

## 2012 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

### INSTRUCTIONS

**DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN**

- Use  $g = 10 \text{ N/kg}$  throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers will result in a deduction of  $\frac{1}{4}$  point. There is no penalty for leaving an answer blank.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2012.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

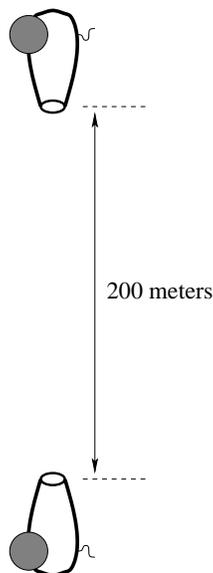
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- Consider a dripping faucet, where the faucet is 10 cm above the sink. The time between drops is such that when one drop hits the sink, one is in the air and another is about to drop. At what height above the sink will the drop in the air be right as a drop hits the sink?
  - Between 0 and 2 cm,
  - Between 2 and 4 cm,
  - Between 4 and 6 cm,
  - Between 6 and 8 cm, ← **CORRECT**
  - Between 8 and 10 cm,
- A cannonball is launched with initial velocity of magnitude  $v_0$  over a horizontal surface. At what minimum angle  $\theta_{\min}$  above the horizontal should the cannonball be launched so that it rises to a height  $H$  which is larger than the horizontal distance  $R$  that it will travel when it returns to the ground?
  - $\theta_{\min} = 76^\circ$  ← **CORRECT**
  - $\theta_{\min} = 72^\circ$
  - $\theta_{\min} = 60^\circ$
  - $\theta_{\min} = 45^\circ$
  - There is no such angle, as  $R > H$  for all range problems.
- An equilateral triangle is sitting on an inclined plane. Friction is too high for it to slide under any circumstance, but if the plane is sloped enough it can “topple” down the hill. What angle incline is necessary for it to start toppling?
  - 30 degrees
  - 45 degrees
  - 60 degrees ← **CORRECT**
  - It will topple at any angle more than zero
  - It can never topple if it cannot slide
- A particle at rest explodes into three particles of equal mass in the absence of external forces. Two particles emerge at a right angle to each other with equal speed  $v$ . What is the speed of the third particle?
  - $v$
  - $\sqrt{2}v$  ← **CORRECT**
  - $2v$
  - $2\sqrt{2}v$
  - The third particle can have a range of different speeds.
- A 12 kg block moving east at 4 m/s collides head on with a 6 kg block that is moving west at 2 m/s. The two blocks move together after the collision. What is the *loss* in kinetic energy in this collision?
  - 36 J
  - 48 J
  - 60 J
  - 72 J ← **CORRECT**
  - 96 J

**The following information applies to questions 6 and 7**

Two cannons are arranged vertically, with the lower cannon pointing upward (towards the upper cannon) and the upper cannon pointing downward (towards the lower cannon), 200m above the lower cannon. Simultaneously, they both fire. The muzzle velocity of the lower cannon is 25m/s and the muzzle velocity of the upper cannon is 55m/s.



6. How long after the cannons fire do the projectiles collide?
- (A) 2.2 s
  - (B) 2.5 s ← **CORRECT**
  - (C) 3.6 s
  - (D) 6.7 s
  - (E) 8.0 s
7. How far beneath the top cannon do the projectiles collide?
- (A) 31 m
  - (B) 67 m
  - (C) 110 m
  - (D) 140 m
  - (E) 170 m ← **CORRECT**
8. A block of mass  $m = 3.0$  kg is moving on a horizontal surface towards a massless spring with spring constant  $k = 80.0$  N/m. The coefficient of kinetic friction between the block and the surface is  $\mu_k = 0.50$ . The block has a speed of 2.0 m/s when it first comes in contact with the spring. How far will the spring be compressed?
- (A) 0.19 m
  - (B) 0.24 m ← **CORRECT**
  - (C) 0.39 m
  - (D) 0.40 m
  - (E) 0.61 m

9. A uniform spherical planet has radius  $R$  and the acceleration due to gravity at its surface is  $g$ . What is the escape velocity of a particle from the planet's surface?

- (A)  $\frac{1}{2}\sqrt{gR}$
- (B)  $\sqrt{gR}$
- (C)  $\sqrt{2gR}$  ← **CORRECT**
- (D)  $2\sqrt{gR}$
- (E) The escape velocity cannot be expressed in terms of  $g$  and  $R$  alone.

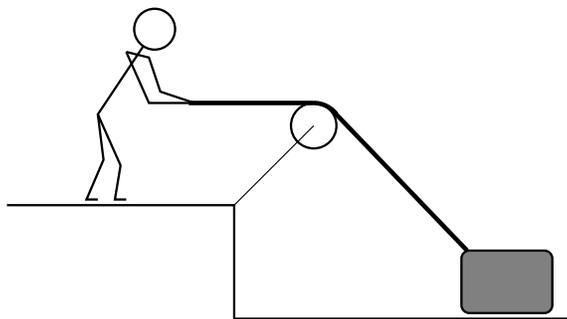
10. Four objects are placed at rest at the top of an inclined plane and allowed to roll without slipping to the bottom in the absence of rolling resistance and air resistance.

- Object A is a solid brass ball of diameter  $d$ .
- Object B is a solid brass ball of diameter  $2d$ .
- Object C is a hollow brass sphere of diameter  $d$ .
- Object D is a solid aluminum ball of diameter  $d$ . (Aluminum is less dense than brass.)

The balls are placed so that their centers of mass all travel the same distance. In each case, the time of motion  $T$  is measured. Which of the following statements is correct?

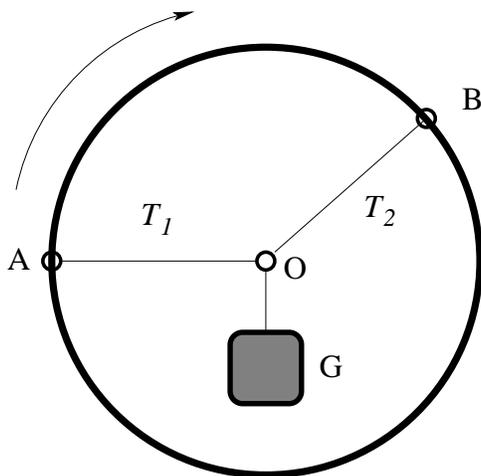
- (A)  $T_B > T_C > T_A = T_D$
- (B)  $T_A = T_B = T_C > T_D$
- (C)  $T_B > T_A = T_C = T_D$
- (D)  $T_C > T_A = T_B = T_D$  ← **CORRECT**
- (E)  $T_A = T_B = T_C = T_D$

11. As shown below, Lily is using the rope through a fixed pulley to move a box with constant speed  $v$ . The kinetic friction coefficient between the box and the ground is  $\mu < 1$ ; assume that the fixed pulley is massless and there is no friction between the rope and the fixed pulley. Then, while the box is moving, which of the following statements is correct?



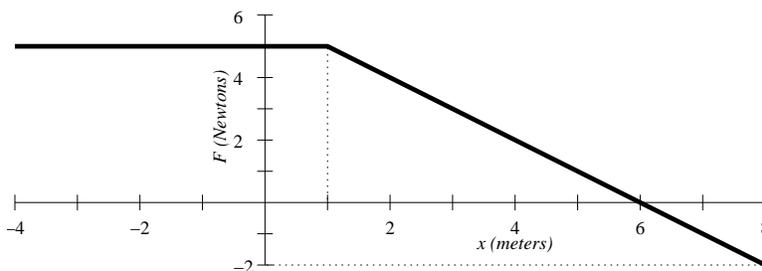
- (A) The magnitude of the force on the rope is constant.
- (B) The magnitude of friction between the ground and the box is decreasing. ← **CORRECT**
- (C) The magnitude of the normal force of the ground on the box is increasing.
- (D) The pressure of the box on the ground is increasing.
- (E) The pressure of the box on the ground is constant.

12. A rigid hoop can rotate about the center. Two massless strings are attached to the hoop, one at A, the other at B. These strings are tied together at the center of the hoop at O, and a weight G is suspended from that point. The strings have a fixed length, regardless of the tension, and the weight G is only supported by the strings. Originally OA is horizontal.



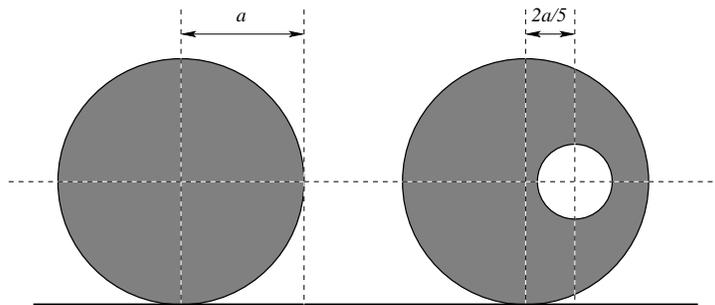
Now, the outer hoop will start to slowly rotate  $90^\circ$  clockwise until OA will become vertical, while keeping the angle between the strings constant and keeping the object static. Which of the following statements about the tensions  $T_1$  and  $T_2$  in the two strings is correct?

- (A)  $T_1$  always decreases.  
 (B)  $T_1$  always increases.  
 (C)  $T_2$  always increases.  
 (D)  $T_2$  will become zero at the end of the rotation. ← **CORRECT**  
 (E)  $T_2$  first increases and then decreases.
13. Shown below is a graph of the  $x$  component of force versus position for a 4.0 kg cart constrained to move in one dimension on the  $x$  axis. At  $x = 0$  the cart has a velocity of  $-3.0$  m/s (in the negative direction). Which of the following is closest to the maximum speed of the cart?



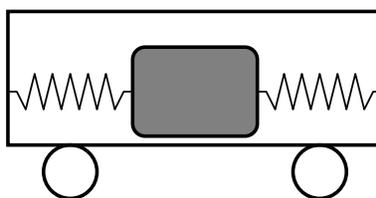
- (A) 1.6 m/s  
 (B) 2.5 m/s  
 (C) 3.0 m/s  
 (D) 4.0 m/s  
 (E) 4.2 m/s ← **CORRECT**

14. A uniform cylinder of radius  $a$  originally has a weight of 80 N. After an off-axis cylinder hole at  $2a/5$  was drilled through it, it weighs 65 N. The axes of the two cylinders are parallel and their centers are at the same height.



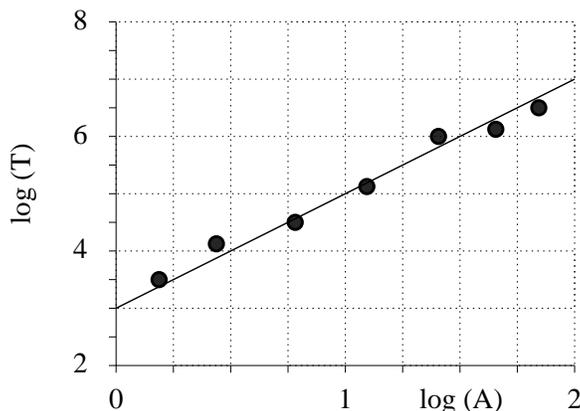
A force  $T$  is applied to the top of the cylinder horizontally. In order to keep the cylinder is at rest, the magnitude of the force is closest to:

- (A) 6 N ← **CORRECT**  
 (B) 10 N  
 (C) 15 N  
 (D) 30 N  
 (E) 38 N
15. A car of mass  $m$  has an engine that provides a constant power output  $P$ . Assuming no friction, what is the maximum constant speed  $v_{\max}$  that this car can drive up a long incline that makes an angle  $\theta$  with the horizontal?
- (A)  $v_{\max} = P/(mg \sin \theta)$  ← **CORRECT**  
 (B)  $v_{\max} = P^2 \sin \theta / mg$   
 (C)  $v_{\max} = \sqrt{2P/mg} / \sin \theta$   
 (D) There is no maximum constant speed.  
 (E) The maximum constant speed depends on the length of the incline.
16. Inside a cart that is accelerating horizontally at acceleration  $\vec{a}$ , there is a block of mass  $M$  connected to two light springs of force constants  $k_1$  and  $k_2$ . The block can move without friction horizontally. Find the vibration frequency of the block.



- (A)  $\frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{M}} + a$   
 (B)  $\frac{1}{2\pi} \sqrt{\frac{k_1 k_2}{(k_1 + k_2)M}}$   
 (C)  $\frac{1}{2\pi} \sqrt{\frac{k_1 k_2}{(k_1 + k_2)M}} + a$   
 (D)  $\frac{1}{2\pi} \sqrt{\frac{|k_1 - k_2|}{M}}$   
 (E)  $\frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{M}}$  ← **CORRECT**

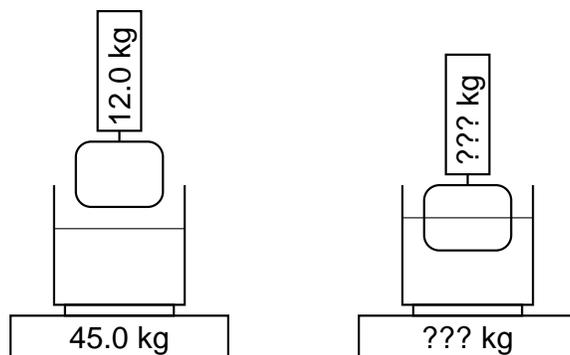
17. Shown below is a log/log plot for the data collected of amplitude and period of oscillation for certain non-linear oscillator.



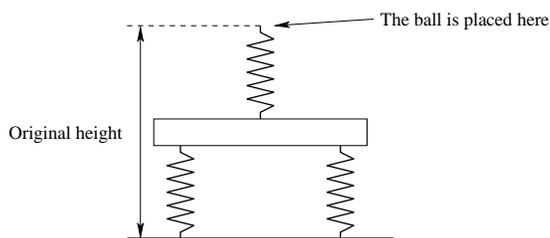
According to the data, the relationship between period  $T$  and amplitude  $A$  is best given by

- (A)  $T = 1000A^2$  ← **CORRECT**  
 (B)  $T = 100A^3$   
 (C)  $T = 2A + 3$   
 (D)  $T = 3\sqrt{A}$   
 (E) Period is independent of amplitude for oscillating systems
18. A mass hangs from the ceiling of a box by an ideal spring. With the box held fixed, the mass is given an initial velocity and oscillates with purely vertical motion. When the mass reaches the lowest point of its motion, the box is released and allowed to fall. To an observer inside the box, which of the following quantities **does not** change when the box is released? Ignore air resistance.
- (A) The amplitude of the oscillation  
 (B) The period of the oscillation ← **CORRECT**  
 (C) The maximum speed reached by the mass  
 (D) The height at which the mass reaches its maximum speed  
 (E) The maximum height reached by the mass
19. A 1,500 Watt motor is used to pump water a vertical height of 2.0 meters out of a flooded basement through a cylindrical pipe. The water is ejected through the end of the pipe at a speed of 2.5 m/s. Ignoring friction and assuming that all of the energy of the motor goes to the water, which of the following is the closest to the radius of the pipe? The density of water is  $\rho = 1000 \text{ kg/m}^3$ .
- (A) 1/3 cm  
 (B) 1 cm  
 (C) 3 cm  
 (D) 10 cm ← **CORRECT**  
 (E) 30 cm

20. A container of water is sitting on a scale. Originally, the scale reads  $M_1 = 45$  kg. A block of wood is suspended from a second scale; originally the scale read  $M_2 = 12$  kg. The density of wood is  $0.60$  g/cm<sup>3</sup>; the density of the water is  $1.00$  g/cm<sup>3</sup>. The block of wood is lowered into the water until half of the block is beneath the surface. What is the resulting reading on the scales?

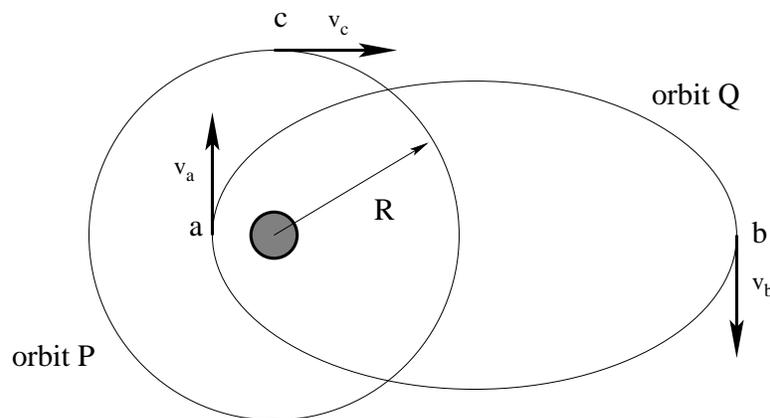


- (A)  $M_1 = 45$  kg and  $M_2 = 2$  kg.  
 (B)  $M_1 = 45$  kg and  $M_2 = 6$  kg.  
 (C)  $M_1 = 45$  kg and  $M_2 = 10$  kg.  
 (D)  $M_1 = 55$  kg and  $M_2 = 6$  kg.  
 (E)  $M_1 = 55$  kg and  $M_2 = 2$  kg. ← **CORRECT**
21. A spring system is set up as follows: a platform with a weight of  $10$  N is on top of two springs, each with spring constant  $75$  N/m. On top of the platform is a third spring with spring constant  $75$  N/m. If a ball with a weight of  $5.0$  N is then fastened to the top of the third spring and then slowly lowered, by how much does the height of the spring system change?



- (A)  $0.033$  m  
 (B)  $0.067$  m  
 (C)  $0.100$  m ← **CORRECT**  
 (D)  $0.133$  m  
 (E)  $0.600$  m
22. The softest audible sound has an intensity of  $I_0 = 10^{-12}$  W/m<sup>2</sup>. In terms of the fundamental units of kilograms, meters, and seconds, this is equivalent to
- (A)  $I_0 = 10^{-12}$  kg/s<sup>3</sup> ← **CORRECT**  
 (B)  $I_0 = 10^{-12}$  kg/s  
 (C)  $I_0 = 10^{-12}$  kg<sup>2</sup>m/s  
 (D)  $I_0 = 10^{-12}$  kg<sup>2</sup>m/s<sup>2</sup>  
 (E)  $I_0 = 10^{-12}$  kg/m · s<sup>3</sup>

23. Which of the following sets of equipment **cannot** be used to measure the local value of the acceleration due to gravity ( $g$ )?
- (A) A spring scale (which reads in force units) and a known mass.
  - (B) A rod of known length, an unknown mass, and a stopwatch.
  - (C) An inclined plane of known inclination, several carts of different known masses, and a stopwatch. ← **CORRECT**
  - (D) A launcher which launches projectiles at a known speed, a projectile of known mass, and a meter stick.
  - (E) A motor with a known output power, a known mass, a piece of string of unknown length, and a stopwatch.
24. Three point masses  $m$  are attached together by identical springs. When placed at rest on a horizontal surface the masses form a triangle with side length  $l$ . When the assembly is rotated about its center at angular velocity  $\omega$ , the masses form a triangle with side length  $2l$ . What is the spring constant  $k$  of the springs?
- (A)  $2m\omega^2$
  - (B)  $\frac{2}{\sqrt{3}}m\omega^2$
  - (C)  $\frac{2}{3}m\omega^2$  ← **CORRECT**
  - (D)  $\frac{1}{\sqrt{3}}m\omega^2$
  - (E)  $\frac{1}{3}m\omega^2$
25. Consider the two orbits around the sun shown below. Orbit P is circular with radius  $R$ , orbit Q is elliptical such that the farthest point b is between  $2R$  and  $3R$ , and the nearest point a is between  $R/3$  and  $R/2$ . Consider the magnitudes of the velocity of the circular orbit  $v_c$ , the velocity of the comet in the elliptical orbit at the farthest point  $v_b$ , and the velocity of the comet in the elliptical orbit at the nearest point  $v_a$ . Which of the following rankings is correct?



- (A)  $v_b > v_c > 2v_a$
- (B)  $2v_c > v_b > v_a$
- (C)  $10v_b > v_a > v_c$  ← **CORRECT**
- (D)  $v_c > v_a > 4v_b$
- (E)  $2v_a > \sqrt{2}v_b > v_c$

## 1 Solutions

1d	6b	11b	16e	21c
2a	7e	12d	17a	22a
3c	8b	13e	18b	23c
4b	9c	14a	19d	24c
5d	10d	15a	20e	25c