AAPT UNITED STATES PHYSICS TEAM
AIP CEE 2013

Semifinal Exam

DO NOT DISTRIBUTE THIS PAGE

Important Instructions for the Exam Supervisor

- This examination consists of two parts.
- Part A has four questions and is allowed 90 minutes.
- Part B has two questions and is allowed 90 minutes.
- The first page that follows is a cover sheet. Examinees may keep the cover sheet for both parts of the exam.
- The parts are then identified by the center header on each page. Examinees are only allowed to do one part at a time, and may not work on other parts, even if they have time remaining.
- Allow 90 minutes to complete Part A. Do not let students look at Part B. Collect the answers to Part A before allowing the examinee to begin Part B. Examinees are allowed a 10 to 15 minutes break between parts A and B.
- Allow 90 minutes to complete Part B. Do not let students go back to Part A.
- Ideally the test supervisor will divide the question paper into 3 parts: the cover sheet (page 2), Part A (pages 3-5), and Part B (pages 7-8). Examinees should be provided parts A and B individually, although they may keep the cover sheet.
- The supervisor *must* collect all examination questions, including the cover sheet, at the end of the exam, as well as any scratch paper used by the examinees. Examinees may *not* take the exam questions. The examination questions may be returned to the students after April 1, 2013.
- Examinees are allowed calculators, but they may not use symbolic math, programming, or graphic features of these calculators. Calculators may not be shared and their memory must be cleared of data and programs. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. Examinees may not use any tables, books, or collections of formulas.
- **Please provide the examinees with graph paper for Part A. A straight edge or ruler could also be useful.**

AAPT UNITED STATES PHYSICS TEAM
AIP CEE 2013

Semifinal Exam

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems. Each question is worth 25 points. Do not look at Part B during this time.
- After you have completed Part A you may take a break.
- Then work Part B. You have 90 minutes to complete both problems. Each question is worth 50 points. Do not look at Part A during this time.
- Show all your work. Partial credit will be given. Do not write on the back of any page. Do not write anything that you wish graded on the question sheets.
- Start each question on a new sheet of paper. Put your AAPT ID number, your name, the question number and the page number/total pages for this problem, in the upper right hand corner of each page. For example,

  AAPT ID #
  Doe, Jamie
  A1 - 1/3

- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers/solutions) on this contest until after April 1, 2013.**

Possibly Useful Information. You may use this sheet for both parts of the exam.

$g = 9.8\ \mathrm{N/kg}$	$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$	$k_\mathrm{m} = \mu_0/4\pi = 10^{-7}\ \mathrm{T \cdot m/A}$
$c = 3.00 \times 10^8\ \mathrm{m/s}$	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J/K}$
$N_\mathrm{A} = 6.02 \times 10^{23}\ (\mathrm{mol})^{-1}$	$R = N_\mathrm{A} k_\mathrm{B} = 8.31\ \mathrm{J/(mol \cdot K)}$
$\sigma = 5.67 \times 10^{-8}\ \mathrm{J/(s \cdot m^2 \cdot K^4)}$	$e = 1.602 \times 10^{-19}\ \mathrm{C}$
$1\mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$	$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s} = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}$
$m_e = 9.109 \times 10^{-31}\ \mathrm{kg} = 0.511\ \mathrm{MeV/c^2}$	$(1+x)^n \approx 1 + nx$ for $\lvert x \rvert \ll 1$
$\sin\theta \approx \theta - \frac{1}{6}\theta^3$ for $\lvert\theta\rvert \ll 1$	$\cos\theta \approx 1 - \frac{1}{2}\theta^2$ for $\lvert\theta\rvert \ll 1$

Part A

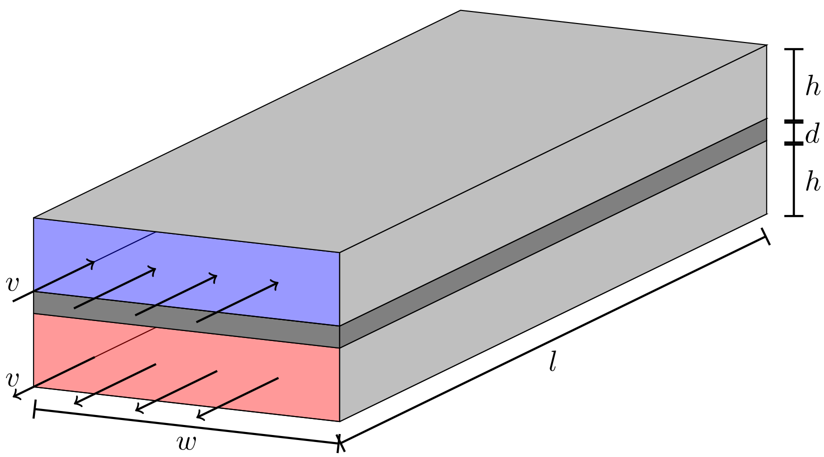
Question A1

The flow of heat through a material can be described via the *thermal conductivity* κ. If the two faces of a slab of material with thermal conductivity κ, area A, and thickness d are held at temperatures differing by ΔT, the thermal power P transferred through the slab is

$$P = \frac{\kappa A \Delta T}{d}$$

A *heat exchanger* is a device which transfers heat between a hot fluid and a cold fluid; they are common in industrial applications such as power plants and heating systems. The heat exchanger shown below consists of two rectangular tubes of length l, width w, and height h. The tubes are separated by a metal wall of thickness d and thermal conductivity κ. Originally hot fluid flows through the lower tube at a speed v from right to left, and originally cold fluid flows through the upper tube in the opposite direction (left to right) at the same speed. The heat capacity per unit volume of both fluids is c.

The hot fluid enters the heat exchanger at a higher temperature than the cold fluid; the difference between the temperatures of the entering fluids is ΔT_i. When the fluids exit the heat exchanger the difference has been reduced to ΔT_f. (It is possible for the exiting originally cold fluid to have a *higher* temperature than the exiting originally hot fluid, in which case $\Delta T_f < 0$.)



Assume that the temperature in each pipe depends only on the lengthwise position, and consider transfer of heat only due to conduction in the metal and due to the bulk movement of fluid. Under the assumptions in this problem, while the temperature of each fluid varies along the length of the exchanger, the temperature *difference* across the wall is the same everywhere. You need not prove this.

Find ΔT_f in terms of the other given parameters.

Question A2

A solid round object of radius R can roll down an incline that makes an angle θ with the horizontal. Assume that the rotational inertia about an axis through the center of mass is given by $I = \beta m R^2$. The coefficient of kinetic and static friction between the object and the incline is μ. The object moves from rest through a vertical distance h.

a. If the angle of the incline is sufficiently large, then the object will slip and roll; if the angle of the incline is sufficiently small, then the object with roll without slipping. Determine the angle θ_c that separates the two types of motion.

b. Derive expressions for the linear acceleration of the object down the ramp in the case of

   i. Rolling without slipping, and

   ii. Rolling and slipping.

Question A3

A beam of muons is maintained in a circular orbit by a uniform magnetic field. Neglect energy loss due to electromagnetic radiation.

The mass of the muon is 1.88×10^{-28} kg, its charge is -1.602×10^{-19} C, and its half-life is $1.523\ \mu$s.

a. The speed of the muons is much less than the speed of light. It is found that half of the muons decay during each full orbit. What is the magnitude of the magnetic field?

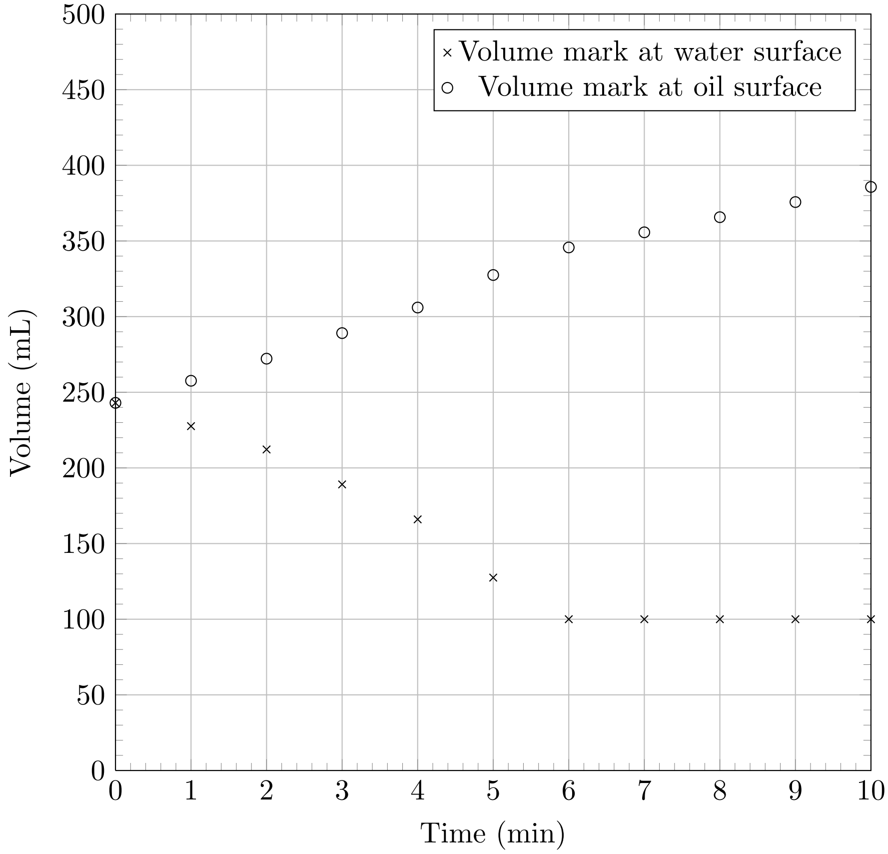
b. The experiment is repeated with the same magnetic field, but the speed of the muons is increased; it is no longer much less than the speed of light. Does the fraction of muons which decay during each full orbit increase, decrease, or stay the same?

   The following facts about special relativity may be useful:

   - The Lorentz factor for a particle moving at speed v is
   $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$
   - The Lorentz factor gives the magnitude of *time dilation*; that is, a clock moving at speed v in a given reference frame runs slow by a factor γ in that frame.
   - The momentum of a particle is given by
   $$\vec{p} = \gamma m \vec{v}$$
   where m does not depend on v.
   - The Lorentz force law in the form
   $$\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$
   continues to hold.

Question A4

A graduated cylinder is partially filled with water; a rubber duck floats at the surface. Oil is poured into the graduated cylinder at a slow, constant rate, and the volume marks corresponding to the surface of the water and the surface of the oil are recorded as a function of time.



Water has a density of 1.00 g/mL; the density of air is negligible, as are surface effects. Find the density of the oil.

STOP: Do Not Continue to Part B

If there is still time remaining for Part A, you should review your work for Part A, but do not continue to Part B until instructed by your exam supervisor.

Part B

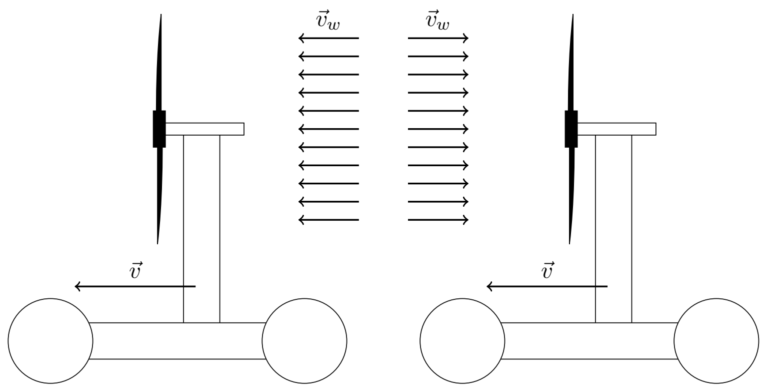
Question B1

Shown below is the *Blackbird*, a vehicle built in 2009.

There is no source of stored energy such as a battery or gasoline engine; all of the power used to move the car comes from the wind. The only important mechanism in the car is a gearbox that can transfer power between the wheels and the propeller.

The Blackbird was driven both directly downwind and directly upwind, as shown below. In each case the car remained exactly parallel (or anti-parallel) to the wind without turning. The tests were conducted on level ground, in steady, uniform wind, and continued long enough to reach the steady state.

Source: fasterthanthewind.org



Downwind Upwind

When driving downwind, the builders claim that they were able to drive "faster than the wind": that is, with $|\vec{v}| > |\vec{v}_w|$, so that the car experienced a relative headwind while traveling. Commenters on the Internet claimed, often angrily, that this was physically impossible and that the Blackbird was a hoax. Some commenters also claimed that the upwind case was physically impossible.

a. Consider first the **downwind faster than the wind** case.

- Is the motion actually possible as claimed? If not, offer a brief explanation!
- If the motion is possible, is power transferred from the propeller to the wheels or vice versa?
- If the motion is possible, what ground speed is attained? For this question, suppose that when transferring power in either direction between the propeller and the wheels, a fraction α of the useful work is lost; let the wind speed be v_w. Neglect all other losses of energy.

b. Answer the previous questions for the **upwind** case.

Question B2

This problem concerns three situations involving the transfer of energy into a region of space by electromagnetic fields. In the first case, that energy is stored in the kinetic energy of a charged object; in the second and third cases, the energy is stored in an electric or magnetic field.

In general, whenever an electric and a magnetic field are at an angle to each other, energy is transferred; for example, this principle is the reason electromagnetic radiation transfers energy. The power transferred per unit area is given by the *Poynting vector*:

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

In each part of this problem, the last subpart asks you to verify that the rate of energy transfer agrees with the formula for the Poynting vector. Therefore, you should not use the formula for the Poynting vector before the last subpart!

a. A long, insulating cylindrical rod has radius R and carries a uniform volume charge density ρ. A uniform external electric field E exists in the direction of its axis. The rod moves in the direction of its axis at speed v.

   i. What is the power per unit length $\mathcal{P}$ delivered to the rod?
   ii. What is the magnetic field B at the surface of the rod? Draw the direction on a diagram.
   iii. Compute the Poynting vector, draw its direction on a diagram, and verify that it agrees with the rate of energy transfer.

b. A parallel plate capacitor consists of two discs of radius R separated by a distance $d \ll R$. The capacitor carries charge Q, and is being charged by a small, constant current I.

   i. What is the power P delivered to the capacitor?
   ii. What is the magnetic field B just inside the edge of the capacitor? Draw the direction on a diagram. (Ignore fringing effects in the electric field for this calculation.)
   iii. Compute the Poynting vector, draw its direction on a diagram, and verify that it agrees with the rate of energy transfer.

c. A long solenoid of radius R has $\mathcal{N}$ turns of wire per unit length. The solenoid carries current I, and this current is increased at a small, constant rate $\frac{dI}{dt}$.

   i. What is the power per unit length $\mathcal{P}$ delivered to the solenoid?
   ii. What is the electric field E just inside the surface of the solenoid? Draw its direction on a diagram.
   iii. Compute the Poynting vector, draw its direction on a diagram, and verify that it agrees with the rate of energy transfer.