AAPT UNITED STATES PHYSICS TEAM
AIP CEE 2013

Semifinal Exam

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Important Instructions for the Exam Supervisor

- This examination consists of two parts.
- Part A has four questions and is allowed 90 minutes.
- Part B has two questions and is allowed 90 minutes.
- The first page that follows is a cover sheet. Examinees may keep the cover sheet for both parts of the exam.
- The parts are then identified by the center header on each page. Examinees are only allowed to do one part at a time, and may not work on other parts, even if they have time remaining.
- Allow 90 minutes to complete Part A. Do not let students look at Part B. Collect the answers to Part A before allowing the examinee to begin Part B. Examinees are allowed a 10 to 15 minutes break between parts A and B.
- Allow 90 minutes to complete Part B. Do not let students go back to Part A.
- Ideally the test supervisor will divide the question paper into 3 parts: the cover sheet (page 2), Part A (pages 3-9), and Part B (pages 11-17). Examinees should be provided parts A and B individually, although they may keep the cover sheet.
- The supervisor *must* collect all examination questions, including the cover sheet, at the end of the exam, as well as any scratch paper used by the examinees. Examinees may *not* take the exam questions. The examination questions may be returned to the students after April 1, 2013.
- Examinees are allowed calculators, but they may not use symbolic math, programming, or graphic features of these calculators. Calculators may not be shared and their memory must be cleared of data and programs. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. Examinees may not use any tables, books, or collections of formulas.
- **Please provide the examinees with graph paper for Part A. A straight edge or ruler could also be useful.**

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Semifinal Exam

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems. Each question is worth 25 points. Do not look at Part B during this time.
- After you have completed Part A you may take a break.
- Then work Part B. You have 90 minutes to complete both problems. Each question is worth 50 points. Do not look at Part A during this time.
- Show all your work. Partial credit will be given. Do not write on the back of any page. Do not write anything that you wish graded on the question sheets.
- Start each question on a new sheet of paper. Put your AAPT ID number, your name, the question number and the page number/total pages for this problem, in the upper right hand corner of each page. For example,

  AAPT ID #
  Doe, Jamie
  A1 - 1/3

- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers/solutions) on this contest until after April 1, 2013.**

Possibly Useful Information. You may use this sheet for both parts of the exam.

$g = 9.8$ N/kg	$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$	$k_\mathrm{m} = \mu_0/4\pi = 10^{-7}\ \mathrm{T \cdot m/A}$
$c = 3.00 \times 10^8$ m/s	$k_\mathrm{B} = 1.38 \times 10^{-23}$ J/K
$N_\mathrm{A} = 6.02 \times 10^{23}\ (\mathrm{mol})^{-1}$	$R = N_\mathrm{A} k_\mathrm{B} = 8.31\ \mathrm{J/(mol \cdot K)}$
$\sigma = 5.67 \times 10^{-8}\ \mathrm{J/(s \cdot m^2 \cdot K^4)}$	$e = 1.602 \times 10^{-19}$ C
$1\mathrm{eV} = 1.602 \times 10^{-19}$ J	$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s} = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}$
$m_e = 9.109 \times 10^{-31}\ \mathrm{kg} = 0.511\ \mathrm{MeV/c^2}$	$(1+x)^n \approx 1 + nx$ for $\lvert x \rvert \ll 1$
$\sin\theta \approx \theta - \frac{1}{6}\theta^3$ for $\lvert\theta\rvert \ll 1$	$\cos\theta \approx 1 - \frac{1}{2}\theta^2$ for $\lvert\theta\rvert \ll 1$

Part A

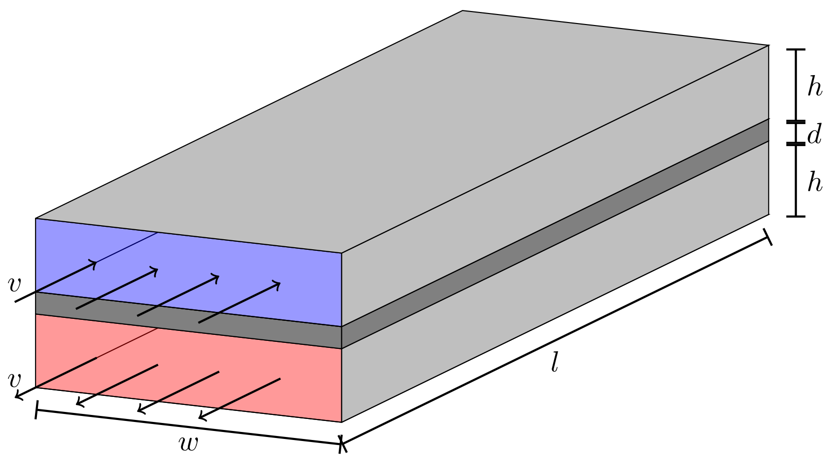
Question A1

The flow of heat through a material can be described via the *thermal conductivity* κ. If the two faces of a slab of material with thermal conductivity κ, area A, and thickness d are held at temperatures differing by ΔT, the thermal power P transferred through the slab is

$$P = \frac{\kappa A \Delta T}{d}$$

A *heat exchanger* is a device which transfers heat between a hot fluid and a cold fluid; they are common in industrial applications such as power plants and heating systems. The heat exchanger shown below consists of two rectangular tubes of length l, width w, and height h. The tubes are separated by a metal wall of thickness d and thermal conductivity κ. Originally hot fluid flows through the lower tube at a speed v from right to left, and originally cold fluid flows through the upper tube in the opposite direction (left to right) at the same speed. The heat capacity per unit volume of both fluids is c.

The hot fluid enters the heat exchanger at a higher temperature than the cold fluid; the difference between the temperatures of the entering fluids is ΔT_i. When the fluids exit the heat exchanger the difference has been reduced to ΔT_f. (It is possible for the exiting originally cold fluid to have a *higher* temperature than the exiting originally hot fluid, in which case $\Delta T_f < 0$.)



Assume that the temperature in each pipe depends only on the lengthwise position, and consider transfer of heat only due to conduction in the metal and due to the bulk movement of fluid. Under the assumptions in this problem, while the temperature of each fluid varies along the length of the exchanger, the temperature *difference* across the wall is the same everywhere. You need not prove this.

Find ΔT_f in terms of the other given parameters.

Solution

Suppose the temperature difference across the wall is ΔT_w. Since the total area of the wall is simply lw, the power transferred across the wall is

$$P = \frac{\kappa l w}{d} \Delta T_w$$

In a time dt, the energy transferred is therefore

$$dE = P\ dt = \frac{\kappa l w}{d} \Delta T_w\ dt$$

Meanwhile, suppose the red fluid enters at temperature T_r and the blue fluid at temperature T_b. The red fluid then exits at temperature $T_b + \Delta T_w$, so the overall temperature change of the red fluid is

$$\Delta T_r = T_r - (T_b + \Delta T_w) = \Delta T_i - \Delta T_w$$

In a time dt, a volume of red fluid $vwh\ dt$ flows through the pipe; the heat capacity of this amount of fluid is $vwhc\ dt$, so the energy transferred out of it is therefore

$$dE = vwhc\ dt\ \Delta T_r = vwhc(\Delta T_i - \Delta T_w)\ dt$$

(Note that the same result is obtained for the heat transferred into the blue fluid; if the flow rates or heat capacities were not the same, this would not hold, exposing the fact that ΔT_w is not constant in that case.)

Equating,

$$\frac{\kappa l w}{d} \Delta T_w = vwhc(\Delta T_i - \Delta T_w)$$

$$\Delta T_w = \frac{\Delta T_i}{1 + \frac{\kappa l}{dvhc}}$$

Because the red fluid exits at $T_b + \Delta T_w$, and the blue fluid at $T_r - \Delta T_w$,

$$\Delta T_f = (T_b + \Delta T_w) - (T_r - \Delta T_w) = -\Delta T_i + 2\Delta T_w$$

$$\Delta T_f = \Delta T_i \left(\frac{2}{1 + \frac{\kappa l}{dvhc}} - 1 \right)$$

The performance of the heat exchanger is controlled entirely by the dimensionless parameter $\frac{\kappa l}{dvhc}$; as might be intuitive, a long tube and high conductivity are beneficial, whereas a thick wall, high flow rate, and high heat capacity is not. The poorest performance, unsurprisingly, reflects essentially no heat transfer, with the red fluid and blue fluid exiting with the same temperatures they started with. Interestingly, the limit of performance is a complete reversal in the temperatures of the two fluids, with $\Delta T_f \to -\Delta T_i$.

Question A2

A solid round object of radius R can roll down an incline that makes an angle θ with the horizontal. Assume that the rotational inertia about an axis through the center of mass is given by $I = \beta m R^2$. The coefficient of kinetic and static friction between the object and the incline is μ. The object moves from rest through a vertical distance h.

a. If the angle of the incline is sufficiently large, then the object will slip and roll; if the angle of the incline is sufficiently small, then the object with roll without slipping. Determine the angle θ_c that separates the two types of motion.

b. Derive expressions for the linear acceleration of the object down the ramp in the case of

   i. Rolling without slipping, and

   ii. Rolling and slipping.

Solution

As the object rolls down the incline, there is a torque about the center of mass given by

$$\tau = Rf$$

where f is the force of friction. The angular acceleration is then

$$\alpha = \frac{\tau}{I} = \frac{f}{\beta m R}$$

or, as will be more useful,

$$f = \alpha \beta m R$$

The object experience a linear acceleration down the incline given by

$$ma = mg \sin\theta - f$$

We have to cases to consider. Either the object rolls without slipping so that $f \leq \mu mg \cos\theta$ and $a = \alpha R$, or the object rolls while slipping so that $f = \mu mg \cos\theta$ and $a > \alpha R$.

Rolling without slipping

Combining the equalities, we get

$$ma = mg \sin\theta - \beta m a$$

or

$$a = g \frac{\sin\theta}{1+\beta}$$

Rolling while slipping

Combining the equalities, we get

$$ma = mg \sin\theta - \mu mg \cos\theta$$

or

$$a = g\left(\sin\theta - \mu \cos\theta\right)$$

The motion is changes at an angle where the static friction is greatest, or when *both* conditions are equalities:

$$f = \mu mg \cos\theta$$

and

$$a = \alpha R$$

In that case

$$\frac{\sin\theta_c}{1+\beta} = \sin\theta_c - \mu\cos\theta_c$$

or

$$\tan\theta_c = \left(\frac{1}{\beta} + 1\right)\mu$$

Question A3

A beam of muons is maintained in a circular orbit by a uniform magnetic field. Neglect energy loss due to electromagnetic radiation.

The mass of the muon is 1.88×10^{-28} kg, its charge is -1.602×10^{-19} C, and its half-life is 1.523 μs.

a. The speed of the muons is much less than the speed of light. It is found that half of the muons decay during each full orbit. What is the magnitude of the magnetic field?

b. The experiment is repeated with the same magnetic field, but the speed of the muons is increased; it is no longer much less than the speed of light. Does the fraction of muons which decay during each full orbit increase, decrease, or stay the same?

   The following facts about special relativity may be useful:

   - The Lorentz factor for a particle moving at speed v is
   $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$
   - The Lorentz factor gives the magnitude of *time dilation*; that is, a clock moving at speed v in a given reference frame runs slow by a factor γ in that frame.
   - The momentum of a particle is given by
   $$\vec{p} = \gamma m \vec{v}$$
   where m does not depend on v.
   - The Lorentz force law in the form
   $$\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$
   continues to hold.

Solution

For brevity we simply present the full relativistic solution.

The relationships for circular motion

$$\left|\frac{d\vec{v}}{dt}\right| = \frac{|\vec{v}|^2}{r}$$

$$2\pi r = |\vec{v}|\, T$$

are purely a matter of mathematics, and thus continue to hold under special relativity. Meanwhile, since $|\vec{v}|$ is constant for circular motion, γ is constant as well. Thus we can take magnitudes in the Lorentz force law (and set $\vec{E} = 0$) to find

$$\gamma m \left|\frac{d\vec{v}}{dt}\right| = q\,|\vec{v}|\,B$$

Combining these relationships,

$$\frac{2\pi}{T}\gamma m = qB$$

If half of the muons decay during each orbit, in the muons' frame of reference each orbit takes one half-life $T_{1/2}$. In the lab frame, then,

$$T = \gamma T_{1/2}$$

and so

$$\frac{2\pi}{\gamma T_{1/2}}\gamma m = qB$$

$$B = \frac{2\pi m}{q T_{1/2}}$$

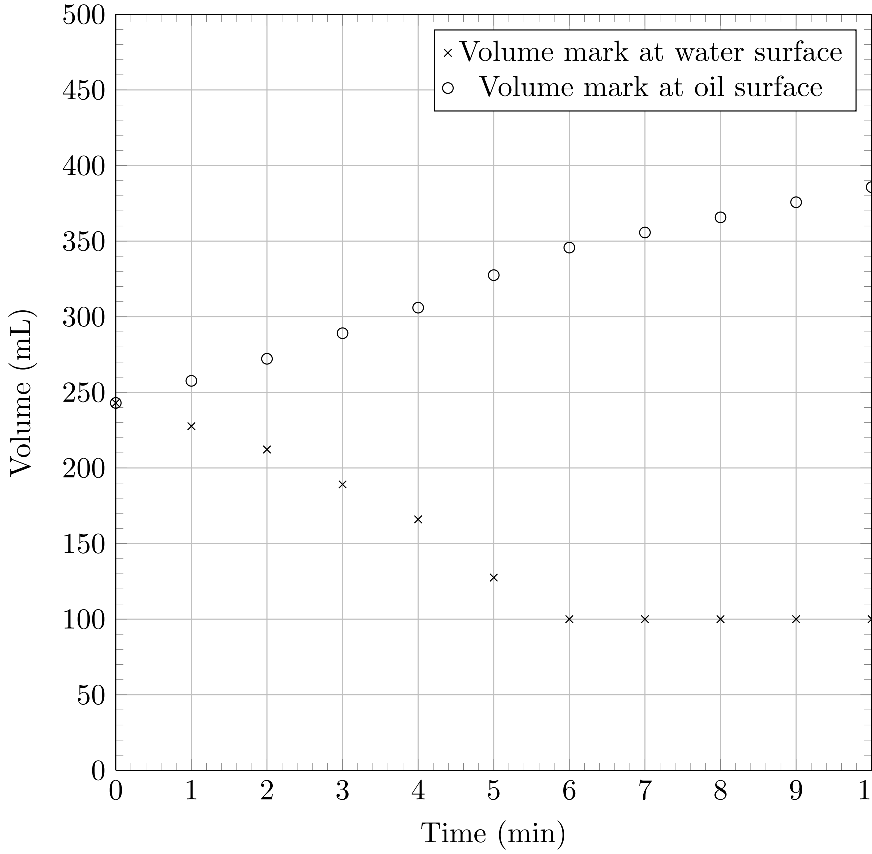
Numerically,

$$B = 4.85 \text{ mT}$$

The speed of the muons is irrelevant to the fraction which decay per orbit, even in the relativistic case. (The orbits take longer, but the muons live longer, both by the same factor γ.)

Question A4

A graduated cylinder is partially filled with water; a rubber duck floats at the surface. Oil is poured into the graduated cylinder at a slow, constant rate, and the volume marks corresponding to the surface of the water and the surface of the oil are recorded as a function of time.



Water has a density of 1.00 g/mL; the density of air is negligible, as are surface effects. Find the density of the oil.

Solution

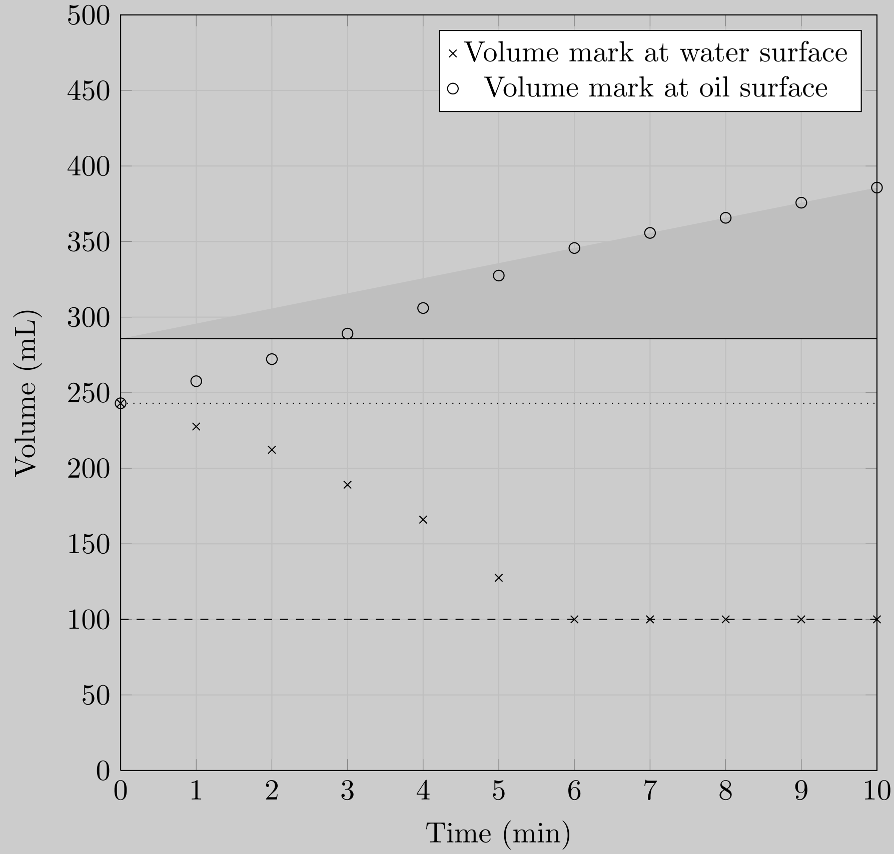
As the oil is poured in, more and more of the weight of the duck is supported by oil, and it rises out of the water, reducing the water level. Eventually this stops, either because the duck is fully submerged in oil or because it is floating entirely above the water. At all times, the weight of the water that is no longer displaced equals the weight of the newly displaced oil:

$$\rho_o g \Delta V_o = \rho_w g \Delta V_w$$

With this understanding many approaches are possible; we illustrate one. The change in the volume of displaced water is easily read off the graph as the distance between the dotted and dashed lines; it is 143 mL. Finding the volume of displaced oil requires us to take into account the increasing amount of oil in the cylinder. We know there is no oil at $t = 0$, because the oil level and water level coincide, and we know that the rate of change of the oil level for $t > 6$ min is the pour rate, because the water level is not changing. Extrapolating to $t = 0$ we conclude that the volume of oil in the container at any time is given by the height of the shaded region. The volume of displaced oil can then be read as the distance between the solid and dashed lines; it is 186 mL. The density of the oil is

$$\rho_o = \rho_w \frac{\Delta V_w}{\Delta V_o} = (1.00 \text{ g/mL}) \frac{143 \text{ mL}}{186 \text{ mL}}$$

$\rho_o = 0.77$ g/mL


× Volume mark at water surface
○ Volume mark at oil surface
Volume (mL)
Time (min)
0
50
100
150
200
250
300
350
400
450
500
0
1
2
3
4
5
6
7
8
9
10

STOP: Do Not Continue to Part B

If there is still time remaining for Part A, you should review your work for Part A, but do not continue to Part B until instructed by your exam supervisor.

Part B

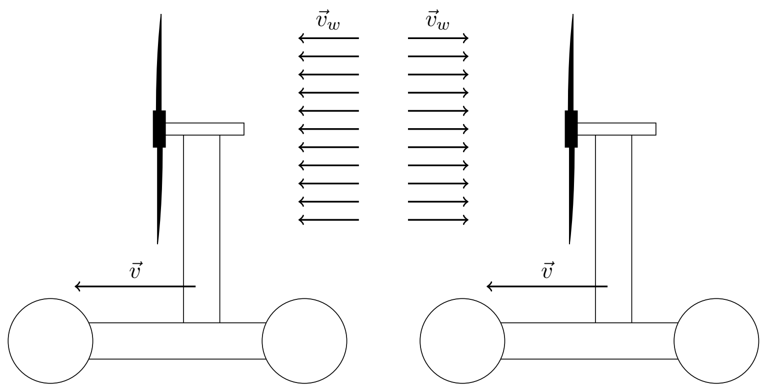
Question B1

Shown below is the *Blackbird*, a vehicle built in 2009.

There is no source of stored energy such as a battery or gasoline engine; all of the power used to move the car comes from the wind. The only important mechanism in the car is a gearbox that can transfer power between the wheels and the propeller.

The Blackbird was driven both directly downwind and directly upwind, as shown below. In each case the car remained exactly parallel (or anti-parallel) to the wind without turning. The tests were conducted on level ground, in steady, uniform wind, and continued long enough to reach the steady state.

Source: fasterthanthewind.org



Downwind Upwind

When driving downwind, the builders claim that they were able to drive "faster than the wind": that is, with $|\vec{v}| > |\vec{v}_w|$, so that the car experienced a relative headwind while traveling. Commenters on the Internet claimed, often angrily, that this was physically impossible and that the Blackbird was a hoax. Some commenters also claimed that the upwind case was physically impossible.

a. Consider first the **downwind faster than the wind** case.

- Is the motion actually possible as claimed? If not, offer a brief explanation!
- If the motion is possible, is power transferred from the propeller to the wheels or vice versa?
- If the motion is possible, what ground speed is attained? For this question, suppose that when transferring power in either direction between the propeller and the wheels, a fraction α of the useful work is lost; let the wind speed be v_w. Neglect all other losses of energy.

b. Answer the previous questions for the **upwind** case.

Solution

Both modes are possible as claimed.

When a force exists between two bodies in relative motion, the net power produced or consumed is proportional to the relative velocity between them:

$$P = F v_r$$

This value is independent of the reference frame in which the power is measured, even though the power delivered to each body separately is not. This is a consequence of Newton's third law; if the velocities of the bodies in a particular reference frame are v_1 and v_2, the net power is

$$P = F v_1 + (-F) v_2$$
$$P = F(v_1 - v_2)$$

The relative speed of the Blackbird and the air is different from the relative speed of the Blackbird and the ground; if the ground speed is v, the airspeed is $v - v_w$ in the downwind mode and $v + v_w$ in the upwind mode. Thus, even though the force due to the air and the force due to the ground must balance, one force can produce more power than the other one absorbs.

Power should always be produced by the force corresponding to the larger relative velocity. Thus **in the downwind case, power is transferred from the wheels to the propeller**, whereas **in the upwind case, power is transferred from the propeller to the wheels**. (In practice this reversal required a lengthy reconfiguration of the Blackbird between the two trials.)

In the **downwind** case, the wheel power is

$$P_w = F v$$

and the propeller power is

$$P_p = F(v - v_w)$$

Power is transferred to the propeller, so

$$P_p = (1 - \alpha) P_w$$
$$F(v - v_w) = (1 - \alpha) F v$$
$$v = \frac{v_w}{\alpha}$$

At high energy loss one moves close to the wind speed (as expected); with sufficiently low energy loss, any speed is possible.

In the **upwind** case, the wheel power is still

$$P_w = F v$$

but the propeller power is

$$P_p = F(v + v_w)$$

Power is transferred to the wheels, so

$$P_w = (1 - \alpha) P_p$$
$$F v = (1 - \alpha) F(v + v_w)$$
$$v = v_w \left(\frac{1}{\alpha} - 1 \right)$$

At high energy loss one cannot make any progress at all (as expected); again, with sufficiently low energy loss, any speed is possible.

Question B2

This problem concerns three situations involving the transfer of energy into a region of space by electromagnetic fields. In the first case, that energy is stored in the kinetic energy of a charged object; in the second and third cases, the energy is stored in an electric or magnetic field.

In general, whenever an electric and a magnetic field are at an angle to each other, energy is transferred; for example, this principle is the reason electromagnetic radiation transfers energy. The power transferred per unit area is given by the *Poynting vector*:

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

In each part of this problem, the last subpart asks you to verify that the rate of energy transfer agrees with the formula for the Poynting vector. Therefore, you should not use the formula for the Poynting vector before the last subpart!

a. A long, insulating cylindrical rod has radius R and carries a uniform volume charge density ρ. A uniform external electric field E exists in the direction of its axis. The rod moves in the direction of its axis at speed v.

   i. What is the power per unit length $\mathcal{P}$ delivered to the rod?

   ii. What is the magnetic field B at the surface of the rod? Draw the direction on a diagram.

   iii. Compute the Poynting vector, draw its direction on a diagram, and verify that it agrees with the rate of energy transfer.

b. A parallel plate capacitor consists of two discs of radius R separated by a distance $d \ll R$. The capacitor carries charge Q, and is being charged by a small, constant current I.

   i. What is the power P delivered to the capacitor?

   ii. What is the magnetic field B just inside the edge of the capacitor? Draw the direction on a diagram. (Ignore fringing effects in the electric field for this calculation.)

   iii. Compute the Poynting vector, draw its direction on a diagram, and verify that it agrees with the rate of energy transfer.

c. A long solenoid of radius R has $\mathcal{N}$ turns of wire per unit length. The solenoid carries current I, and this current is increased at a small, constant rate $\frac{dI}{dt}$.

   i. What is the power per unit length $\mathcal{P}$ delivered to the solenoid?

   ii. What is the electric field E just inside the surface of the solenoid? Draw its direction on a diagram.

   iii. Compute the Poynting vector, draw its direction on a diagram, and verify that it agrees with the rate of energy transfer.

Solution

a. i. A length l of the rod has charge $q = \pi R^2 l\rho$; the force on it is $F = qE$ and the power delivered is $P = Fv$. Combining these,

$$P = \pi R^2 l\rho E v$$

$$\mathcal{P} = \pi R^2 \rho E v$$

ii. The length l of the rod moves past a point in a time $t = \frac{l}{v}$, so the current carried by the rod is

$$I = \frac{q}{t} = \pi R^2 \rho v$$

Applying Ampere's law to a loop of radius R,

$$\oint B\, dl = \mu_0 I_{enc}$$

$$2\pi R B = \mu_0 \pi R^2 \rho v$$

$$B = \frac{1}{2}\mu_0 R\rho v$$

The field is circumferential as given by the right-hand rule.

iii. The electric and magnetic fields are perpendicular, so the Poynting vector has magnitude

$$S = \frac{1}{\mu_0} E B$$

$$S = \frac{1}{2} R\rho v E$$

A quick application of the right hand rule indicates that it points inward along the surface of the cylinder, as it ought. The cylinder has area per unit length $2\pi r$, so the rate of energy transfer per unit length is

$$\mathcal{P} = 2\pi r S = \pi R^2 \rho v E$$

in agreement with the previous result.

b. i. The capacitance is given by the standard parallel-plate capacitor formula:

$$C = \frac{\epsilon_0 \pi R^2}{d}$$

The voltage on the capacitor is thus

$$V = \frac{Q}{C} = \frac{Qd}{\epsilon_0 \pi R^2}$$

and the power is

$$P = IV$$

$$P = \frac{IQd}{\epsilon_0 \pi R^2}$$

Students may choose instead to apply the formula for the volume energy density,

$$\mathcal{U} = \frac{1}{2}\epsilon_0 E^2$$

ii. Consider an Amperian loop encircling the edge of the capacitor, and use a flat Gaussian surface through the center of the capacitor. The electric field here is perpendicular to the surface and has magnitude

$$E = \frac{V}{d} = \frac{Q}{\epsilon_0 \pi R^2}$$

The electric flux through the surface is thus

$$\phi_E = \pi R^2 E = \frac{Q}{\epsilon_0}$$

This can also be determined directly using Gauss's law and appropriate symmetries. There is no current through the surface, so from Ampere's Law

$$\oint B\, dl = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

$$2\pi R B = \mu_0 \frac{dQ}{dt}$$

$$B = \frac{\mu_0 I}{2\pi R}$$

The field is circumferential as given by the right-hand rule.

Note that we could instead use a curved Gaussian surface that avoids the center of the capacitor and intersects one of the charging wires! In this case we have directly

$$\oint B\, dl = \mu_0 I$$

and the calculation proceeds as before.

iii. The electric and magnetic fields are perpendicular, so again

$$S = \frac{1}{\mu_0} EB$$

$$S = \frac{IQ}{2\epsilon_0 \pi^2 R^3}$$

A quick application of the right hand rule indicates that it points inward along the edge of the capacitor, as it ought. The area of this region is $2\pi Rd$, so the power delivered is

$$P = 2\pi R d S = \frac{IQd}{\epsilon_0 \pi R^2}$$

in agreement with the previous result.

c. i. Suppose that the solenoid has length l. The inductance is

$$L = \mu_0 \mathcal{N}^2 \pi R^2 l$$

Students may quote this formula directly, or derive it as follows. Consider an Amperian loop of length d intersecting the solenoid. This loop encloses $\mathcal{N}d$ turns of wire, so from Ampere's law (remembering that the magnetic field exists entirely within the solenoid)

$$\oint B\, dl = \mu_0 I_{enc}$$

$$Bd = \mu_0 \mathcal{N} dI$$

$$B = \mu_0 \mathcal{N} I$$

There are $\mathcal{N}l$ loops, so the total flux is

$$\Phi = \mathcal{N} l B \pi R^2$$

$$\Phi = \mu_0 \mathcal{N}^2 I \pi R^2 l$$

and since $\Phi = LI$,

$$L = \mu_0 \mathcal{N}^2 \pi R^2 l$$

as quoted above.

The voltage across the inductor is thus

$$V = L\frac{dI}{dt}$$

$$V = \mu_0 \mathcal{N}^2 \pi R^2 l \frac{dI}{dt}$$

and the power delivered is

$$P = IV$$

$$P = \mu_0 \mathcal{N}^2 \pi R^2 l I \frac{dI}{dt}$$

or, dividing by l,

$$\mathcal{P} = \mu_0 \mathcal{N}^2 \pi R^2 I \frac{dI}{dt}$$

Students may choose instead to apply the formula for the volume energy density,

$$\mathcal{U} = \frac{1}{2\mu_0} B^2$$

ii. Consider an Amperian loop just inside the surface of the solenoid. From above, the magnetic field through this loop is $B = \mu_0 \mathcal{N} I$; thus (working in magnitudes)

$$\oint E\, dl = \frac{d\phi_B}{dt}$$

$$2\pi R E = \mu_0 \mathcal{N} \pi R^2 \frac{dI}{dt}$$

$$E = \frac{1}{2} \mu_0 \mathcal{N} R \frac{dI}{dt}$$

The field is circumferential as given by Lenz's law and the right-hand rule.

iii. The electric and magnetic fields are perpendicular, so again

$$S = \frac{1}{\mu_0} EB$$

$$S = \frac{1}{2} \mu_0 \mathcal{N}^2 R I \frac{dI}{dt}$$

A quick application of the right hand rule indicates that it points inward towards the axis of the solenoid, as it ought. The area per unit length is just $2\pi R$, so the power per unit length is

$$\mathcal{P} = 2\pi R S$$

$$\mathcal{P} = \mu_0 \mathcal{N}^2 \pi R^2 I \frac{dI}{dt}$$

in agreement with the previous result.