

## USA Physics Olympiad Exam

DO NOT DISTRIBUTE THIS PAGE

### Important Instructions for the Exam Supervisor

- This examination consists of two parts.
- Part A has four questions and is allowed 90 minutes.
- Part B has two questions and is allowed 90 minutes.
- The first page that follows is a cover sheet. Examinees may keep the cover sheet for both parts of the exam.
- The parts are then identified by the center header on each page. Examinees are only allowed to do one part at a time, and may not work on other parts, even if they have time remaining.
- Allow 90 minutes to complete Part A. Do not let students look at Part B. Collect the answers to Part A before allowing the examinee to begin Part B. Examinees are allowed a 10 to 15 minutes break between parts A and B.
- Allow 90 minutes to complete Part B. Do not let students go back to Part A.
- Ideally the test supervisor will divide the question paper into 4 parts: the cover sheet (page 2), Part A (pages 3-7), Part B (pages 9-11), and several answer sheets for one of the questions in Part A (pages 13-13). Examinees should be provided parts A and B individually, although they may keep the cover sheet. The answer sheets should be printed single sided!
- The supervisor *must* collect all examination questions, including the cover sheet, at the end of the exam, as well as any scratch paper used by the examinees. Examinees may *not* take the exam questions. The examination questions may be returned to the students after April 15, 2015.
- Examinees are allowed calculators, but they may not use symbolic math, programming, or graphic features of these calculators. Calculators may not be shared and their memory must be cleared of data and programs. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. Examinees may not use any tables, books, or collections of formulas.



## USA Physics Olympiad Exam

### INSTRUCTIONS

#### DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems. Each question is worth 25 points. Do not look at Part B during this time.
- After you have completed Part A you may take a break.
- Then work Part B. You have 90 minutes to complete both problems. Each question is worth 50 points. Do not look at Part A during this time.
- Show all your work. Partial credit will be given. Do not write on the back of any page. Do not write anything that you wish graded on the question sheets.
- Start each question on a new sheet of paper. Put your AAPT ID number, your name, the question number and the page number/total pages for this problem, in the upper right hand corner of each page. For example,

AAPT ID #

Doe, Jamie

A1 - 1/3

- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers/solutions) on this contest until after April 15, 2015.**

**Possibly Useful Information. You may use this sheet for both parts of the exam.**

$$g = 9.8 \text{ N/kg}$$

$$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

$$c = 3.00 \times 10^8 \text{ m/s}$$

$$N_A = 6.02 \times 10^{23} \text{ (mol)}^{-1}$$

$$\sigma = 5.67 \times 10^{-8} \text{ J}/(\text{s} \cdot \text{m}^2 \cdot \text{K}^4)$$

$$1\text{eV} = 1.602 \times 10^{-19} \text{ J}$$

$$m_e = 9.109 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV}/c^2$$

$$\sin \theta \approx \theta - \frac{1}{6}\theta^3 \text{ for } |\theta| \ll 1$$

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$

$$k_m = \mu_0/4\pi = 10^{-7} \text{ T} \cdot \text{m/A}$$

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

$$R = N_A k_B = 8.31 \text{ J}/(\text{mol} \cdot \text{K})$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$(1+x)^n \approx 1+nx \text{ for } |x| \ll 1$$

$$\cos \theta \approx 1 - \frac{1}{2}\theta^2 \text{ for } |\theta| \ll 1$$

## Part A

### Question A1

Consider a particle of mass  $m$  that elastically bounces off of an infinitely hard horizontal surface under the influence of gravity. The total mechanical energy of the particle is  $E$  and the acceleration of free fall is  $g$ . Treat the particle as a point mass and assume the motion is non-relativistic.

- a. An estimate for the regime where quantum effects become important can be found by simply considering when the deBroglie wavelength of the particle is on the same order as the height of a bounce. Assuming that the deBroglie wavelength is defined by the maximum momentum of the bouncing particle, determine the value of the energy  $E_q$  where quantum effects become important. Write your answer in terms of some or all of  $g$ ,  $m$ , and Planck's constant  $h$ .
- b. A second approach allows us to develop an estimate for the actual allowed energy levels of a bouncing particle. Assuming that the particle rises to a height  $H$ , we can write

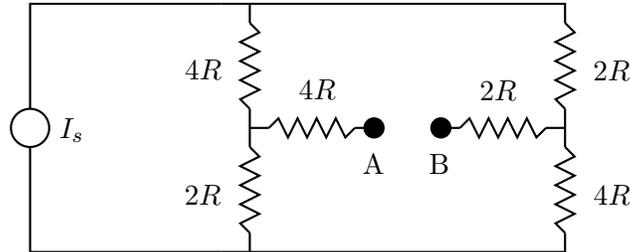
$$2 \int_0^H p \, dx = \left( n + \frac{1}{2} \right) h$$

where  $p$  is the momentum as a function of height  $x$  above the ground,  $n$  is a non-negative integer, and  $h$  is Planck's constant.

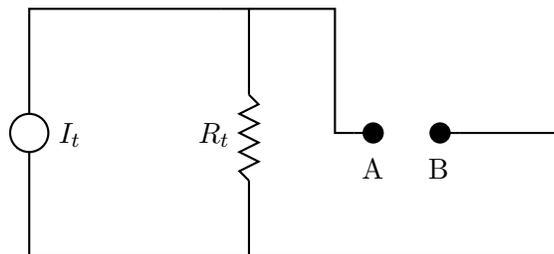
- i. Determine the allowed energies  $E_n$  as a function of the integer  $n$ , and some or all of  $g$ ,  $m$ , and Planck's constant  $h$ .
  - ii. Numerically determine the minimum energy of a bouncing neutron. The mass of a neutron is  $m_n = 1.675 \times 10^{-27} \text{ kg} = 940 \text{ MeV}/c^2$ ; you may express your answer in either Joules or eV.
  - iii. Determine the bounce height of one of these minimum energy neutrons.
- c. Let  $E_0$  be the minimum energy of the bouncing neutron and  $f$  be the frequency of the bounce. Determine an order of magnitude estimate for the ratio  $E/f$ . It only needs to be accurate to within an order of magnitude or so, but you do need to show work!

### Question A2

Consider the circuit shown below.  $I_s$  is a constant current source, meaning that no matter what device is connected between points A and B, the current provided by the constant current source is the same.



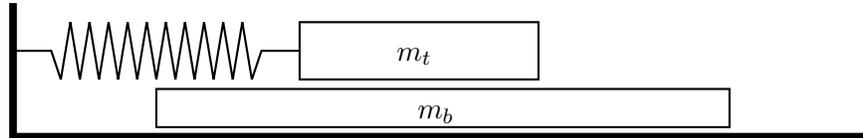
- Connect an ideal voltmeter between  $A$  and  $B$ . Determine the voltage reading in terms of any or all of  $R$  and  $I_s$ .
- Connect instead an ideal ammeter between  $A$  and  $B$ . Determine the current in terms of any or all of  $R$  and  $I_s$ .
- It turns out that it is possible to replace the above circuit with a new circuit as follows:



From the point of view of *any* passive resistance that is connected between  $A$  and  $B$  the circuits are identical. You don't need to prove this statement, but you do need to find  $I_t$  and  $R_t$  in terms of any or all of  $R$  and  $I_s$ .

### Question A3

A large block of mass  $m_b$  is located on a horizontal frictionless surface. A second block of mass  $m_t$  is located on top of the first block; the coefficient of friction (both static and kinetic) between the two blocks is given by  $\mu$ . All surfaces are horizontal; all motion is effectively one dimensional. A spring with spring constant  $k$  is connected to the top block only; the spring obeys Hooke's Law equally in both extension and compression. Assume that the top block never falls off of the bottom block; you may assume that the bottom block is very, very long. The top block is moved a distance  $A$  away from the equilibrium position and then released from rest.



- Depending on the value of  $A$ , the motion can be divided into two types: motion that experiences no frictional energy losses and motion that does. Find the value  $A_c$  that divides the two motion types. Write your answer in terms of any or all of  $\mu$ , the acceleration of gravity  $g$ , the masses  $m_t$  and  $m_b$ , and the spring constant  $k$ .
- Consider now the scenario  $A \gg A_c$ . In this scenario the amplitude of the oscillation of the top block as measured against the original equilibrium position will change with time. Determine the magnitude of the change in amplitude,  $\Delta A$ , after one complete oscillation, as a function of any or all of  $A$ ,  $\mu$ ,  $g$ , and the angular frequency of oscillation of the top block  $\omega_t$ .
- Assume still that  $A \gg A_c$ . What is the maximum speed of the bottom block during the first complete oscillation cycle of the upper block?

**Question A4**

A heat engine consists of a moveable piston in a vertical cylinder. The piston is held in place by a removable weight placed on top of the piston, but piston stops prevent the piston from sinking below a certain point. The mass of the piston is  $m = 40.0$  kg, the cross sectional area of the piston is  $A = 100$  cm<sup>2</sup>, and the weight placed on the piston has a mass of  $m = 120.0$  kg.

Assume that the region around the cylinder and piston is a vacuum, so you don't need to worry about external atmospheric pressure.

- At point **A** the cylinder volume  $V_0$  is completely filled with liquid water at a temperature  $T_0 = 320$  K and a pressure  $P_{\min}$  that would be just sufficient to lift the piston alone, except the piston has the additional weight placed on top.
- Heat energy is added to the water by placing the entire cylinder in a hot bath.
- At point **B** the piston and weight begins to rise.
- At point **C** the volume of the cylinder reaches  $V_{\max}$  and the temperature reaches  $T_{\max}$ . The heat source is removed; the piston stops rising and is locked in place.
- Heat energy is now removed from the water by placing the entire cylinder in a cold bath.
- At point **D** the pressure in the cylinder returns to  $P_{\min}$ . The added weight is removed; the piston is unlocked and begins to move down.
- The cylinder volume returns to  $V_0$ . The cylinder is removed from the cold bath, the weight is placed back on top of the piston, and the cycle repeats.

Because the liquid water can change to gas, there are several important events that take place

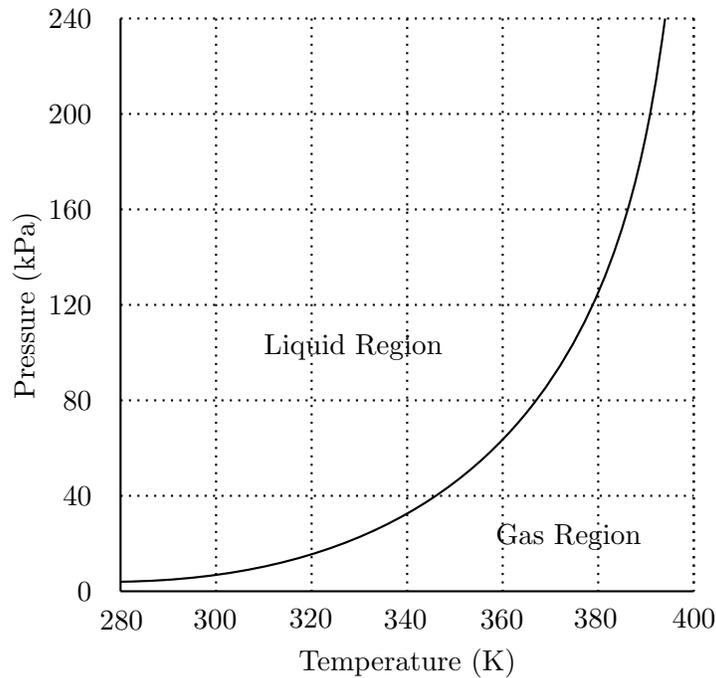
- At point **W** the liquid begins changing to gas.
- At point **X** all of the liquid has changed to gas. This occurs at the same point as point **C** described above.
- At point **Y** the gas begins to change back into liquid.
- At point **Z** all of the gas has changed back into liquid.

When in the liquid state you need to know that for water kept at constant volume, a change in temperature  $\Delta T$  is related to a change in pressure  $\Delta P$  according to

$$\Delta P \approx (10^6 \text{ Pa/K})\Delta T$$

When in the gas state you should assume that water behaves like an ideal gas.

Of relevance to this question is the pressure/temperature phase plot for water, showing the regions where water exists in liquid form or gaseous form. The curve shows the coexistence condition, where water can exist simultaneously as gas or liquid.



The following graphs should be drawn on the answer sheet provided.

- Sketch a PT diagram for this cycle on the answer sheet. The coexistence curve for the liquid/gas state is shown. Clearly and accurately label the locations of points **B** through **D** and **W** through **Z** on this cycle.
- Sketch a PV diagram for this cycle on the answer sheet. You should estimate a reasonable value for  $V_{\max}$ , note the scale is logarithmic. Clearly and accurately label the locations of points **B** through **D** on this cycle. Provide reasonable approximate locations for points **W** through **Z** on this cycle.

# **STOP: Do Not Continue to Part B**

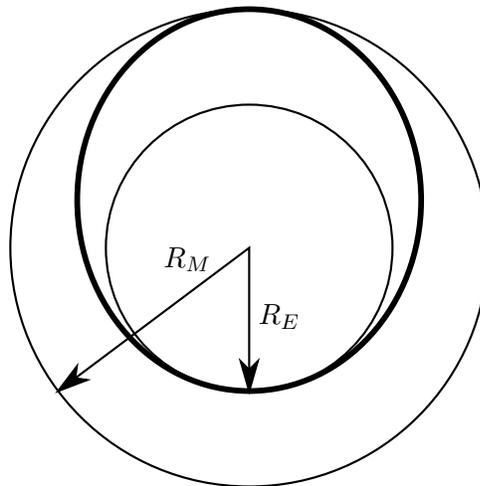
If there is still time remaining for Part A, you should review your work for Part A, but do not continue to Part B until instructed by your exam supervisor.

## Part B

### Question B1

This problem is divided into three parts. It is possible to solve these three parts independently, but they are not equally weighted.

- a. An ideal rocket when empty of fuel has a mass  $m_r$  and will carry a mass of fuel  $m_f$ . The fuel burns and is ejected with an exhaust speed of  $v_e$  relative to the rocket. The fuel burns at a constant mass rate for a total time  $T_b$ . Ignore gravity; assume the rocket is far from any other body.
  - i. Determine an equation for the acceleration of the rocket as a function of time  $t$  in terms of any or all of  $t$ ,  $m_f$ ,  $m_r$ ,  $v_e$ ,  $T_b$ , and any relevant fundamental constants.
  - ii. Assuming that the rocket starts from rest, determine the final speed of the rocket in terms of any or all of  $m_r$ ,  $m_f$ ,  $v_e$ ,  $T_b$ , and any relevant fundamental constants.
  
- b. The ship starts out in a circular orbit around the sun very near the Earth and has a goal of moving to a circular orbit around the Sun that is very close to Mars. It will make this transfer in an elliptical orbit as shown in bold in the diagram below. This is accomplished with an initial velocity boost near the Earth  $\Delta v_1$  and then a second velocity boost near Mars  $\Delta v_2$ . Assume that both of these boosts are from instantaneous impulses, and ignore mass changes in the rocket as well as gravitational attraction to either Earth or Mars. Don't ignore the Sun! Assume that the Earth and Mars are both in circular orbits around the Sun of radii  $R_E$  and  $R_M = R_E/\alpha$  respectively. The orbital speeds are  $v_E$  and  $v_M$  respectively.

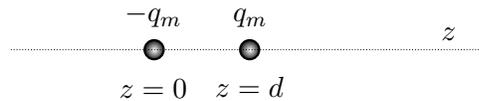


- i. Derive an expression for the velocity boost  $\Delta v_1$  to change the orbit from circular to elliptical. Express your answer in terms of  $v_E$  and  $\alpha$ .
- ii. Derive an expression for the velocity boost  $\Delta v_2$  to change the orbit from elliptical to circular. Express your answer in terms of  $v_E$  and  $\alpha$ .
- iii. What is the angular separation between Earth and Mars, as measured from the Sun, at the time of launch so that the rocket will start from Earth and arrive at Mars when it reaches the orbit of Mars? Express your answer in terms of  $\alpha$ .

## Question B2

The nature of magnetic dipoles.

- a. A “Gilbert” dipole consists of a pair of magnetic monopoles each with a magnitude  $q_m$  but opposite magnetic charges separated by a distance  $d$ , where  $d$  is small. In this case, assume that  $-q_m$  is located at  $z = 0$  and  $+q_m$  is located at  $z = d$ .



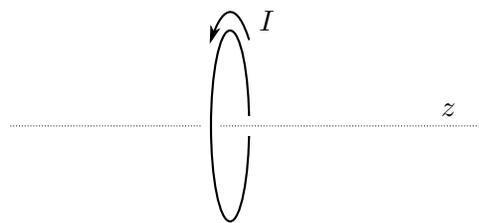
Assume that magnetic monopoles behave like electric monopoles according to a coulomb-like force

$$F = \frac{\mu_0}{4\pi} \frac{q_{m1}q_{m2}}{r^2}$$

and the magnetic field obeys

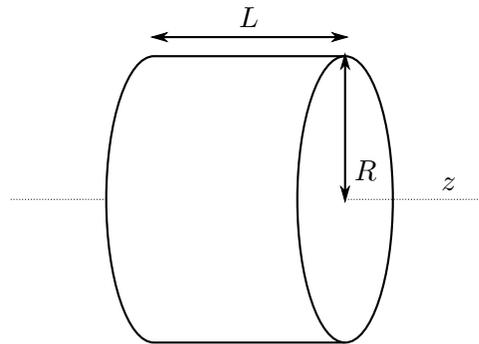
$$B = F/q_m.$$

- i. What are the dimensions of the quantity  $q_m$ ?
  - ii. Write an exact expression for the magnetic field strength  $B(z)$  along the  $z$  axis as a function of  $z$  for  $z > d$ . Write your answer in terms of  $q_m$ ,  $d$ ,  $z$ , and any necessary fundamental constants.
  - iii. Evaluate this expression in the limit as  $d \rightarrow 0$ , assuming that the product  $q_m d = p_m$  is kept constant, keeping only the lowest non-zero term. Write your answer in terms of  $p_m$ ,  $z$ , and any necessary fundamental constants.
- b. An “Ampère” dipole is a magnetic dipole produced by a current loop  $I$  around a circle of radius  $r$ , where  $r$  is small. Assume that the  $z$  axis is the axis of rotational symmetry for the circular loop, and the loop lies in the  $xy$  plane at  $z = 0$ .

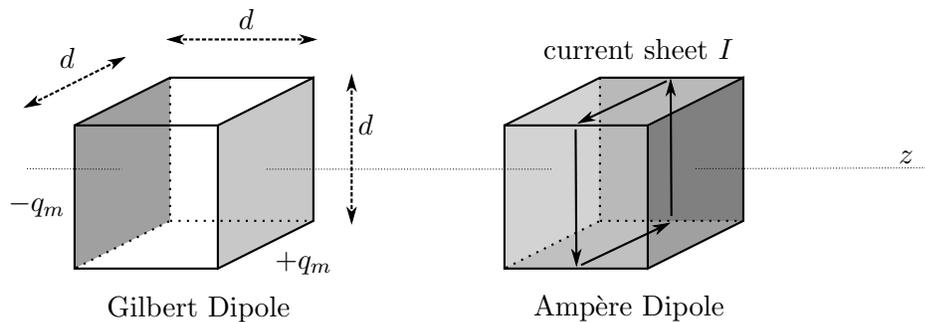


- i. Write an exact expression for the magnetic field strength  $B(z)$  along the  $z$  axis as a function of  $z$  for  $z > 0$ . Write your answer in terms of  $I$ ,  $r$ ,  $z$ , and any necessary fundamental constants.
- ii. Let  $kIr^\gamma$  have dimensions equal to that of the quantity  $p_m$  defined above in Part aiii, where  $k$  and  $\gamma$  are dimensionless constants. Determine the value of  $\gamma$ .
- iii. Evaluate the expression in Part bi in the limit as  $r \rightarrow 0$ , assuming that the product  $kIr^\gamma = p'_m$  is kept constant, keeping only the lowest non-zero term. Write your answer in terms of  $k$ ,  $p'_m$ ,  $z$ , and any necessary fundamental constants.
- iv. Assuming that the two approaches are equivalent,  $p_m = p'_m$ . Determine the constant  $k$  in Part bii.

- c. Now we try to compare the two approaches if we model a physical magnet as being composed of densely packed microscopic dipoles.



A cylinder of this uniform magnetic material has a radius  $R$  and a length  $L$ . It is composed of  $N$  magnetic dipoles that could be either all Ampère type or all Gilbert type.  $N$  is a *very* large number. The axis of rotation of the cylinder and all of the dipoles are all aligned with the  $z$  axis and all point in the same direction as defined above so that the magnetic field *outside* the cylinder is the same in either dipole case as you previously determined. Below is a picture of the two dipole models; they are cubes of side  $d \ll R$  and  $d \ll L$  with volume  $v_m = d^3$ .



- i. Assume that  $R \gg L$  and only Gilbert type dipoles, determine the magnitude and direction of  $B$  at the center of the cylinder in terms of any or all of  $p_m$ ,  $R$ ,  $L$ ,  $v_m$ , and any necessary fundamental constants.
- ii. Assume that  $R \ll L$  and only Ampère type dipoles, determine the magnitude and direction of  $B$  at the center of the cylinder in terms of any or all of  $p_m$ ,  $R$ ,  $L$ ,  $v_m$ , and any necessary fundamental constants.

# Answer Sheets

Following are answer sheets for some of the graphical portions of the test.

