Acceleration of Light at Earth’s Surface

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General relativity requires that light traveling upward or downward
at the earth’s surface has an acceleration equal to +2\(g\).

The radial acceleration of an object at the earth’s surface in a gravitational field
\(g = GM/r^2\) is equal to
\[
a_r = -g(1 - 3\beta_r^2) \quad (1)
\]
where \(\beta_r\) is the radial velocity \(v_r\), divided by \(c\). When \(\beta_r = 0\), the object will accelerate
downward with acceleration \(g\) as expected. When the object in question is light, then
\(\beta_r = \pm 1\) and \(a_r\) is equal to \(+2g\). If the object is traveling with a radial velocity \(\beta_r = \pm 1/\sqrt{3}\),
then it will not experience any radial acceleration at all.

We know that the velocity of light above the surface of the earth is greater than \(c\)
relative to a surface observer, and it is less than \(c\) (in a matter free depression) below the
surface. So when traveling from below to above the surface, light will accelerate upward;
and when traveling from above to below the surface, it will also accelerate upward.
Equation 1 gives the quantitative amount of that acceleration.

This equation is only valid when the gravitational potential \(\phi\) is considerably less
that \(c^2\). More precisely, Eq. 1 is given by
\[
a_r = -\sigma g(1 - 3\beta_r^2/\sigma^2) \quad (2)
\]
where \(\sigma = 1 + 2\phi/c^2\), with \(\phi = -GM/r\) [Ref. 1, Eq. 12.37].

Equation 2 is derived from an even more general relationship in contravariant
spatial coordinates \(x^i = (x^1, x^2, x^3)\) given by
\[
a^i = -\Gamma^i_{\mu\nu}v^\mu v^\nu + (v^i/c)\Gamma^4_{\mu\nu}v^\mu v^\nu \quad (3)
\]
where \(a^i = d^2x^i/dt^2\), \(v^i = dx^i/dt\), and \(v^\mu = (v^i, c)\) is not a four-vector. We call \(x^4 = ct = \tau\).
Greek indices sum over four coordinates. The acceleration \(a_r\) in Eq. 1 is \(a^1\) in Eq. 3.
Equation 3 is derived in Appendix D of Ref. 1.

To evaluate this equation for our case the Schwarzschild metric
\[
ds^2 = \sigma^1 dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) - \sigma dt^2
\]
is used to find the Christoffel symbols
\[
\Gamma^1_{11} = -GM/\sigma r^2c^2 \quad \Gamma^1_{44} = \sigma GM/r^2c^2
\]
\[
\Gamma^4_{14} = \Gamma^4_{41} = GM/\sigma r^2c^2
\]
