Student Understanding (and Misunderstanding) of Important Concepts in Relativity

Stamatis Vokos

Department of Physics
Seattle Pacific University

AAPT Workshop on Teaching GR to Undergraduates
July 21, 2006

Special thanks

- Rachel Scherr, Peter Shaffer, Andrew Boudreaux
- Collaborating instructors at UW and elsewhere, especially
  - James Bardeen, Norval Fortson
  - Jim Freericks (Georgetown), Greg Kilcup (Ohio), Tevian Dray (Oregon), Mark Haugan (Purdue)
- Edwin F. Taylor, Arnold B. Arons
- Graduate student interview volunteers

Relativity: *Publications*

- R.E. Scherr, P.S. Shaffer, and S. Vokos, *"Student understanding of time in special relativity: Simultaneity and reference frames,"* Phys. Educ. Res., Am. J. Phys. Suppl. **69**, S24-S35 (2001)
- R.E. Scherr, P.S. Shaffer, and S. Vokos, *"The challenge of changing deeply-held student beliefs about the physical world: An example from special relativity,"* Am. J. Phys. **70**, 1238-1248 (2002)

Populations investigated

- Students in Honors introductory physics course
- Students in regular introductory physics course
- Students in second-year modern physics course
- Students in third-year course in special and general relativity
- Prospective and practicing physics teachers
- First-year physics graduate students

Research methods

“Pretests”: Short ungraded questions

given after lecture and textbook instruction, but before special instruction

“Post-tests”: Examination questions

given after all instruction

Individual student interviews

Observations of students during instruction

“Every schoolboy in the streets of Göttingen understands more about four-dimensional geometry than Einstein.”

Quote by David Hilbert
in *Exploring Black Holes*, E.F. Taylor and J.A. Wheeler

Relevant Findings from PER

- Weak mathematical preparation often causes students to view mathematical expressions as symbolic identities rather than relationships among quantities that have physical meaning.
- Conversely, facility with mathematical manipulations can hide lack of conceptual understanding rendering meaningful learning and application very unlikely.
- Above all, persistent conceptual difficulties with foundational ideas may render any application of formalism instructionally irrelevant.

Relevant Findings from PER

- Weak mathematical preparation often causes students to view mathematical expressions as symbolic identities rather than relationships among quantities that have physical meaning.
- Conversely, facility with mathematical manipulations can hide lack of conceptual understanding rendering meaningful learning and application very unlikely.
- **Above all, persistent conceptual difficulties with foundational ideas may render any accretion of formalism instructionally irrelevant.**

Ball in gravitational field – Pretest

Administered after lecture instruction in junior Relativity *course*

A small ball moving radially away from a compact, spherically symmetric star of mass 1 km* passes an observer, Shelly, who is at rest at a Schwarzschild radial coordinate r_0 = 8/3 km.

The ball passes Shelly's location at r_0 = 8/3 km at $t = t_0$ (event 1) and a nearby location r = 8/3 km + 0.1 m at $t = t_0 + dt_0$ (event 2).

The metric for events occurring in the $\theta = \pi/2$ plane, expressed in terms of the far-away time coordinate t, the radial coordinate (reduced circumference) r, and the angular coordinate ϕ, is

$$d\tau^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} - r^2 d\phi^2.$$

* In units with $G/c^2 = 1$.

Ball in gravitational field – Pretest

Administered after lecture instruction in junior Relativity *course*

$$d\tau^2 = \left(1-\frac{2M}{r}\right)dt^2 - \frac{dr^2}{\left(1-\frac{2M}{r}\right)} - r^2 d\phi^2.$$

A small ball that moves radially away from a compact, spherically symmetric star of mass 1 km passes an observer, Shelly, who is at rest at a Schwarzschild radial coordinate r_0 = 8/3 km.

The ball passes Shelly's location at r_0 = 8/3 km at $t = t_0$ (event 1) and a nearby location r = 8/3 km + 0.1 m at $t = t_0 + dt_0$ (event 2).

Shelly measures the ball's velocity to be 1/2 (*i.e., c/*2).

Evaluate dt_0 (in meters). Show all your work.

Ball in gravitational field – Pretest

To give a correct answer, students need to

- ***recognize*** that all measurement is inherently local.
- ***apply*** an appropriate measurement procedure in a given observer's frame,
  e.g., Shelly's determination of the velocity of the ball

$$v_{Shelly} = \frac{dr_{Shelly}}{dt_{Shelly}} = \frac{1}{2}.$$

- ***relate*** the measurements of the local observer to those of a distant observer,
  e.g., Shelly's coordinates to the Schwarzschild coordinates

$$\frac{dr_{Shelly}}{dt_{Shelly}} = dr_0\left(1-\frac{2M}{r_0}\right)^{-\frac{1}{2}} \Big/ dt_0\left(1-\frac{2M}{r_0}\right)^{\frac{1}{2}} = \frac{1}{\left(1-\frac{2M}{r_0}\right)}\frac{dr_0}{dt_0}.$$

Ball in gravitational field
Pretest results

	Junior *Relativity* course (1997, 1998) (N = 49)
Correct approach	**5%**
dr_{Shelly}/dt_{Shelly} = 1/2	
Incorrect approach	**95%**
dr_0/dt_0 = 1/2	25%
$dr_0/d\tau$ = 1/2	25%
Other	45%

Results after lecture and textbook instruction on Schwarzschild metric

Fundamentals of General Relativity

Matter tells spacetime how to curve.

Field equations (need tensor calculus) or metric

Spacetime tells matter how to move.

Geodesic equation (need understanding of accelerating frames)

At every spacetime point x, we can choose a frame such that $g_{\mu\nu}(x)=\eta_{\mu\nu}$.

Need firm grounding in SR

Fundamentals of Special Relativity

Students need to relate the results of measurements made in different reference frames.

Events form the basis for *all* measurements.

- *Events have frame-independent existence.*
- *Events have coordinate labels that depend on the frame in which measurements are made.*

Operational definitions

“Spreading time over space” *P.W. Bridgman*

“An event occurs at time t_0 in a given frame.”

A local observer (situated near the location of event)

- measures time of the event as t_0
- sends time information (*e.g.,* via light signal)

All observers in the same inertial frame

- receive time information at a later time
- correct for signal delay during transit
- *agree* that the event occurred at time t_0

The process by which clocks are synchronized is crucial for the construction of the concept of a reference frame.

Ball in gravitational field – Pretest

To give a correct answer, students need to

- ***recognize*** that all measurement is inherently local.
- ***apply*** an appropriate measurement procedure in a given observer's frame,
  e.g., Shelly's determination of the velocity of the ball

$$v_{Shelly} = \frac{dr_{Shelly}}{dt_{Shelly}} = \frac{1}{2}.$$

- ***relate*** the measurements of the local observer to those of a distant observer,
  e.g., Shelly's coordinates to the Schwarzschild coordinates

$$\frac{dr_{Shelly}}{dt_{Shelly}} = dr_0\left(1-\frac{2M}{r_0}\right)^{-\frac{1}{2}} \Big/ dt_0\left(1-\frac{2M}{r_0}\right)^{\frac{1}{2}} = \frac{1}{\left(1-\frac{2M}{r_0}\right)}\frac{dr_0}{dt_0}.$$

“Hey Yogi, what time is it?”

"Hey Yogi, what time is it?"

"You mean *now*?"

Research Questions

After traditional instruction on special relativity, do students have a functional understanding of important basic concepts?

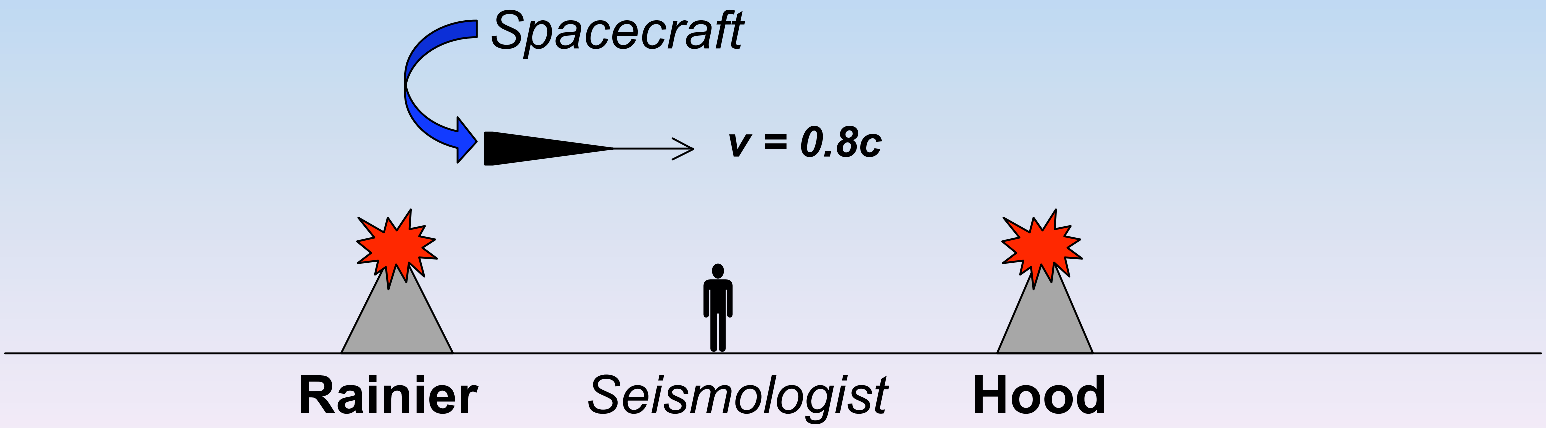
- identification of *relevant* events
- relating events to coordinate labels in a specific frame
- synchronization of clocks

Does student reasoning reflect the workings of a "latticework" of identical metersticks and synchronized clocks?

Spacecraft question

Mt. Rainier and Mt. Hood erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between them. A spacecraft flies past Rainier towards Hood at v=0.8c.

(Students are asked about the order of the eruptions in the spacecraft frame.)



Spacecraft question: a correct response

Application of the Lorentz transformations:

$$\Delta t_{RH}' = \gamma(\Delta t_{RH} - v\Delta x_{RH}/c^2)$$

Time between eruptions in ground frame (zero)

Velocity of spacecraft (positive)

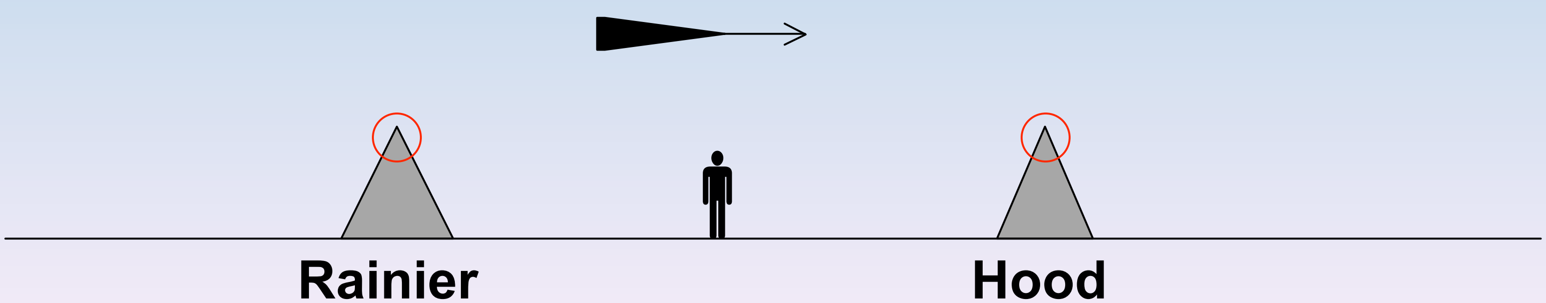
Distance from Rainier's eruption to Hood's eruption (positive)

Negative: ***Hood erupts before Rainier.***

Incomplete reasoning *introductory students*

“Mt. Hood erupted first because the spacecraft is moving towards it, so the **wavefront of the eruption of Mt. Hood will reach the craft first.”**

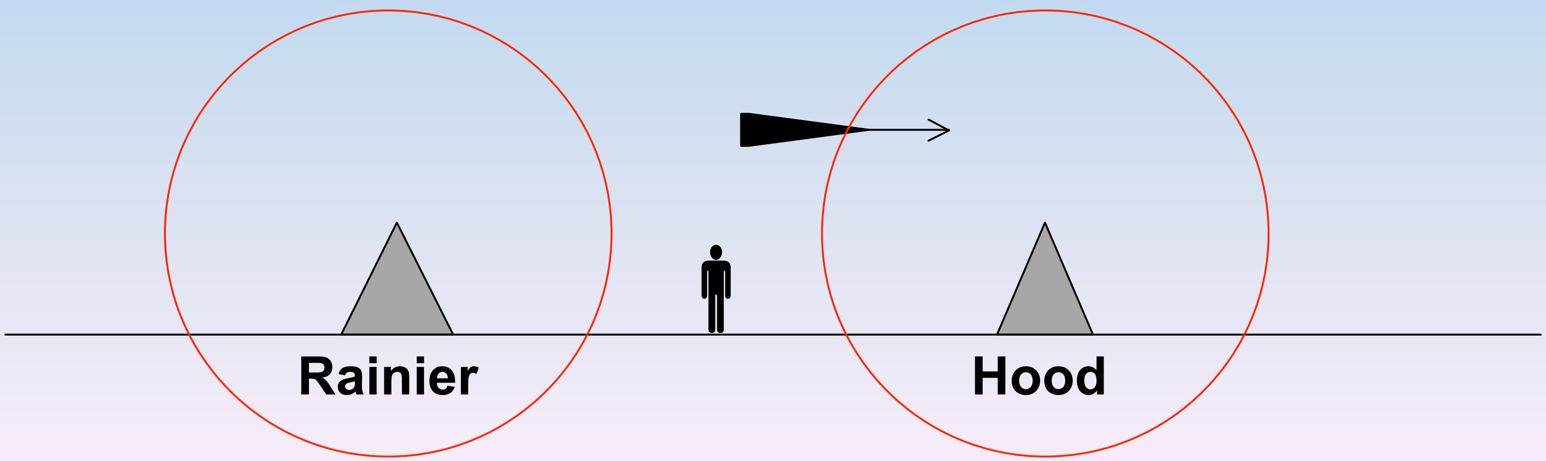
“Hood first, because the spacecraft will **encounter those wavefronts first.”**



Incomplete reasoning *introductory students*

"Mt. Hood erupted first because the spacecraft is moving towards it, so the **wavefront of the eruption of Mt. Hood will reach the craft first."**

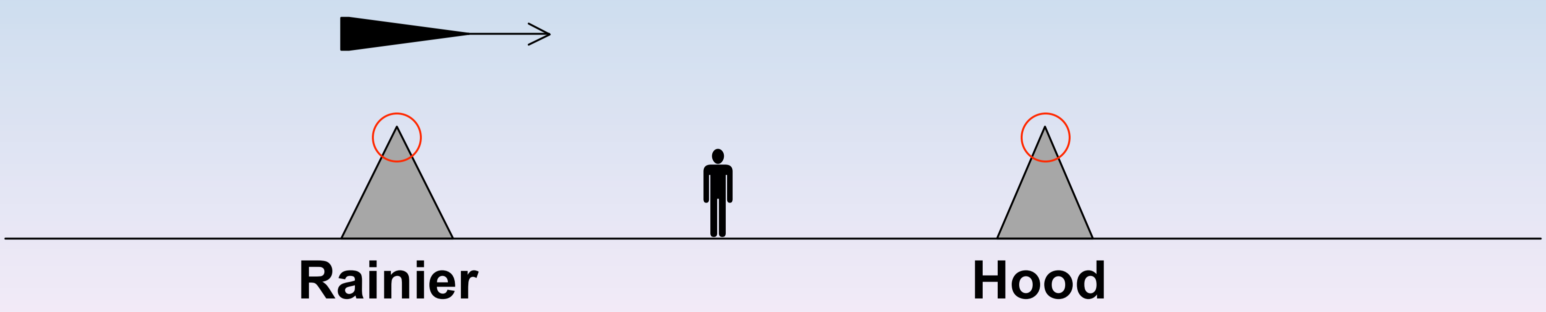
"Hood first, because the spacecraft will **encounter those wavefronts first."**



Incomplete reasoning *introductory students*

“Mt. Hood erupted first because the spacecraft is moving towards it, so the **wavefront of the eruption of Mt. Hood will reach the craft first.”**

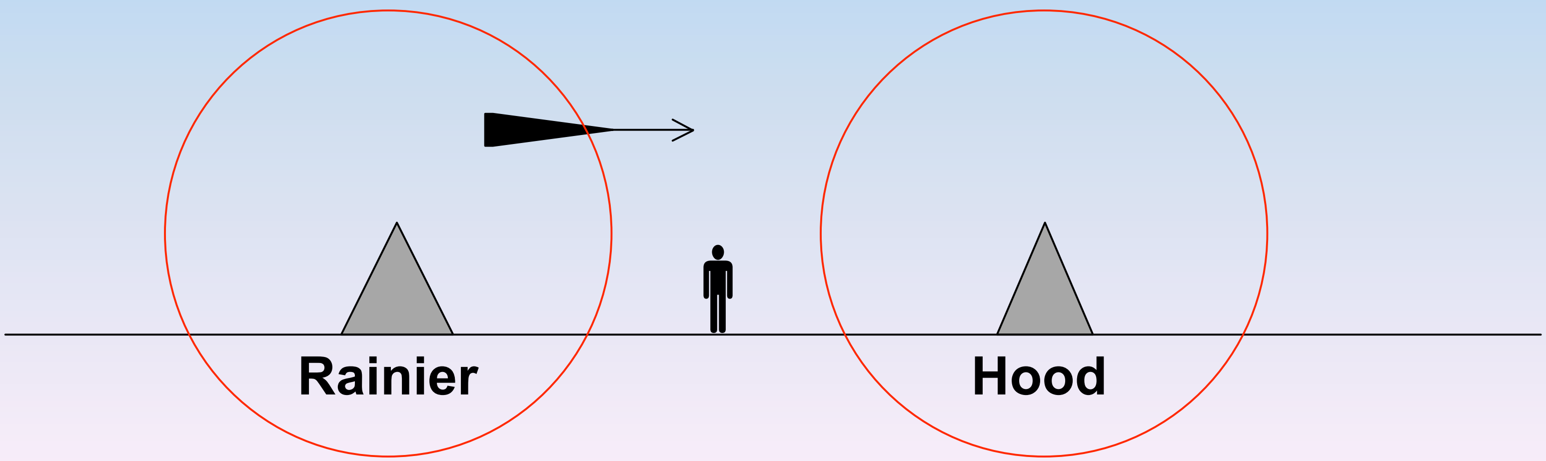
“Hood first, because the spacecraft will **encounter those wavefronts first.”**



Incomplete reasoning *introductory students*

“Mt. Hood erupted first because the spacecraft is moving towards it, so the **wavefront of the eruption of Mt. Hood will reach the craft first.”**

“Hood first, because the spacecraft will **encounter those wavefronts first.”**



Location-specific version

Do students believe that an observer's location affects the time ordering of events?

Mt. Rainier and Mt. Hood erupt at the same time in the frame of a seismologist at rest in a laboratory midway between them. A spacecraft flying past Rainier towards Hood at v=0.8c **is directly over Mt. Rainier when it erupts.**

Let Event 1 be "Mt. Rainier erupts," and Event 2 be "Mt. Hood erupts."

In the spacecraft frame, does Event 1 occur before, after, or at the same time as Event 2?

- Over 300 students in 10 different classes

Results of location-specific version

Introductory students	**Before instruction** Sp98, Au99 (N=67)	**After instruction** Sp97, Au98, Sp99 (N=73)
Correct answer with correct reasoning *or* incomplete reasoning	5% (3)	10% (8)
Closer event occurs first	65% (46)	75% (55)
Simultaneous eruptions (absolute simultaneity)	20% (12)	5% (5)

Results of location-specific version

Graduate students	**Interview task** Sp99 (N=11)
Correct answer with correct reasoning *or* incomplete reasoning	25% (3)
Closer event occurs first	55% (6)
Simultaneous eruptions (absolute simultaneity)	0

Results of location-specific version

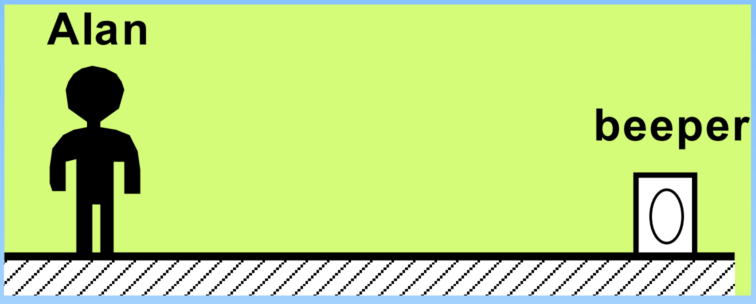
Graduate students	**Interview task** Sp99 (N=11)	**Qualifying exam** Au00 (N=23)
Correct answer with correct reasoning *or* incomplete reasoning	25% (3)	30% (7)
Closer event occurs first	55% (6)	60% (14)
Simultaneous eruptions (absolute simultaneity)	0	10% (2)

Tutorials in Physics: Special relativity

Topics include:

- **Events and reference frames**
- **Relativity of simultaneity**
- **Spatial measurements**
- **Length contraction and time dilation**
- **Applications of the Lorentz transformations**
- **Lorentz invariance**
- **Energy, momentum, and relativistic collisions**
- **The Schwarzschild metric**
- **Geodesics in curved spacetime**

Excerpt: Construction of a reference frame



Alan, equipped with meter sticks, clocks, and assistants, hears a beep.

How can he determine the time at which the beep was emitted:

– Using his knowledge of v_{sound}?

– *Without* knowing v_{sound}?

Excerpt: Definition of simultaneity

Alan hears a beep and a honk at the same time.

In Alan's frame, is the beep emitted
before, after, or *at the same time as* the honk?

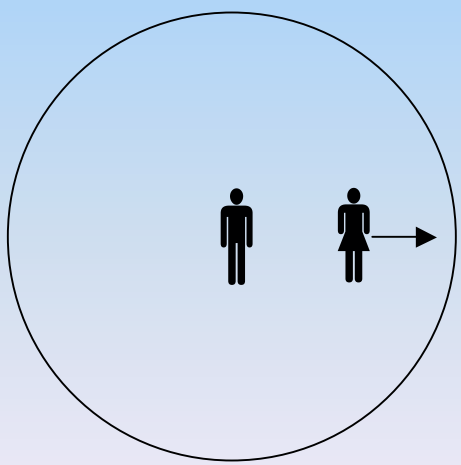
"Intelligent observer"

Excerpt: Applying invariance of speed of light

At the instant Alan and Beth pass, a spark jumps between them, emitting light.

Sketch the light wavefront:

<u>in Alan's frame</u>



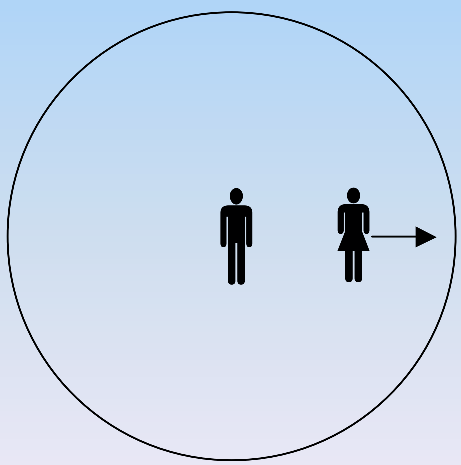
<u>in Beth's frame</u>

Excerpt: Applying invariance of speed of light

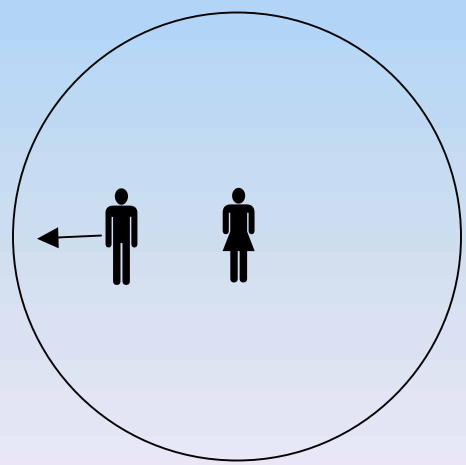
At the instant Alan and Beth pass, a spark jumps between them, emitting light.

Sketch the light wavefront:

<u>in Alan's frame</u>



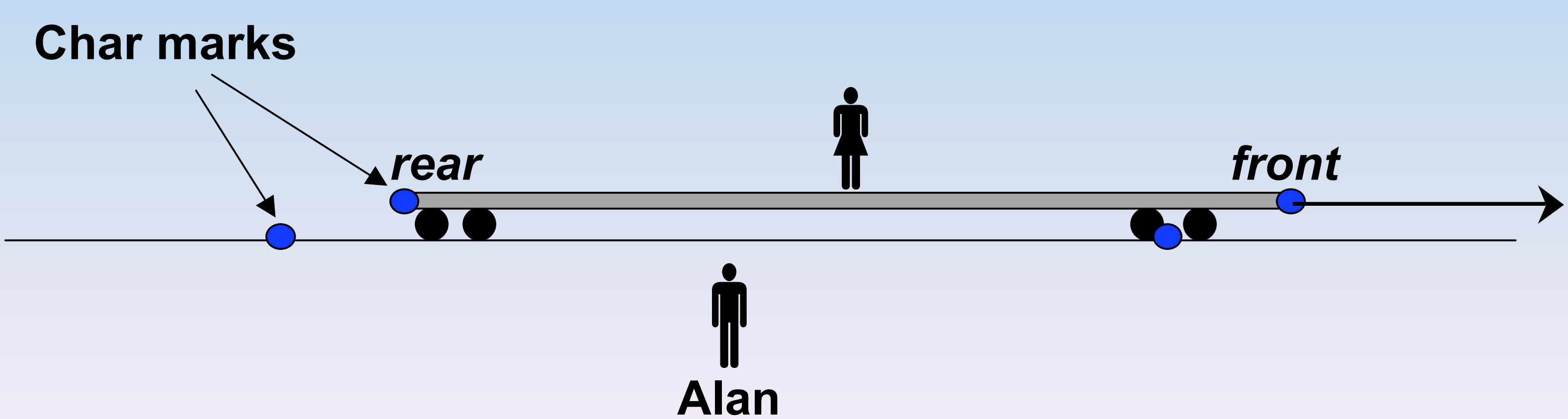
<u>in Beth's frame</u>



Excerpt: Relating ideas to physical context

Alan stands at a train track; Beth stands on the train. Sparks jump at front and rear of train, simultaneously in Alan's frame, leaving char marks.

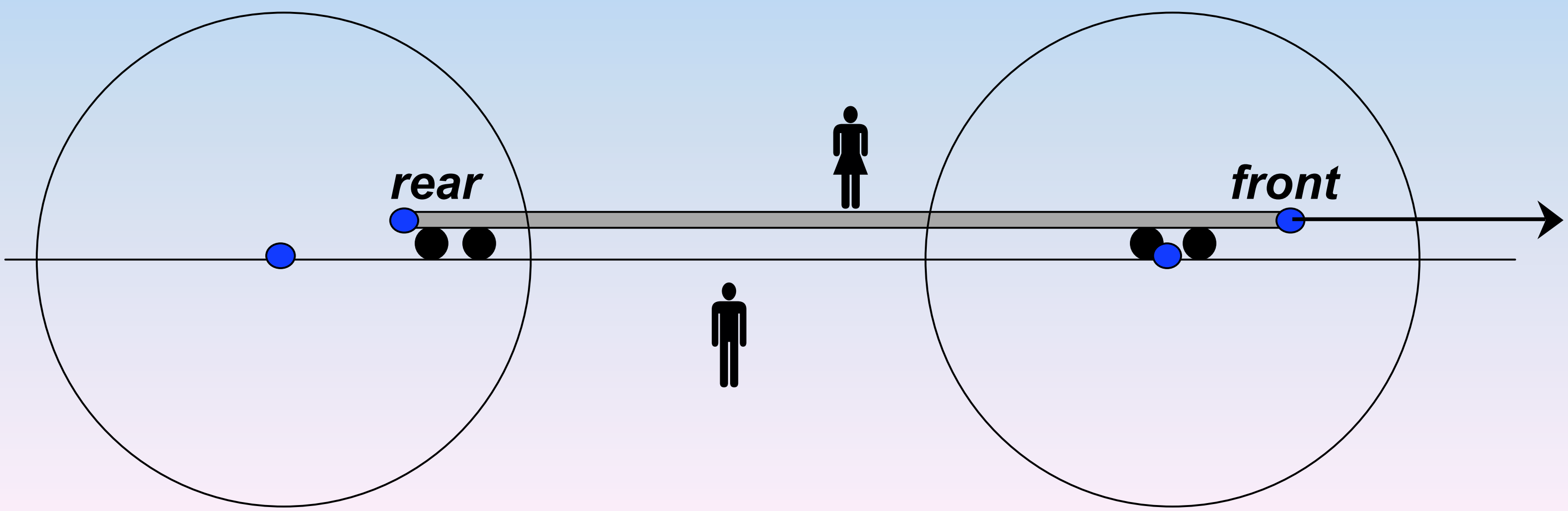
- **Draw the light wavefronts in Alan's frame.**
- **In Alan's frame, do the wavefronts hit Beth at the same instant?**



Excerpt: Relating ideas to physical context

Alan stands at a train track; Beth stands on the train. Sparks jump at front and rear of train, simultaneously in Alan's frame, leaving char marks.

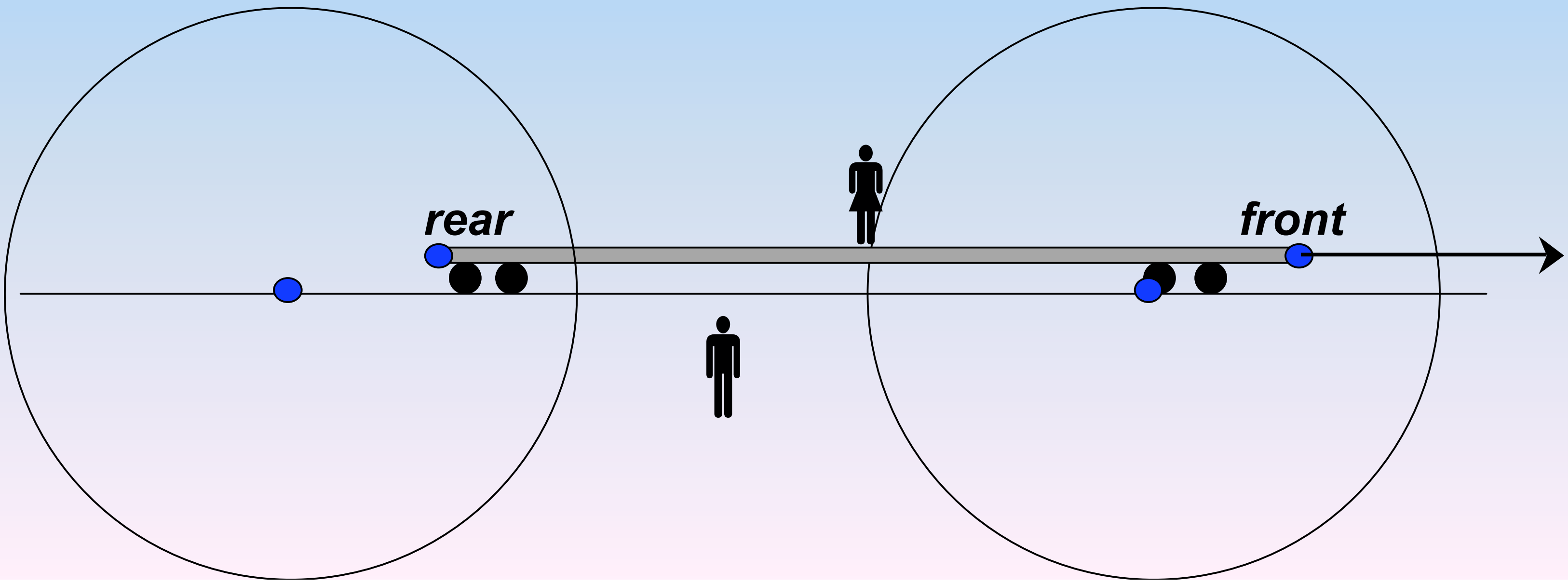
- **Draw the light wavefronts in Alan's frame.**
- **In Alan's frame, do the wavefronts hit Beth at the same instant?**



Excerpt: Relating ideas to physical context

Alan stands at a train track; Beth stands on the train. Sparks jump at front and rear of train, simultaneously in Alan's frame, leaving char marks.

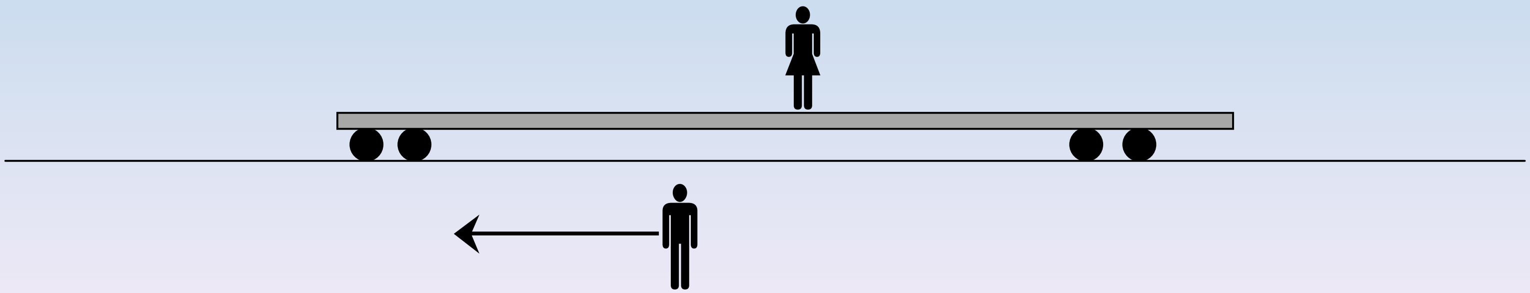
- **Draw the light wavefronts in Alan's frame.**
- **In Alan's frame, do the wavefronts hit Beth at the same instant?**



Excerpt: Determining order of events in train frame

A cassette player sits at Beth's feet. If the front wavefront hits it first, it plays Beethoven's Fifth at top volume; if the wavefronts hit simultaneously, it does not play.

- **Does the player play in Alan's frame?**
  In *Beth's* frame?

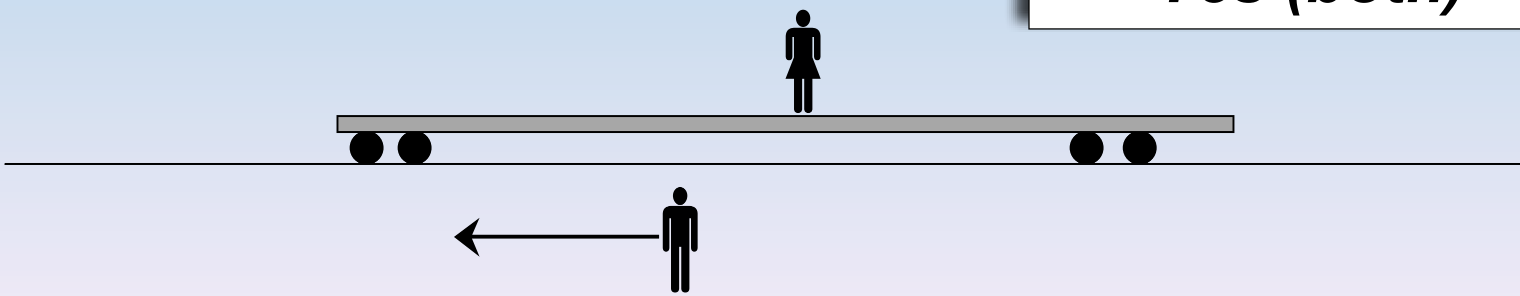


Excerpt:
Determining order of events in train frame

A cassette player sits at Beth's feet. If the front wavefront hits it first, it plays Beethoven's Fifth at top volume; if the wavefronts hit simultaneously, it does not play.

- **Does the player play in Alan's frame?
  In *Beth's* frame?**

**Correct answer:
*Yes (both)***



Response to tape player task

S1: We just figured out that the tape player plays in Alan's frame.

S2: But it can't. In Beth's frame they hit her at the same time. So she won't hear it.

Response to tape player task

S4: Wait, so Alan hears it and Beth doesn't? That's one awesome tape player!

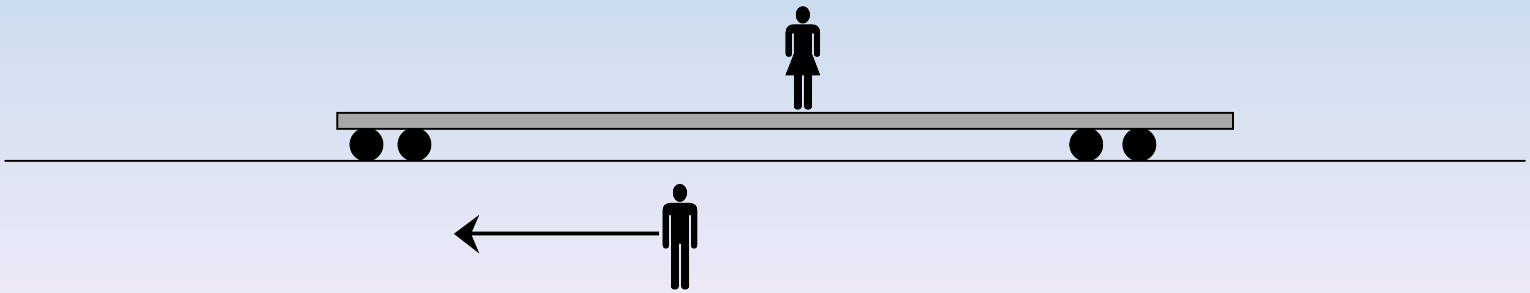
S5: That's so cool! This is what [the instructor was] telling us last week! That in some universe Sara was wearing purple and in another one she was wearing blue or something,

S4: Oh oh oh, **parallel universes!** Yeah!

Excerpt:
Determining order of events in train frame

Later in the day, Beth ejects the tape from the player. She descends from the train, and she and Alan examine the tape together.

- **Will the tape have wound at all from its starting position?**



Response to tape player task

S1: We just figured out that the tape player plays in Alan's frame.

S2: But it can't. In Beth's frame they hit her at the same time. So she won't hear it.

S3: But look down here, it's asking if she hears it and if the tape will have wound from its starting position. **If the tape is going to play, that's it, it's going to play.**

Response to tape player task

S2: But it can't play for Beth! She's in the middle! They hit her at the same time!

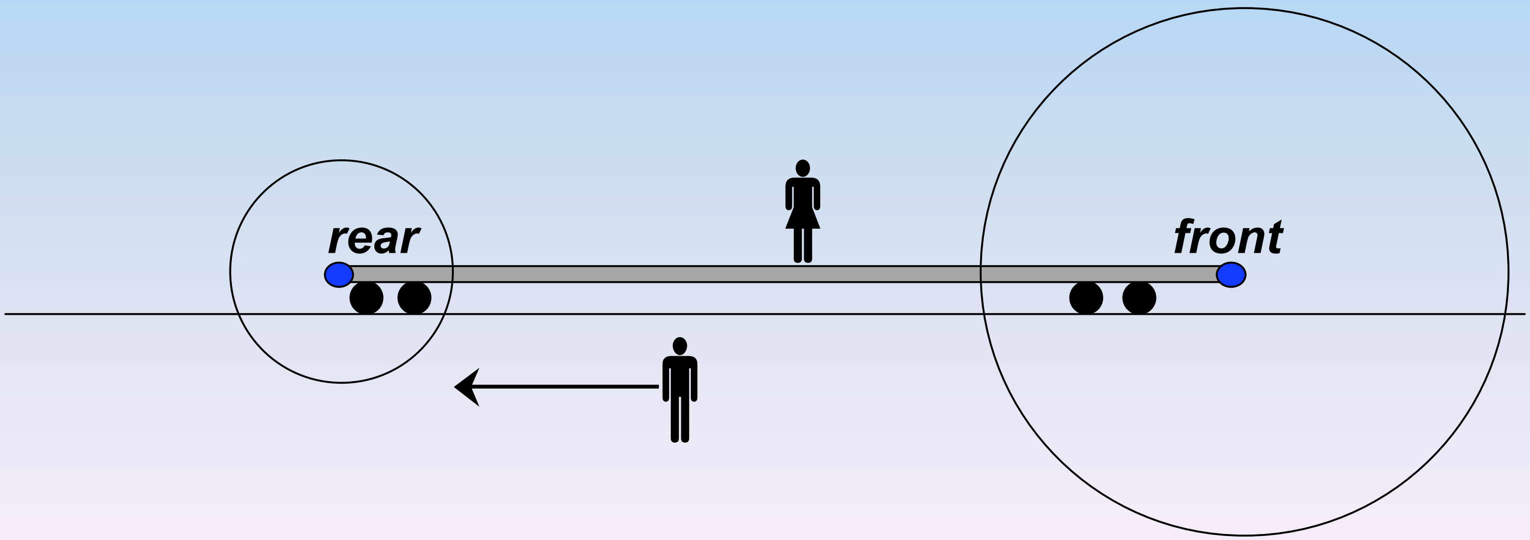
S1: But we just figured out that it plays!

S2: Right! And then a black hole opens up! And God steps out! and he points his finger and says [shouting] "YOU CAN'T DO THAT!"

Excerpt:
Determining order of events in train frame

Later in the day, Beth ejects the tape from the player. She descends from the train, and she and Alan examine the tape together.

- **Will the tape have wound at all from its starting position?**



Spacecraft question *(location-specific)*

Mt. Rainier and Mt. Hood erupt at the same time in the frame of a seismologist at rest in a laboratory midway between them. A spacecraft flying past Rainier towards Hood at v=0.8c
is directly over Mt. Rainier when it erupts*.*

Let Event 1 be "Mt. Rainier erupts," and Event 2 be "Mt. Hood erupts."

In the spacecraft frame, does Event 1 occur before, after, or at the same time as Event 2?

Correct answer: *Hood erupts first*
(event 1 occurs after event 2)

Results: *Spacecraft* question

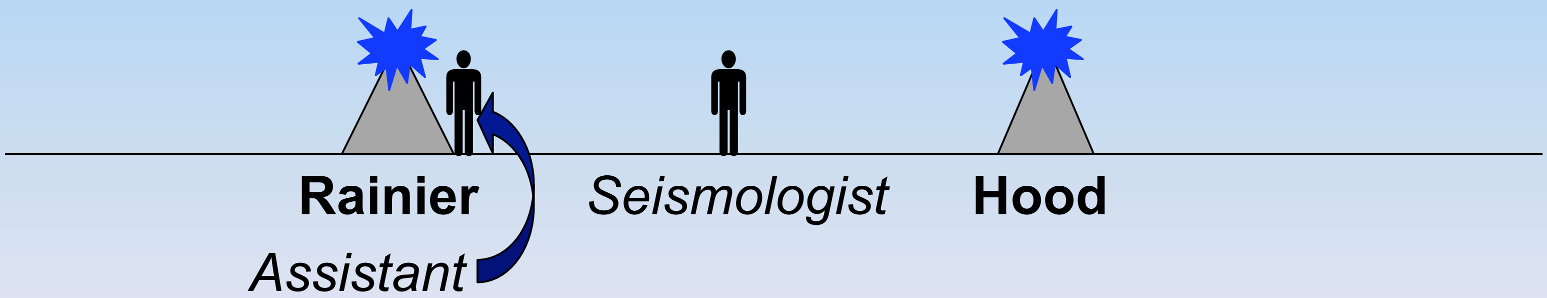
	Traditional instruction		Tutorial instruction
	Introductory students Sp97, Au98, Sp99 (N=73)	**Graduate students** Au00 (N=23)	**Introductory students** Sp97, Sp98, Au99 (N=117)
Correct answer with correct *or* incomplete reasoning	10% (8)	30% (7)	50% (59)
Closer event occurs first	75% (55)	60% (14)	40% (45)
Simultaneous eruptions	5% (5)	10% (2)	<5% (1)

Seismologists question

Mt. Rainier and Mt. Hood erupt at the same time in the frame of a seismologist at rest in a laboratory midway between them.

The seismologist's assistant is at rest near the base of Mt. Rainier.

In the reference frame of the assistant, does Rainier erupt before, after, or at the same time as Hood?



Correct answer: ***At the same time***

Results: *Seismologist* question

Introductory students	**Traditional instruction** Sp97, Au98, Sp99 (N=79)	**Tutorial instruction** Sp97, Sp98, Au98, Au99 (N=141)
Correct (simultaneous) regardless of reasoning	30% (25)	90% (124)
Rainier erupts first	60% (49)	10% (14)
Other (*e.g.*, Hood erupts first, not enough information)	5% (5)	<5% (3)

Concluding remarks

- ✦ Students' beliefs about reference frames, events, and the nature of time strongly affect their study of relativity.
- ✦ Their difficulties include a combination of misinterpretation of technical terms and deeply-rooted conceptual difficulties.
- ✦ Research can guide the development of curriculum to effectively engage students with the intellectual issues of relativity.

Unless students develop a coherent framework for the key ideas, many students will never gain a deep understanding or a true appreciation of science.

The role of physics education research– at the introductory level and beyond