

## 2010 $F = ma$ Contest

Entia non multiplicanda sunt praeter necessitatem

25 QUESTIONS - 75 MINUTES

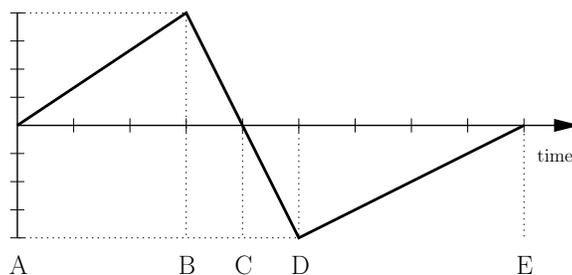
### INSTRUCTIONS

**DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN**

- Use  $g = 10 \text{ N/kg}$  throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers will result in a deduction of  $\frac{1}{4}$  point. There is no penalty for leaving an answer blank.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 8, 2010.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

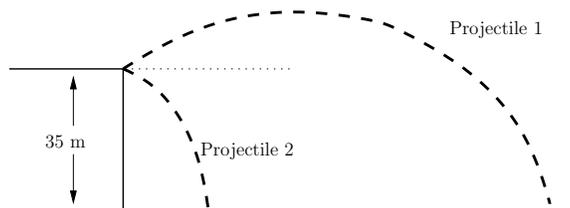
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Questions 1 to 3 refer to the figure below which shows a representation of the motion of a squirrel as it runs in a straight-line along a telephone wire. The letters A through E refer to the indicated times.



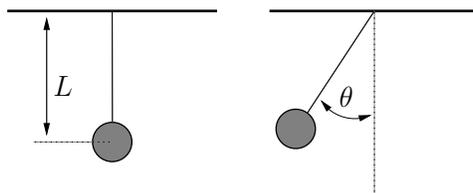
- If the graph is a graph of POSITION vs. TIME, then the squirrel has the greatest speed at what time(s) or during what time interval(s)?
  - From A to B
  - From B to C only
  - From B to D
  - From C to D only
  - From D to E
- If, instead, the graph is a graph of VELOCITY vs. TIME, then the squirrel has the greatest speed at what time(s) or during what time interval(s)?
  - at B
  - at C
  - at D
  - at both B and D
  - From C to D
- If, instead, the graph is a graph of ACCELERATION vs. TIME and the squirrel starts from rest, then the squirrel has the greatest speed at what time(s) or during what time interval?
  - at B
  - at C
  - at D
  - at both B and D
  - From C to D
- Two teams of movers are lowering a piano from the window of a 10 floor apartment building. The rope breaks when the piano is 30 meters above the ground. The movers on the ground, alerted by the shouts of the movers above, first notice the piano when it is 14 meters above the ground. How long do they have to get out of the way before the piano hits the ground?
  - 0.66 sec
  - 0.78 sec
  - 1.67 sec
  - 1.79 sec
  - 2.45 sec

5. Two projectiles are launched from a 35 meter ledge as shown in the diagram. One is launched from a 37 degree angle above the horizontal and the other is launched from 37 degrees below the horizontal. Both of the launches are given the same initial speed of  $v_0 = 50$  m/s.

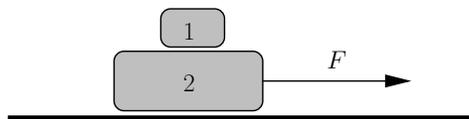


- The difference in the times of flight for these two projectiles,  $t_1 - t_2$ , is closest to
- (A) 3 s  
 (B) 5 s  
 (C) 6 s  
 (D) 8 s  
 (E) 10 s
6. A projectile is launched across flat ground at an angle  $\theta$  to the horizontal and travels in the absence of air resistance. It rises to a maximum height  $H$  and lands a horizontal distance  $R$  away. What is the ratio  $H/R$ ?
- (A)  $\tan \theta$   
 (B)  $2 \tan \theta$   
 (C)  $\frac{2}{\tan \theta}$   
 (D)  $\frac{1}{2} \tan \theta$   
 (E)  $\frac{1}{4} \tan \theta$
7. Harry Potter is sitting 2.0 meters from the center of a merry-go-round when Draco Malfoy casts a spell that glues Harry in place and then makes the merry-go-round start spinning on its axis. Harry has a mass of 50.0 kg and can withstand 5.0  $g$ 's of acceleration before passing out. What is the magnitude of Harry's angular momentum when he passes out?
- (A) 200  $\text{kg}\cdot\text{m}^2/\text{s}$   
 (B) 330  $\text{kg}\cdot\text{m}^2/\text{s}$   
 (C) 660  $\text{kg}\cdot\text{m}^2/\text{s}$   
 (D) 1000  $\text{kg}\cdot\text{m}^2/\text{s}$   
 (E) 2200  $\text{kg}\cdot\text{m}^2/\text{s}$
8. A car attempts to accelerate up a hill at an angle  $\theta$  to the horizontal. The coefficient of static friction between the tires and the hill is  $\mu > \tan \theta$ . What is the maximum acceleration the car can achieve (in the direction upwards along the hill)? Neglect the rotational inertia of the wheels.
- (A)  $g \tan \theta$   
 (B)  $g(\mu \cos \theta - \sin \theta)$   
 (C)  $g(\mu - \sin \theta)$   
 (D)  $g\mu \cos \theta$   
 (E)  $g(\mu \sin \theta - \cos \theta)$

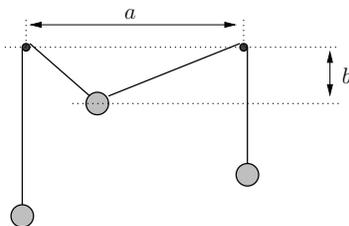
9. A point object of mass  $M$  hangs from the ceiling of a car from a massless string of length  $L$ . It is observed to make an angle  $\theta$  from the vertical as the car accelerates uniformly from rest. Find the acceleration of the car in terms of  $\theta$ ,  $M$ ,  $L$ , and  $g$ .



- (A)  $Mg \sin \theta$   
 (B)  $MgL \tan \theta$   
 (C)  $g \tan \theta$   
 (D)  $g \cot \theta$   
 (E)  $Mg \tan \theta$
10. A block of mass  $m_1$  is on top of a block of mass  $m_2$ . The lower block is on a horizontal surface, and a rope can pull horizontally on the lower block. The coefficient of kinetic friction for all surfaces is  $\mu$ . What is the resulting acceleration of the lower block if a force  $F$  is applied to the rope? Assume that  $F$  is sufficiently large so that the top block slips on the lower block.



- (A)  $a_2 = (F - \mu g(2m_1 + m_2))/m_2$   
 (B)  $a_2 = (F - \mu g(m_1 + m_2))/m_2$   
 (C)  $a_2 = (F - \mu g(m_1 + 2m_2))/m_2$   
 (D)  $a_2 = (F + \mu g(m_1 + m_2))/m_2$   
 (E)  $a_2 = (F - \mu g(m_2 - m_1))/m_2$
11. The three masses shown in the accompanying diagram are equal. The pulleys are small, the string is lightweight, and friction is negligible. Assuming the system is in equilibrium, what is the ratio  $a/b$ ? The figure is *not* drawn to scale!

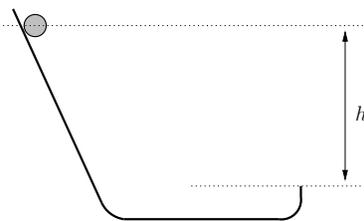


- (A)  $1/2$   
 (B)  $1$   
 (C)  $\sqrt{3}$   
 (D)  $2$   
 (E)  $2\sqrt{3}$

12. A ball with mass  $m$  projected horizontally off the end of a table with an initial kinetic energy  $K$ . At a time  $t$  after it leaves the end of the table it has kinetic energy  $3K$ . What is  $t$ ? Neglect air resistance.

- (A)  $(3/g)\sqrt{K/m}$
- (B)  $(2/g)\sqrt{K/m}$
- (C)  $(1/g)\sqrt{8K/m}$
- (D)  $(K/g)\sqrt{6/m}$
- (E)  $(2K/g)\sqrt{1/m}$

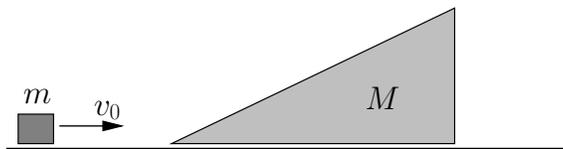
13. A ball of mass  $M$  and radius  $R$  has a moment of inertia of  $I = \frac{2}{5}MR^2$ . The ball is released from rest and rolls down the ramp with no frictional loss of energy. The ball is projected vertically upward off a ramp as shown in the diagram, reaching a maximum height  $y_{\max}$  above the point where it leaves the ramp. Determine the maximum height of the projectile  $y_{\max}$  in terms of  $h$ .



- (A)  $h$
  - (B)  $\frac{25}{49}h$
  - (C)  $\frac{2}{5}h$
  - (D)  $\frac{5}{7}h$
  - (E)  $\frac{7}{5}h$
14. A 5.0 kg block with a speed of 8.0 m/s travels 2.0 m along a horizontal surface where it makes a head-on, perfectly elastic collision with a 15.0 kg block which is at rest. The coefficient of kinetic friction between both blocks and the surface is 0.35. How far does the 15.0 kg block travel before coming to rest?

- (A) 0.76 m
- (B) 1.79 m
- (C) 2.29 m
- (D) 3.04 m
- (E) 9.14 m

The following figure is used for questions 15 and 16.



A small block of mass  $m$  is moving on a horizontal table surface at initial speed  $v_0$ . It then moves smoothly onto a sloped big block of mass  $M$ . The big block can also move on the table surface. Assume that everything moves without friction.

15. A small block moving with initial speed  $v_0$  moves smoothly onto a sloped big block of mass  $M$ . After the small block reaches the height  $h$  on the slope, it slides down. Find the height  $h$ .

- (A)  $h = \frac{v_0^2}{2g}$   
 (B)  $h = \frac{1}{g} \frac{Mv_0^2}{m+M}$   
 (C)  $h = \frac{1}{2g} \frac{Mv_0^2}{m+M}$   
 (D)  $h = \frac{1}{2g} \frac{mv_0^2}{m+M}$   
 (E)  $h = \frac{v_0^2}{g}$

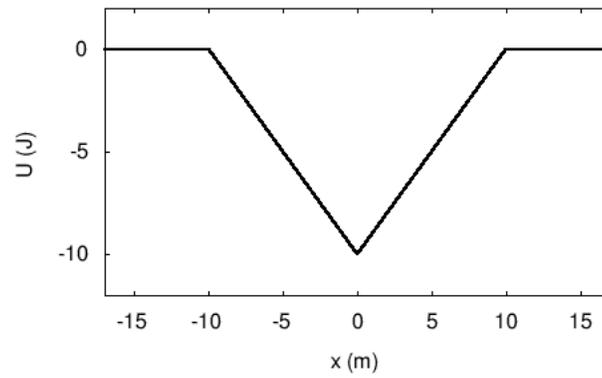
16. Following the previous set up, find the speed  $v$  of the small block after it leaves the slope.

- (A)  $v = v_0$   
 (B)  $v = \frac{m}{m+M} v_0$   
 (C)  $v = \frac{M}{m+M} v_0$   
 (D)  $v = \frac{M-m}{m} v_0$   
 (E)  $v = \frac{M-m}{m+M} v_0$

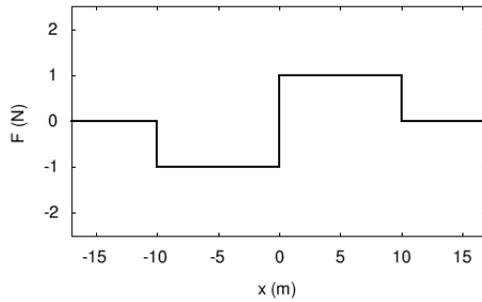
17. Four masses  $m$  are arranged at the vertices of a tetrahedron of side length  $a$ . What is the gravitational potential energy of this arrangement?

- (A)  $-2 \frac{Gm^2}{a}$   
 (B)  $-3 \frac{Gm^2}{a}$   
 (C)  $-4 \frac{Gm^2}{a}$   
 (D)  $-6 \frac{Gm^2}{a}$   
 (E)  $-12 \frac{Gm^2}{a}$

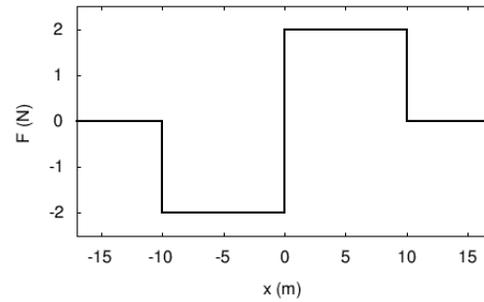
The following graph of potential energy is used for questions 18 through 20.



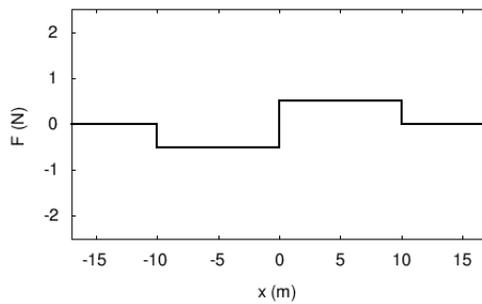
18. Which of the following represents the force corresponding to the given potential?



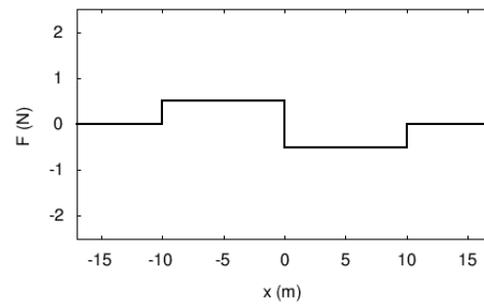
(A)



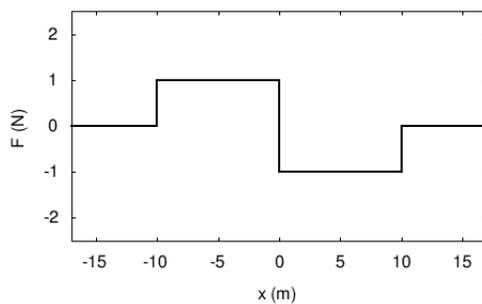
(B)



(C)

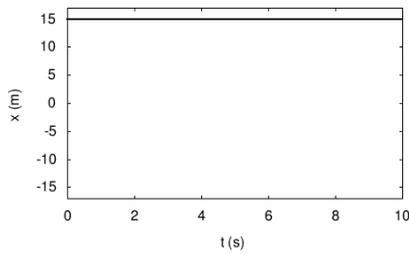


(D)

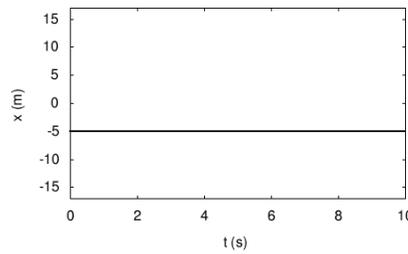


(E)

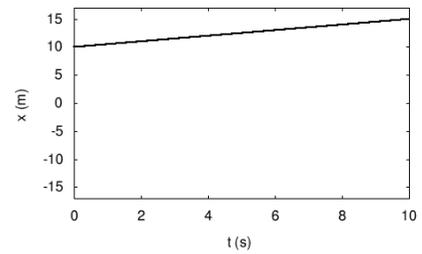
19. Consider the following graphs of position *vs.* time.



I.



II.

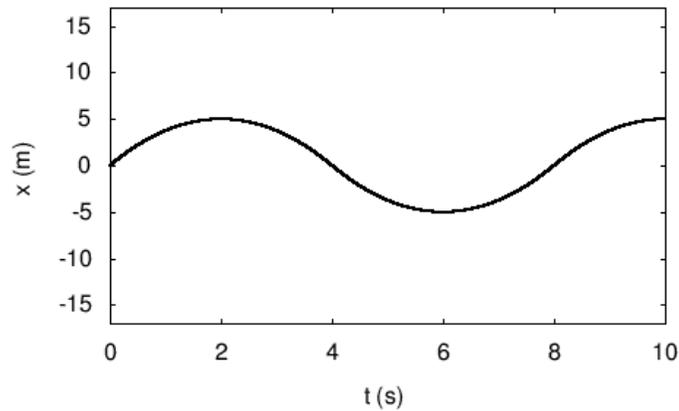


III.

Which of the graphs could be the motion of a particle in the given potential?

- (A) I
- (B) III
- (C) I and II
- (D) I and III
- (E) I, II, and III

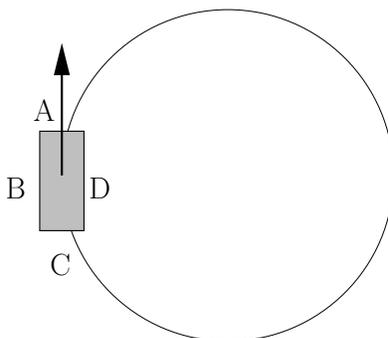
20. Consider the following graph of position *vs.* time, which represents the motion of a certain particle in the given potential.



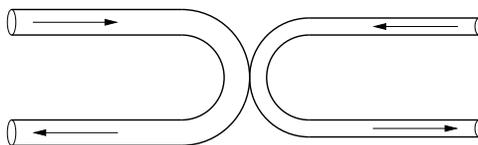
What is the total energy of the particle?

- (A) -5 J
- (B) 0 J
- (C) 5 J
- (D) 10 J
- (E) 15 J

21. The gravitational self potential energy of a solid ball of mass density  $\rho$  and radius  $R$  is  $E$ . What is the gravitational self potential energy of a ball of mass density  $\rho$  and radius  $2R$ ?
- (A)  $2E$   
 (B)  $4E$   
 (C)  $8E$   
 (D)  $16E$   
 (E)  $32E$
22. A balloon filled with helium gas is tied by a light string to the floor of a car; the car is sealed so that the motion of the car does not cause air from outside to affect the balloon. If the car is traveling with constant speed along a circular path, in what direction will the balloon on the string lean towards?

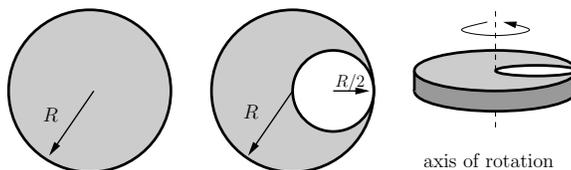


- (A) A  
 (B) B  
 (C) C  
 (D) D  
 (E) Remains vertical
23. Two streams of water flow through the U-shaped tubes shown. The tube on the left has cross-sectional area  $A$ , and the speed of the water flowing through it is  $v$ ; the tube on the right has cross-sectional area  $A' = 1/2A$ . If the net force on the tube assembly is zero, what must be the speed  $v'$  of the water flowing through the tube on the right? Neglect gravity, and assume that the speed of the water in each tube is the same upon entry and exit.

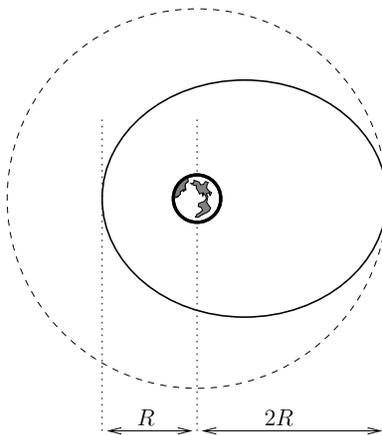


- (A)  $1/2v$   
 (B)  $v$   
 (C)  $\sqrt{2}v$   
 (D)  $2v$   
 (E)  $4v$

24. A uniform circular disk of radius  $R$  begins with a mass  $M$ ; about an axis through the center of the disk and perpendicular to the plane of the disk the moment of inertia is  $I_0 = \frac{1}{2}MR^2$ . A hole is cut in the disk as shown in the diagram. In terms of the radius  $R$  and the mass  $M$  of the original disk, what is the moment of inertia of the resulting object about the axis shown?



- (A)  $(15/32)MR^2$   
 (B)  $(13/32)MR^2$   
 (C)  $(3/8)MR^2$   
 (D)  $(9/32)MR^2$   
 (E)  $(15/16)MR^2$
25. Spaceman Fred's spaceship (which has negligible mass) is in an elliptical orbit about Planet Bob. The minimum distance between the spaceship and the planet is  $R$ ; the maximum distance between the spaceship and the planet is  $2R$ . At the point of maximum distance, Spaceman Fred is traveling at speed  $v_0$ . He then fires his thrusters so that he enters a circular orbit of radius  $2R$ . What is his new speed?



- (A)  $\sqrt{3/2}v_0$   
 (B)  $\sqrt{5}v_0$   
 (C)  $\sqrt{3/5}v_0$   
 (D)  $\sqrt{2}v_0$   
 (E)  $2v_0$