AAPT UNITED STATES PHYSICS TEAM
AIP 2014

2014 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers will result in a deduction of $\frac{1}{4}$ point. There is no penalty for leaving an answer blank.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2014.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

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Contributors to this year's exam include David Fallest, David Jones, Jiajia Dong, Paul Stanley, Warren Turner, Qiuzi Li, and former US Team members Andrew Lin, Matthew Huang, Samuel Zbarsky.

1. A car turning to the right is traveling at constant speed in a circle. From the driver's perspective, the angular momentum vector about the center of the circle points X and the acceleration vector of the car points Y where

   (A) X is left, Y is left.

   (B) X is forward, Y is right.

   (C) X is down, Y is forward.

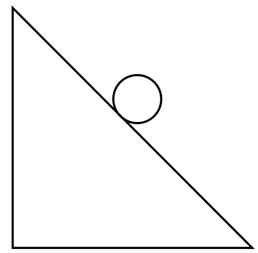
   (D) X is left, Y is right.

   (E) X is down, Y is right. ← **CORRECT**

Solution

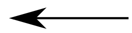
Use right hand rule to find angular momentum, and acceleration is toward center in uniform circular motion.

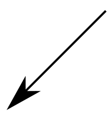
2. A ball rolls without slipping down an inclined plane as shown in the diagram.

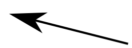


Which of the following vectors best represents the direction of the total force that the ball exerts on the plane?

(A) 

(B) 

(C) 

(D) 

(E) 

CORRECT answer is (E)

Solution

Normal force into plane. Friction must be less than the parallel force, or it wouldn't slide down incline! Correct answer is into plane and down plane; note that if the ball is rolling downward and accelerating, friction must be less than the parallel component of gravity.

3. An object of uniform density floats partially submerged so that 20% of the object is above the water. A 3 N force presses down on the top of the object so that the object becomes fully submerged. What is the volume of the object? The density of water is $\rho_{\mathrm{H_2O}} = 1000\ \mathrm{kg/m^3}$.

   (A) $V_{object} = 0.3$ L
   (B) $V_{object} = 0.67$ L
   (C) $V_{object} = 1.2$ L
   (D) $V_{object} = 1.5$ L ← **CORRECT**
   (E) $V_{object} = 3.0$ L

Solution

Submerging object requires pushing against the additional buoyant force of $0.2\rho_w V g$, so $V = 1.5$ L.

4. What are the correct values of the numbers in the following statements? Assume there are no external forces, and take $N = 1$ to mean that the statement cannot be made for any meaningful number of particles.

   - If a particle at rest explodes into N_1 or fewer particles with known masses, and the total kinetic energy of the new particles is known, the kinetic energy of each of the new particles is completely determined.
   - If a particle at rest explodes into N_2 or fewer particles, the velocities of the new particles must lie in a line.
   - If a particle at rest explodes into N_3 or fewer particles, the velocities of the new particles must lie in a plane.

   (A) $N_1 = 2$, $N_2 = 1$, $N_3 = 1$

   (B) $N_1 = 1$, $N_2 = 2$, $N_3 = 3$

   (C) $N_1 = 2$, $N_2 = 2$, $N_3 = 3$ ← **CORRECT**

   (D) $N_1 = 3$, $N_2 = 2$, $N_3 = 3$

   (E) $N_1 = 2$, $N_2 = 3$, $N_3 = 4$

Solution

Certainly N_2 is at most 2 and N_3 is at most 3; it is certainly possible for three particles to emerge with non-collinear velocities, or four to emerge with non-planar ones. (Consider for example the case where all of the particles have equal mass and they emerge at the corners of a triangle or tetrahedron.) However, note that the total momentum of the daughter particles must be zero; it is impossible for two non-collinear vectors to sum to zero, nor three non-coplanar vectors. Thus $N_2 = 2$ and $N_3 = 3$.

Meanwhile, N_1 is at most 2; consider three identical particles, where (among many other possibilities) any one could emerge at rest with the energy split equally among the other two. In the case of two particles, the velocities must be in opposite direction and (as argued) in a straight line; thus, up to overall direction, they are entirely determined by the kinetic energies. There are two independent equations governing the kinetic energies; the total momentum must be zero, and the total kinetic energy is fixed. Thus the kinetic energies are fully determined in this case, and $N_1 = 2$.

5. A unicyclist goes around a circular track of radius 30 m at a (*amazingly fast!*) constant speed of 10 m/s. At what angle to the left (or right) of vertical must the unicyclist lean to avoid falling? Assume that the height of the unicyclist is much smaller than the radius of the track.

   (A) 9.46°

   (B) 9.59°

   (C) 18.4° ← **CORRECT**

   (D) 19.5°

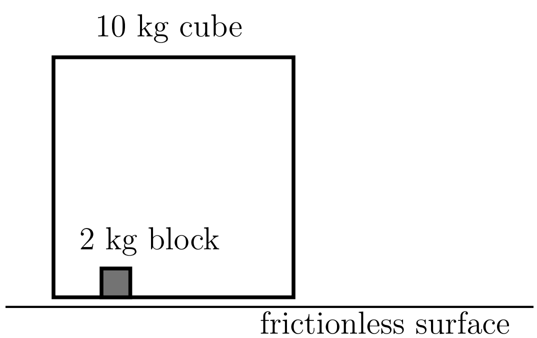
   (E) 70.5°

Solution

Force toward center is mv^2/r. Force toward ground is mg. The force toward the center is from friction and acts at the point in contact with the ground. The normal force is equal I'm magnitude to the force of gravity and acts upward at the point of contact. We are then interested in the tangent angle as defined by

$$\tan\theta = \frac{v^2}{gr}$$

6. A cubical box of mass 10 kg with edge length 5 m is free to move on a frictionless horizontal surface. Inside is a small block of mass 2 kg, which moves without friction inside the box. At time $t = 0$, the block is moving with velocity 5 m/s directly towards one of the faces of the box, while the box is initially at rest. The coefficient of restitution for any collision between the block and box is 90%, meaning that the relative speed between the box and block immediately after a collision is 90% of the relative speed between the box and block immediately before the collision.



After 1 minute, the block is a displacement x from the original position. Which of the following is closest to x?

(A) 0 m
(B) 50 m ← **CORRECT**
(C) 100 m
(D) 200 m
(E) 300 m

Solution

The easiest way to consider this is the motion of the center of mass of the system, which, by conservation of momentum, must move with a speed of

$$v_{\text{cm}} = \frac{(2\text{ kg})(5\text{ m/s})}{(2\text{ kg}) + (10\text{ kg})} = \frac{5}{6}\text{ m/s}.$$

After one minute the center of mass has moved $(\frac{5}{6}\text{ m/s})(60\text{ s}) = 50\text{ m}$.

Now the center of mass must be bounded by the cubical box, so the position of the inside block must be somewhere near the 50 meter mark.

7. A 1.00 m long stick with uniform density is allowed to rotate about a point 30.0 cm from its end. The stick is perfectly balanced when a 50.0 g mass is placed on the stick 20.0 cm from the same end. What is the mass of the stick?

   (A) 35.7 g

   (B) 33.3 g

   (C) 25.0 g ← **CORRECT**

   (D) 17.5 g

   (E) 14.3 g

8. An object of mass M is hung on a vertical spring of spring constant k and is set into vertical oscillations. The period of this oscillation is T_0. The spring is then cut in half and the same mass is attached and the system is set up to oscillate on a frictionless inclined plane making an angle θ to the horizontal. Determine the period of the oscillations on the inclined plane in terms of T_0.

   (A) T_0

   (B) $T_0/2$

   (C) $2T_0 \sin\theta$

   (D) $T_0/\sqrt{2}$ ← **CORRECT**

   (E) $T_0 \sin\theta/\sqrt{2}$

Solution

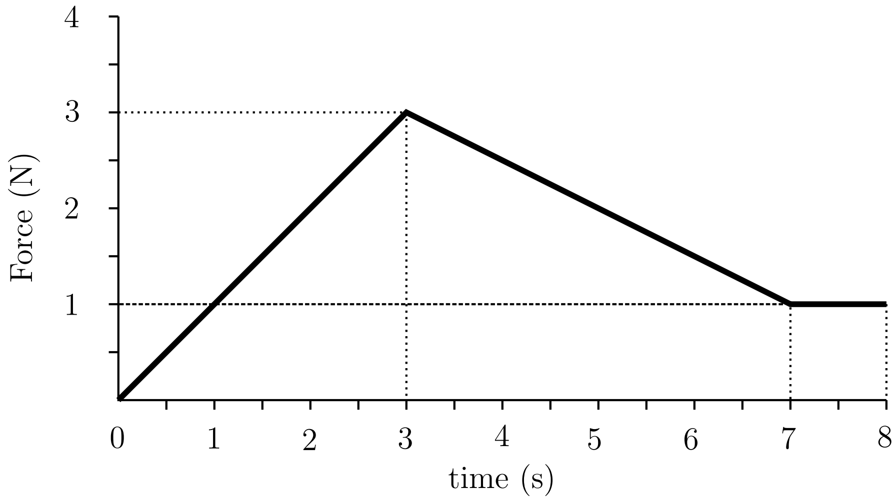
The spring constant of a spring is inversely proportional to the length of the un-stretched spring; cutting a spring in half will double the spring constant.

Placing the system on an incline will change the equilibrium position of the mass, but will not affect the period of oscillation in any way.

As such, the new period of oscillation is given by

$$T = 2\pi\sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}}2\pi\sqrt{\frac{m}{k}} = \frac{T_0}{\sqrt{2}}$$

9. A 5.0 kg object undergoes a time-varying force as shown in the graph below. If the velocity at $t = 0.0$ s is $+1.0$ m/s, what is the velocity of the object at $t = 7$ s?



(A) 2.45 m/s
(B) 2.50 m/s
(C) 3.50 m/s ← **CORRECT**
(D) 12.5 m/s
(E) 15.0 m/s

Solution

The area under the graph is a measure of the change in momentum, so

$$\Delta p = \frac{1}{2}(3\ \mathrm{s})(3\ \mathrm{N}) + \frac{1}{2}(4\ \mathrm{s})(3\ \mathrm{N} + 1\ \mathrm{N}) = 12.5\ \mathrm{N} \cdot \mathrm{s}.$$

The additional velocity is then $\Delta v = 2.5$ m/s.

10. A radio controlled car is attached to a stake in the ground by a 3.00 m long piece of string, and is forced to move in a circular path. The car has an initial angular velocity of 1.00 rad/s and smoothly accelerates at a rate of 4.00 rad/s^2. The string will break if the centripetal acceleration exceeds 2.43×10^2 m/s^2. How long can the car accelerate at this rate before the string breaks?

(A) 0.25 s

(B) 0.50 s

(C) 1.00 s

(D) 1.50 s

(E) 2.00 s ← **CORRECT**

Solution

The string breaks if a_c exceeds the maximum tension, or

$$a_c = r\omega^2 = r\left(\frac{1}{2}\alpha t^2 + \omega_0\right)^2$$

Solve for the time,

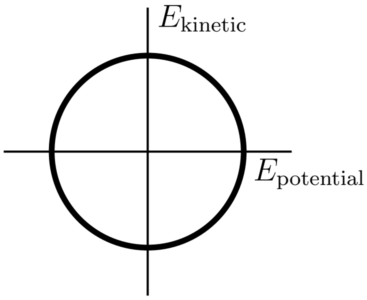
$$t = \sqrt{\frac{2}{\alpha}\left(\sqrt{\frac{a_c}{r}} - \omega_0\right)}$$

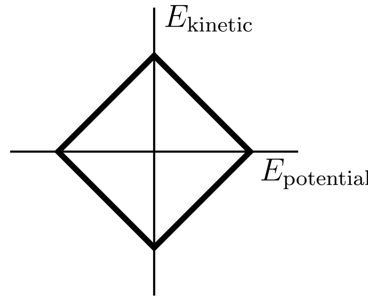
or

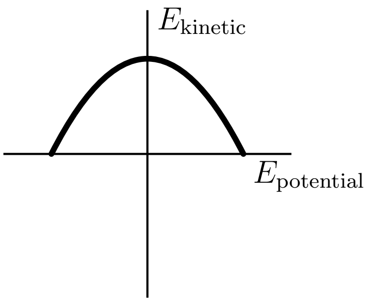
$$t = 2\text{s}$$

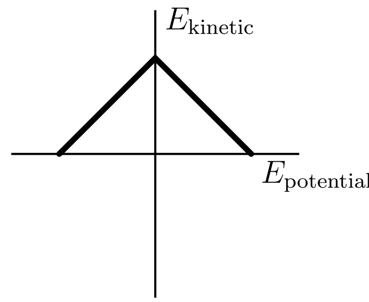
11. A point mass m is connected to an ideal spring on a horizontal frictionless surface. The mass is pulled a short distance and then released.

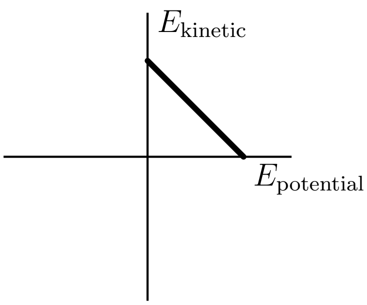
Which of the following is the most correct plot of the kinetic energy as a function of potential energy?

(A) 

(B) 

(C) 

(D) 

(E) 

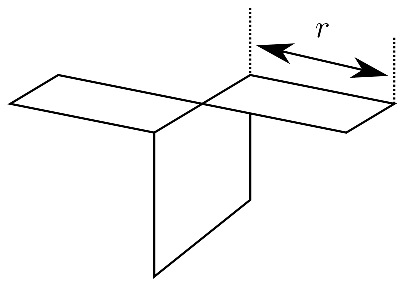
← **CORRECT**

Solution

Energy is conserved, so the total must be constant. So it is the descending diagonal line of slope magnitude one.

The following information applies to questions 12 and 13

A paper helicopter with rotor radius r and weight W is dropped from a height h in air with a density of ρ.



Assuming that the helicopter quickly reaches terminal velocity, a function for the time of flight T can be found in the form

$$T = kh^{\alpha}r^{\beta}\rho^{\delta}W^{\omega}.$$

where k is an unknown dimensionless constant (actually, 1.164). α, β, δ, and ω are constant exponents to be determined.

12. Determine α.

(A) $\alpha = -1$

(B) $\alpha = -1/2$

(C) $\alpha = 0$

(D) $\alpha = 1/2$

(E) $\alpha = 1$ ← **CORRECT**

Solution

If the helicopter is at terminal velocity, then it is falling at constant speed. As such, $h = vT$, where v is the terminal velocity. In that case, $\alpha = 1$.

13. Determine β.

(A) $\beta = 1/3$

(B) $\beta = 1/2$

(C) $\beta = 2/3$

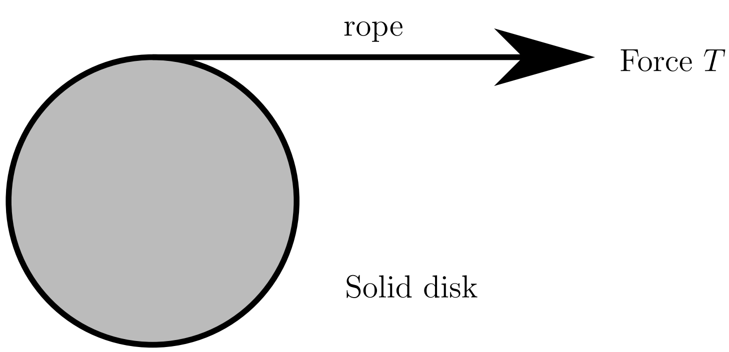
(D) $\beta = 1$ ← **CORRECT**

(E) β can not be uniquely determined without more information.

Solution

Dimensional analysis on mass requires that $\delta = -\omega$. SInce only W has units of time (inverse squared), then $\omega = -1/2$. But $\sqrt{\rho/W}$ has units of length^{-2} time^{-1}, and we know $\alpha = 1$, then $\beta = 1$.

14. A disk of moment of inertia I, mass M, and radius R has a cord wrapped around it tightly as shown in the diagram. The disk is free to slide on its side as shown in the top down view. A constant force of T is applied to the end of the cord and accelerates the disk along a frictionless surface.



After the disk has accelerated some distance, determine the ratio of the translational KE to total KE of the disk,

$$KE_{\text{translational}}/KE_{\text{total}} =$$

(A) $\frac{I}{MR^2}$

(B) $\frac{MR^2}{I}$

(C) $\frac{I}{3MR^2}$

(D) $\frac{I}{MR^2+I}$ ← **CORRECT**

(E) $\frac{MR^2}{MR^2+I}$

Solution

The applied force T accelerates the center of mass of the disk at a rate $a = T/M$. The applied force produces a torque on the disk about the center of mass given by $\tau = RT$, and therefore an angular acceleration of $\alpha = RT/I$.

After a time t the velocity of the disk will be $v = at$ and the angular velocity will be $\omega = \alpha t$. Then

$$KE_{\text{translational}} = \frac{1}{2}M\left(\frac{T}{M}\right)^2 t^2,$$

and

$$KE_{\text{rotational}} = \frac{1}{2}I\left(\frac{RT}{I}\right)^2 t^2,$$

so the desired ratio is then

$$\begin{aligned}\frac{KE_{\text{translational}}}{KE_{\text{total}}} &= \frac{\frac{1}{2}M\left(\frac{T}{M}\right)^2 t^2}{\frac{1}{2}M\left(\frac{T}{M}\right)^2 t^2 + \frac{1}{2}I\left(\frac{RT}{I}\right)^2 t^2},\\ &= \frac{\frac{1}{M}}{\frac{1}{M} + \frac{R^2}{I}},\\ &= \frac{I}{I + MR^2}.\end{aligned}$$

15. The maximum torque output from the engine of a new experimental car of mass m is τ. The maximum rotational speed of the engine is ω. The engine is designed to provide a constant power output P. The engine is connected to the wheels via a perfect transmission that can smoothly trade torque for speed with no power loss. The wheels have a radius R, and the coefficient of static friction between the wheels and the road is μ.

What is the maximum sustained speed v the car can drive up a 30 degree incline? Assume no frictional losses and assume μ is large enough so that the tires do not slip.

(A) $v = 2P/(mg)$ ← **CORRECT**
(B) $v = 2P/(\sqrt{3}mg)$
(C) $v = 2P/(\mu mg)$
(D) $v = \tau\omega/(mg)$
(E) $v = \tau\omega/(\mu mg)$

Solution

The fundamental idea is $P = Fv$ where F is the component of the weight parallel to the incline. Then

$$v = P/mg\sin\theta$$

Since $\theta = 30^\circ$, the answer is $v = 2P/mg$

16. An object of mass m_1 initially moving at speed v_0 collides with an object of mass $m_2 = \alpha m_1$, where $\alpha < 1$, that is initially at rest. The collision could be completely elastic, completely inelastic, or partially inelastic. After the collision the two objects move at speeds v_1 and v_2. Assume that the collision is one dimensional, and that object one cannot pass through object two.

After the collision, the speed ratio $r_1 = v_1/v_0$ of object 1 is bounded by

(A) $(1-\alpha)/(1+\alpha) \le r_1 \le 1$

(B) $(1-\alpha)/(1+\alpha) \le r_1 \le 1/(1+\alpha)$ ← **CORRECT**

(C) $\alpha/(1+\alpha) \le r_1 \le 1$

(D) $0 \le r_1 \le 2\alpha/(1+\alpha)$

(E) $1/(1+\alpha) \le r_1 \le 2/(1+\alpha)$

Solution

Conserving momentum and kinetic energy in the completely elastic collision yields the following quantities.

$$r_1 = \frac{v_1}{v_0} = \frac{1-\alpha}{(1+\alpha)},$$

$$r_2 = \frac{v_2}{v_0} = \frac{2}{(1+\alpha)},$$

Since $\alpha < 1$, object 1 is tricking a less massive object, and therefore continues to move forward.

Conserving momentum in a completely inelastic collision yields

$$r_1 = r_2 = \frac{1}{(1+\alpha)},$$

Note that object 2 will always be moving forward, and since object 1 can't pass through it, object 2 must always move with a more positive (or equal) velocity than object 1. Consequently,

$$1/(1+\alpha) \le r_2 \le 2/(1+\alpha)$$

Sorting out object 1 is a little harder, since under certain circumstances it can bounce backward. But in this case, since $\alpha < 1$, it retains a forward velocity after the collision. Then

$$(1-\alpha)/(1+\alpha) \le r_1 \le 1/(1+\alpha)$$

17. A spherical cloud of dust in space has a uniform density ρ_0 and a radius R_0. The gravitational acceleration of free fall at the surface of the cloud due to the mass of the cloud is g_0.

A process occurs (heat expansion) that causes the cloud to suddenly grow to a radius $2R_0$, while maintaining a uniform (but not constant) density. The gravitational acceleration of free fall at a point R_0 away from the center of the cloud due to the mass of the cloud is now

(A) $g_0/32$

(B) $g_0/16$

(C) $g_0/8$ ← **CORRECT**

(D) $g_0/4$

(E) $g_0/2$

Solution

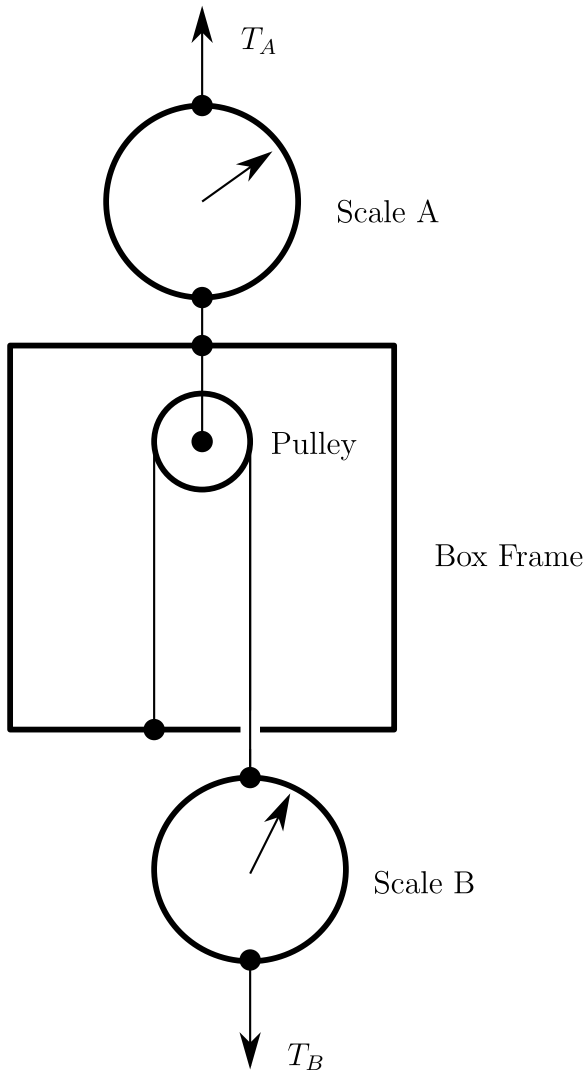
Newton's Law of gravitation for a spherical object depends only on matter located closer to the center than the point of reference.

$$g = \frac{GM}{r^2} = \frac{4\pi}{3} G\rho r,$$

where the last equality is true for uniform density.

Doubling the size cuts the density to 1/8.

18. Consider the following diagram of a box and two weight scales. Scale A supports the box via a massless rope. A pulley is attached to the top of the box; a second massless rope passes over the pulley, one end is attached to the box and the other end to scale B. The two scales read indicate the tensions T_A and T_B in the ropes. Originally scale A reads 30 Newtons and scale B reads 20 Newtons.



If an additional force pulls down on scale B so that the reading increases to 30 Newtons, what will be the new reading on scale A?

(A) 35 Newtons
(B) 40 Newtons ← **CORRECT**
(C) 45 Newtons
(D) 50 Newtons
(E) 60 Newtons

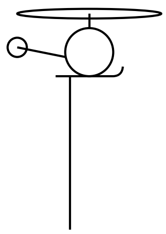
Adapted from a demonstration by Richard Berg.

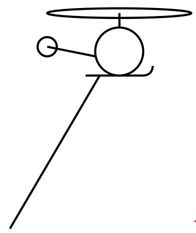
Solution

As cute as the box is, the only thing that matter is that you pull down on scale B with 10 extra newtons, so scale A must increase by 10 to balance.

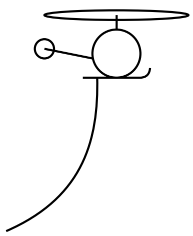
19. A helicopter is flying horizontally at constant speed. A perfectly flexible uniform cable is suspended beneath the helicopter; air friction on the cable is *not* negligible.

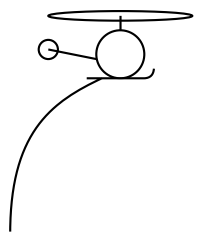
Which of the following diagrams best shows the shape of the cable as the helicopter flies through the air to the right?

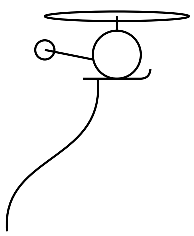
(A) 

(B) 

← **CORRECT**

(C) 

(D) 

(E) 

Solution

Since there is air friction on the cable, then there must be a horizontal component to the force where the cable attaches to the helicopter. Since the amount of air friction would be proportional to the length of the cable hanging beneath *at any point on the cable*, then the cable would be hanging in a straight diagonal line.

This question generated a great deal of controversy. At least two test takers challenged the answer, one who even tried to do the experiment. I'm told that several different "Ph.D" physicists declared that the correct answer was X, but, interestingly enough, couldn't agree on what X should be.

Originally the problem was worded as a cable with a hanging mass. As such, the correct answer would have been D, as explained below.

Since air friction on the hanging object is negligible, the only forces to consider are gravity and the cable. So the cable must be pulling directly upward. Since there is air friction on the cable, then there must be a horizontal component to the force where the cable attaches to the helicopter. The correct answer would then be curving to the left then falling straight down.

20. A crew of scientists has built a new space station. The space station is shaped like a wheel of radius R, with essentially all its mass M at the rim. When the crew arrives, the station will be set rotating at a rate that causes an object at the rim to have radial acceleration g, thereby simulating Earth's surface gravity. This is accomplished by two small rockets, each with thrust T newtons, mounted on the station's rim. How long a time t does one need to fire the rockets to achieve the desired condition?

    (A) $t = \sqrt{gR^3}\, M/(2T)$

    (B) $t = \sqrt{gR}\, M/(2T)$ ← **CORRECT**

    (C) $t = \sqrt{gR}\, M/T$

    (D) $t = \sqrt{gR/\pi}\, M/T$

    (E) $t = \sqrt{gR}\, M/(\pi T)$

Adapted from a problem in *Physics for Scientists and Engineers* by Richard Wolfson

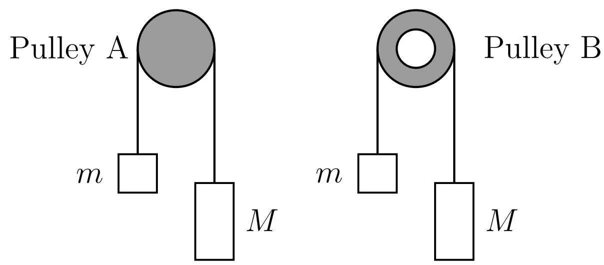
Solution

Desired acceleration: $g = \omega^2 R \rightarrow \omega = \sqrt{g/R}$

The two rockets provide: $2TR = MR^2\alpha \rightarrow \alpha = 2T/MR$

time needed: $t = \omega/\alpha = \dfrac{g^{1/2}R^{1/2}M}{2T}$

21. Two pulleys (shown in figure) are made of the same metal with density ρ. Pulley A is a uniform disk with radius R. Pulley B is identical except a circle of $R/2$ is removed from the center. When two boxes $M = \alpha m$ ($\alpha > 1$) are connected over the pulleys through a massless rope and move without slipping, what is the ratio between the accelerations in system A and B? The mass of pulley A is $M + m$.



(A) $a_A/a_B = 47/48$ ← **CORRECT**

(B) $a_A/a_B = 31/32$

(C) $a_A/a_B = 15/16$

(D) $a_A/a_B = 9/16$

(E) $a_A/a_B = 3/4$

Solution

This is effectively a force balance problem with a generalized Atwood machine.

For mass M, the equations are

$$Ma = Mg - T_M$$

where T_M is tension in rope on the M side.

For mass m, the equations are

$$ma = T_m - mg$$

For the pulley, the equations are

$$I\alpha = R(T_M - t_m)$$

with $\alpha = a/R$ and I the moment of inertia.

Combining,

$$Ia/R^2 = (Mg - Ma) - (ma + mg)$$

or

$$a = g\frac{M - m}{I/R^2 + M + m}$$

Let $I = \beta(M + m)R^2$ for the two pulleys. Then the acceleration ratio is

$$\frac{a_A}{a_B} = \frac{\beta_B + 1}{\beta_A + 1}$$

β_A is easy enough, it is just 1/2, since it is a uniform disk.

β_B requires a little more work, it is $1/2(1-(1/4)(1/4)) = 15/32$ to account for the removal of the interior disk. The ratio of accelerations is then

$$\frac{a_A}{a_B} = \frac{47}{48}$$

22. A body of mass M and a body of mass $m \ll M$ are in circular orbits about their center of mass under the influence of their mutual gravitational attraction to each other. The distance between the bodies is R, which is much larger than the size of either body.

A small amount of matter $\delta m \ll m$ is removed from the body of mass m and transferred to the body of mass M. The transfer is done in such a way so that the orbits of the two bodies remain circular, and remain separated by a distance R. Which of the following statements is correct?

(A) The gravitational force between the two bodies increases.

(B) The gravitational force between the two bodies remains constant.

(C) The total angular momentum of the system increases.

(D) The total angular momentum of the system remains constant.

(E) The period of the orbit of two bodies remains constant. ← **CORRECT**

Solution

The force between two bodies is given by

$$F = G\frac{Mm}{R^2}$$

The new force, F', would be given by

$$F' = G\frac{(M+\delta m)(m-\delta m)}{r^2} = G\frac{Mm-(M-m)\delta m-(\delta m)^2}{R^2}$$

This can be approximated to first order as

$$F' = \left(1-\frac{\delta m}{m}\right)F$$

For centripetal motion,

$$mv^2/r = F$$

where r is the distance from m to the center of mass, which is

$$r = \frac{M}{M+m}R$$

In terms of angular momenta,

$$L_m = mvr = \sqrt{m(mv^2/r)r^3} = \sqrt{mr^3F}$$

or

$$L_m = \sqrt{\frac{mM^3RGmM}{(m+M)^3}} = mM^2\sqrt{\frac{GR}{(m+M)^3}}$$

By symmetry,

$$L_M = m^2M\sqrt{\frac{GR}{(m+M)^3}}$$

and the sum is then

$$L = L_m + L_M = mM\sqrt{\frac{GR}{m+M}}$$

and we already saw that the quantity Mm decreases as δm is transferred from m to M.

Finally, the period of motion can be found from $v = 2\pi r/T$, so

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{mr}{F}}$$

Combining the above,

$$T = 2\pi\sqrt{\frac{mMR}{m+M}\frac{R^2}{GmM}} = 2\pi\sqrt{\frac{R^3}{G(m+M)}}$$

And that's constant.

The following information applies to questions 23 and 24

A 100 kg astronaut carries a launcher loaded with a 10 kg bowling ball; the launcher and the astronaut's spacesuit have negligible mass. The astronaut discovers that firing the launcher results in the ball moving away from her at a relative speed of 50 m/s.

23. What is the impulse delivered to the astronaut when firing the launcher?

(A) 455 N s ← **CORRECT**

(B) 500 N s

(C) 550 N s

(D) 5000 N s

(E) 5500 N s

Solution

Work in the center of mass frame, where the impulse delivered is simply the final momentum of the astronaut. Let the astronaut have mass m_1 and final velocity v_1, and let the launcher have mass m_2 and final velocity v_2. Then from conservation of momentum

$$m_1 v_1 + m_2 v_2 = 0$$

and the relative velocity is (taking $v_1 > 0$)

$$v_r = v_1 - v_2$$

Substituting,

$$m_1 v_1 + m_2(v_1 - v_r) = 0$$

$$m_1 v_1 = \frac{m_1 m_2}{m_1 + m_2} v_r$$

so

$$J = m_1 v_1 = 455 \text{ N s}$$

24. The astronaut in the previous situation is now moving at 10 m/s (as measured in a certain frame of reference). She wishes to fire the launcher so that her velocity turns through as large an angle as possible (in this frame of reference). What is this maximum angle? (*Hint*: a diagram may be useful.)

(A) 24.4°

(B) 26.6°

(C) 27.0° ← **CORRECT**

(D) 30.0°

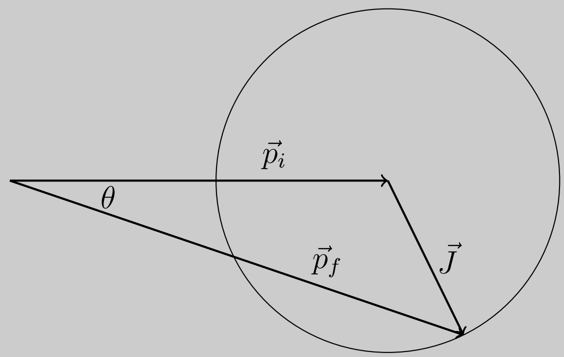
(E) 180.0°

Solution

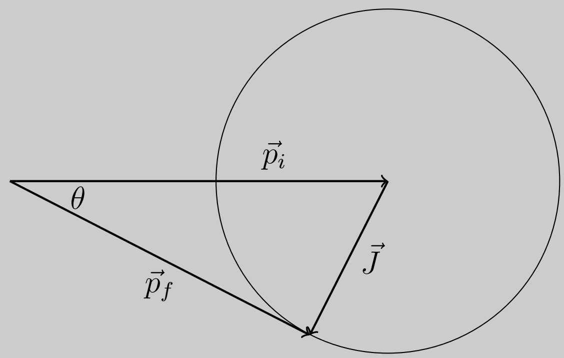
The final momentum of the astronaut is the sum of her initial momentum and the impulse delivered to her. Her initial momentum has magnitude

$$p_i = (100 \text{ kg})(10 \text{ m/s}) = 1000 \text{ kg m/s}$$

and the impulse available has the fixed, smaller magnitude found in the previous problem. (Note that the impulse is the same in all inertial reference frames.) Her final momentum is thus confined to the circle shown below:



The maximum angle is obtained by the tangent to the circle.



The maximum angle is thus

$$\theta = \arcsin \frac{J}{p_i} = 27.0^\circ$$

25. A block with mass m is released from rest at the top of a frictionless ramp. The block starts at a height h_1 above the base of the ramp, slides down the ramp, and then up a second ramp. The coefficient of kinetic friction between the block and the second ramp is μ_k. If both ramps make an angle of θ with the horizontal, to what height h_2 above the base of the second ramp will the block rise?

    (A) $h_2 = (h_1 \sin\theta)/(\mu_k \cos\theta + \sin\theta)$ ← **CORRECT**

    (B) $h_2 = (h_1 \sin\theta)/(\mu_k + \sin\theta)$

    (C) $h_2 = (h_1 \sin\theta)/(\mu_k \cos^2\theta + \sin\theta)$

    (D) $h_2 = (h_1 \sin\theta)/(\mu_k \cos^2\theta + \sin^2\theta)$

    (E) $h_2 = (h_1 \cos\theta)/(\mu_k \sin\theta + \cos\theta)$