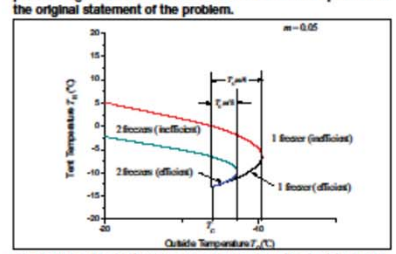


Fig. 2. When the tent wall is less conductive, the tent temperatures resulting from the efficient and inefficient use of the refrigeration units are closer in value. The internal temperature of the freezers is $T_C = -13$ °C, but the values of T_H and T_O are different from those in the original problem. The vertical lines show how the limiting value of T_O depends on T_C and m.

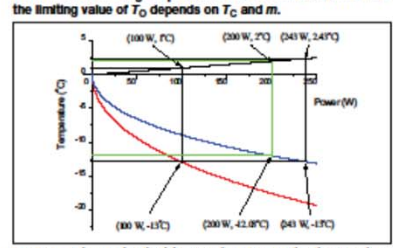


Fig. 3. Variation in the tent temperature (black), the temperature of a single freezer (red), and the cold temperatures for two freezers (blue) as a function of the total electric power delivered to the freezers. The thermal properties of the tent and freezers have been selected so that the power input in the original problem is 100 W. The ratio of the thermal properties has not been changed, so that $m = 0.7534$. The solutions are shown for $T_C = -13$ °C (black rectangles for two different powers) and for $P_{in} = 200$ W (green rectangle).

 DOI: 10.1119/1.4862116

November 2016 Challenge:
deceptively difficult

Boards of education

Three very long boards of equal masses are stacked on a horizontal surface as shown. The coefficients of friction for each pair of surfaces are shown (assume that for each pair of surfaces, the coefficients of static and kinetic friction are the same).

When the top board is given a strike to the right, the system comes to rest after time t. How long would it take the system to come to rest if the bottom board is struck instead? In both cases, the initial speed of the struck board is the same.

the posted solution *runs 12 pages long:*
kinetic vs static friction leads
to many possible cases!

February 2017 Challenge:
thermo always requires assumptions

Think inside the box

An insulated container of mass M and length L is at rest on a horizontal frictionless surface. The container is filled with an ideal gas of unknown mass m and temperature T. The container is divided in half by a light movable insulated piston.

A heater is turned on inside the left part of the container, bringing the temperature of the gas there to $2T$. The temperature of the gas inside the right part of the container remains unchanged. As a result, the container moves a distance x along the surface. Find the mass of the gas m.

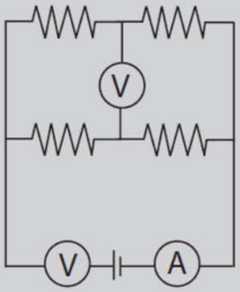
this problem *needs to say* the heating is quasistatic
and that the right side is not isolated but is instead
thermally connected to a bath at *T*

April 2018 Challenge

Why so series?

The circuit shown in the diagram below contains four resistors: two of them have resistance *R*, and the other two –3*R*. The voltmeters are identical.

The readings of the measuring devices are 6 mA, 0.5 V and 3.0 V. Find *R*.



July 2019 issue

PAPER

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iopscience.org/ped

Voltmeter in series?

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CrossMark

Abstract

A recent physics challenge shows a circuit, where a voltmeter is connected in series. Indeed, real voltmeters have finite input resistance, therefore, one may think that they can be used as resistors. In addition, voltmeters measure the voltage difference between their terminals, so it seems to be possible to calculate the current flowing through them. Is it okay? Are there any hidden secrets? The underlying methodology is related to high-quality physics and STEM education, which are increasingly important in the modern world. On one hand, it can be considered as an approval of an improper use that one can never see in any textbook and application, and it may also occur that suggesting and teaching such uncommon solutions can generate an undesired attitude. On the other hand, one can also say that if the students are taught to always follow the application rules, then the development of their creativity can be hindered. We do think it deserves some discussion.

a subsequent article, while not disagreeing with the solution, believes it is bad pedagogy to use a voltmeter as a resistor because it is a *misleading* violation of standard practice: WHAT DO YOU THINK?

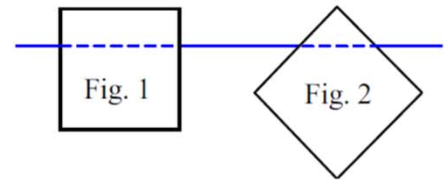
I wonder why this article is in *Physics Education* instead of TPT?

January 2019 Challenge

Spare the rod

A long wooden rod has a square cross section. The rod is placed into a lake. For what range of densities of the rod would it float so that the top surface of the rod remains parallel to the surface of the water?

a hard problem because the rod can rotate to ANY stable angle over a narrow range of densities: in the posted solution, I resort to citing several other publications



1. See, for example, the discussion of ship stability in C. E. Mungan and J. D. Emery, "Rolling the *Black Pearl* over: Analyzing the physics of a movie clip," *Phys. Teach.* **49**, 266–271 (May 2011).

2. B. Lautrup, *Physics of Continuous Matter*, 2nd ed. (CRC Press, Boca Raton FL, 2011), Chap. 3 online at www.cns.gatech.edu/~predrag/courses/PHYS-4421-13/Lautrup/buoyancy.pdf.

3. R. Delbourgo, "The floating plank," *Am. J. Phys.* **55**, 799–802 (Sep. 1987).

4. P. Erdös, G. Schibler, and R.C. Herndon, "Floating equilibrium of symmetrical objects and the breaking of symmetry. Part 1: Prisms," *Am. J. Phys.* **60**, 335–345 (Apr. 1992).

(Submitted by Carl E. Mungan, U. S. Naval Academy, Annapolis, MD)

MUCH THANKS TO BORIS FOR 18 YEARS OF FUN AND COUNTING!