AAPT UNITED STATES PHYSICS TEAM
AIP 2019

2019 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, the answer sheet and the scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 1, 2019.**

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We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

JiaJia Dong, Mark Eichenlaub, Matthew Huang, David Jones, Abijith Krishnan, Daniel Longenecker, Marianna Mao, Joon Pahk, Kye Shi, Brian Skinner, Alex Small, Paul Stanley, Elena Yudovina, and Kevin Zhou.

1. A coin of mass m is dropped straight down from the top of a very tall building. As the coin approaches terminal speed, which is true of the net force on the coin?

   (A) The net force on the coin is upward.

   (B) The net force on the coin is 0.

   (C) The net force on the coin is downward, with a magnitude less than mg. ← **CORRECT**

   (D) The net force on the coin is downward, with a magnitude equal to mg.

   (E) The net force on the coin is downward, with a magnitude greater than mg.

Solution

Before the coin reaches terminal velocity, it is still accelerating downward, so there must be a net force downward on the coin. However, because the coin has nonzero speed, there is a drag force opposing its motion (downward), so the drag force is acting upward on the coin. The only other force acting on the coin (mg) is downwards, so the net force on the coin must be less than mg in magnitude.

2. A mass of $3M$ moving at a speed v collides with a mass of M moving directly toward it, also with a speed v. If the collision is completely elastic, the total kinetic energy after the collision is K_e. If the two masses stick together, the total kinetic energy after the collision is K_s. What is the ratio K_e/K_s?

   (A) 1/2

   (B) 1

   (C) $\sqrt{2}$

   (D) 2

   (E) 4 ← **CORRECT**

Solution

For elastic, total kinetic energy after equals before, so really this question is asking what the ratio of the kinetic energy before to the kinetic energy after in a completely inelastic collision.

Before,

$$K_e = \frac{1}{2}3Mv^2 + \frac{1}{2}Mv^2 = 2Mv^2$$

After, the two masses move as one, so by conservation of momentum,

$$p_f = p_i = 3Mv - Mv = 2Mv,$$

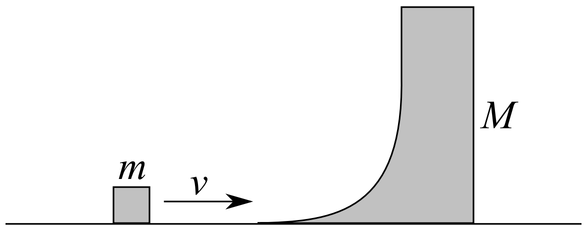
and then

$$K_f = \frac{{p_f}^2}{2(3M + M)} = \frac{1}{2}Mv^2$$

so

$$K_e/K_s = 4$$

3. A block of mass m is launched horizontally onto a curved wedge of mass M at a velocity v. What is the maximum height reached by the block after it shoots off the vertical segment of the wedge? Assume all surfaces are frictionless; both the block and the curved wedge are free to move. The curved wedge does not tilt or topple.



(A) $\dfrac{v^2}{2g}$

(B) $\left(\dfrac{m}{m+M}\right)^2 \cdot \dfrac{v^2}{2g}$

(C) $\left(\dfrac{M}{m+M}\right)^2 \cdot \dfrac{v^2}{2g}$

(D) $\dfrac{m}{m+M} \cdot \dfrac{v^2}{2g}$

(E) $\dfrac{M}{m+M} \cdot \dfrac{v^2}{2g}$ ← **CORRECT**

Solution

Essentially, the difference between the initial kinetic energy and the final kinetic energy of the system is accounted for in the height of the block. The initial kinetic energy of the system is $\frac{1}{2}mv^2$. The final horizontal momentum of the system is again mv, and the block and wedge have the same horizontal velocity. The final kinetic energy is thus $\frac{p^2}{2m_{\text{tot}}} = \frac{m^2v^2}{2(M+m)}$. Then, the leftover energy is

$$\Delta E = \frac{v^2 Mm}{2(M+m)}.$$

Therefore, the final height of the block is $\Delta E/(mg) = \frac{v^2 M}{2(M+m)g}$.

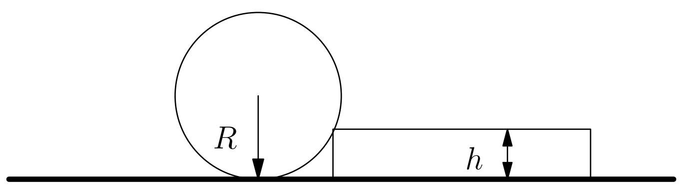
4. A massless spring hangs from the ceiling, and a mass is hung from the bottom of it. The mass is supported so that initially the tension in the spring is zero. The mass is then suddenly released. At the bottom of its trajectory, the mass is 5 centimeters from its original position. Find its oscillation period.

   (A) 0.05 s
   (B) 0.07 s
   (C) 0.31 s $\leftarrow$ **CORRECT**
   (D) 0.44 s
   (E) Not enough information is given.

Solution

The new equilibrium point is midway between the old equilibrium point and the bottom of its oscillating trajectory, i.e. $y = 2.5$cm from the old equilibrium point. At that new equilibrium point $mg = ky$. The frequency ω is equal to $\sqrt{k/m} = \sqrt{g/y}$, which is also equal to $2\pi/T$. We thus get that $T = 2\pi/\sqrt{g/y} = 2\pi\sqrt{y/g} = 2\pi\sqrt{0.025\text{m}/10\text{m/s}^2} = 0.314$ s.

5. A cylinder has a radius R and weight G. You try to roll it over a step of height $h < R$. The minimum force needed to roll the cylinder over is:



   (A) $\dfrac{\sqrt{2Rh - h^2}}{R - h}G$
   (B) $\dfrac{\sqrt{2Rh - h^2}}{2R - h}G$
   (C) $\dfrac{\sqrt{2Rh - h^2}}{2R}G$ $\leftarrow$ **CORRECT**
   (D) $\dfrac{\sqrt{2Rh - h^2}}{R}G$
   (E) $\dfrac{\sqrt{Rh - h^2}}{2R}G$

Solution

Rolling the cylinder over requires overcoming the torque due to gravity. We choose the corner of the step as our pivot point. The force from gravity is mg. The distance between the corner and the vertical line bisecting the cylinder is given by $\sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$. This is our moment arm (i.e., the perpendicular component of the radius). Then, the torque from gravity is $\tau_g = mg\sqrt{2Rh - h^2}$.

To counteract this torque with the smallest force possible, we need the largest moment arm possible. The largest moment arm possible is $2R$, so $F_{\text{min}} = \tau_g/(2R)$, which gives us the desired answer.

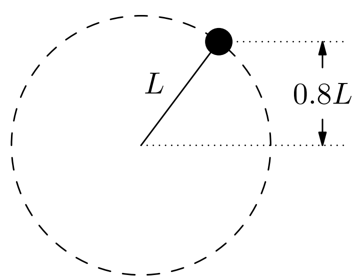
6. A ball is released from rest above an inclined plane and bounces elastically down the plane. As the ball progresses down the plane, the time and the distance between each collision will:

   (A) remain the same, and increase. ← **CORRECT**
   (B) increase, and remain the same.
   (C) decrease, and increase.
   (D) decrease, and remain the same.
   (E) both remain the same.

Solution

Work in the reference frame with x-axis along the plane and y-axis perpendicular to the plane. Then in the y-direction, the ball simply bounces regularly up and down, so the collisions are uniformly spaced in time. However, gravity provides an acceleration in the x-direction, so the distance increases.

7. An object of mass m is attached to the end of a massless rod of length L. The other end of the rod is attached to a frictionless pivot. The object is raised so that its height is $0.8L$ above the pivot, as shown in the figure. After the object is released from rest, what is the tension in the rod when it is horizontal?



   (A) 0.6 mg
   (B) 1.6 mg ← **CORRECT**
   (C) 2.6 mg
   (D) 3.6 mg
   (E) 5.36 mg

Solution

The object has square velocity $v^2 = 2gh = 1.6gL$ when the rod is horizontal. The tension in the rod supplies the centripetal acceleration of the object, so $T = mv^2/L = 1.6mg$.

8. The mass and the radius of the Earth are M and R. If an object starting at a distance of R from the Earth's surface is moving at a velocity $v_0 = \sqrt{\frac{2GM}{3R}}$ tangentially, what is its trajectory?

   (A) A parabola or a hyperbola

   (B) A circle around Earth

   (C) An ellipse whose minimum distance from the Earth's surface is R ← **CORRECT**

   (D) An ellipse whose maximum distance from the Earth's surface is R

   (E) A straight line along the direction of the initial velocity

Solution

The orbit shape will depend on the choices of r_0 and v_0. The escape speed of an object at a radius r from the center of Earth is

$$v_{esc} = \sqrt{\frac{2GM}{r}}.$$

If

$$v > \sqrt{\frac{2GM}{r}},$$

the object will have a hyperbolic escape trajectory.

If

$$v = \sqrt{\frac{GM}{r}},$$

the object will have a circular orbit.

If

$$\sqrt{\frac{2GM}{r}} > v > \sqrt{\frac{GM}{r}},$$

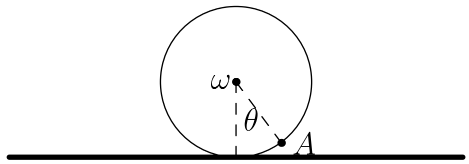
the object has an elliptical orbit starting closest to Earth.

If

$$\sqrt{\frac{GM}{r}} > v > 0,$$

the object has an elliptical orbit starting furthest from Earth. Note that $r = 2R$, so $v = \sqrt{\frac{4MG}{3r}}$, so the answer is C.

9. A wheel of radius R is rolling without slipping with angular velocity ω.



For point A on the wheel at an angle θ with respect to the vertical, shown in the figure, what is the magnitude of its velocity with respect to the ground?

(A) ωR

(B) $\omega R \sin(|\theta|/2)$

(C) $\sqrt{2}\omega R \sin(|\theta|/2)$

(D) $2\omega R \sin(|\theta|)$

(E) $2\omega R \sin(|\theta|/2)$ ← **CORRECT**

Solution

The velocity at point A with respect to the surface is the sum of the velocity of the center of mass (CM) and the velocity at A with respect to CM. The contact point between the wheel and the surface is the instantaneous axis of rotation. Therefore:

$$\vec{v}_A = \vec{v}_{\text{cm}} + \vec{\omega} \times \vec{R} \Rightarrow v_A^2 = v_{\text{cm}}^2 + \omega^2 R^2 - 2v_{\text{cm}}\omega r \cos\theta$$

Meanwhile, at the contact point, $\vec{v}_{\text{con}} = \vec{v}_{\text{cm}} + \vec{\omega} \times \vec{R} = 0$, thus $v_{\text{cm}} = \omega R \Rightarrow v_A = 2\omega R \sin\theta/2$

10. A flat uniform disk of radius $2R$ has a hole of radius R removed from the center. The resulting annulus is then cut in half along the diameter. The remaining shape has mass M. What is the moment of inertia of this shape, about the axis of rotational symmetry of the original disk?

(A) $\frac{45}{32}MR^2$

(B) $\frac{7}{6}MR^2$

(C) $\frac{8}{5}MR^2$

(D) $\frac{5}{2}MR^2$ ← **CORRECT**

(E) $\frac{15}{8}MR^2$

Solution

Start with $I = (1/2)mr^2$, then the annulus has moment of inertia of

$$I_a = \frac{1}{2}m(2R)^2 - \frac{1}{2}\frac{m}{4}R^2 = \frac{1}{2}\frac{15}{4}mR^2$$

assuming m is the mass of the original disk before removing the center. But in doing so, the new mass is 3/4 the original, so

$$I_a = \frac{1}{2}\frac{15}{4}\frac{4}{3}(2M)R^2 = 5MR^2.$$

Cutting the annulus in half reduces the mass to half, but doesn't change the geometry, so

$$I_f = \frac{5}{2}MR^2$$

11. To test the speed of a model car, you time the car with a stopwatch as it travels a distance of 100 m. You record a time of 5.0 s, and your measurement has an uncertainty of 0.2 s. What is the uncertainty in your estimate of the car's speed? Assume that the car travels at a constant speed and the distance of 100 m is known very precisely.

    (A) $v = 20 \pm 0.16$ m/s
    (B) $v = 20 \pm 0.8$ m/s ← **CORRECT**
    (C) $v = 20 \pm 1.0$ m/s
    (D) $v = 20 \pm 1.25$ m/s
    (E) $v = 20 \pm 4.0$ m/s

Solution

The speed v is related to the distance x and the time t by $v = x/t$. The uncertainty Δv in the speed is described by

$$\Delta v = \left| \frac{dv}{dt} \right| \times \Delta t, \tag{1}$$

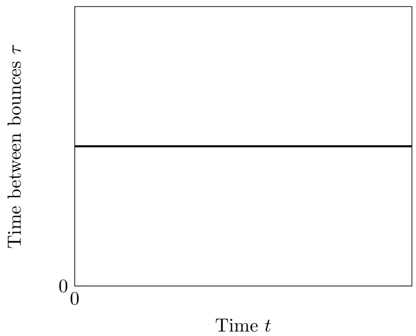
where Δt is the uncertainty in the time and $dv/dt = -x/t^2$ is the derivative of v with respect to t. Inserting $x = 100\,\text{m}$, $t = 5.0\,\text{s}$, and $\Delta t = 0.2\,\text{s}$ gives

$$\Delta v = \frac{x}{t} \times \frac{\Delta t}{t} = 0.8\,\text{m/s}. \tag{2}$$

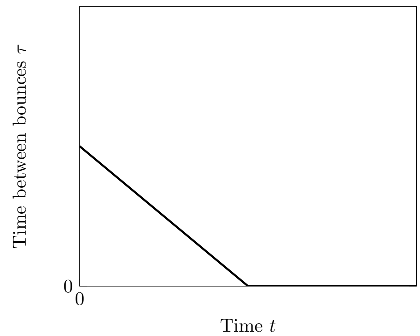
You may refer to more information on error propagation in the solutions for $F = ma$ 2018 A.

12. A steel ball bearing bounces vertically on a steel plate. If the speed of the ball just before a bounce is v_i, the speed of the ball immediately afterward is $v_f = \alpha v_i$, with $\alpha < 1$. Which one of the following graphs best shows the time between successive bounces, τ, as a function of time?

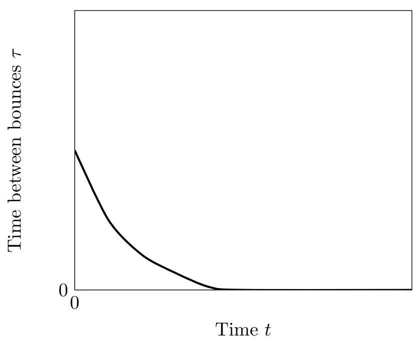
(A)



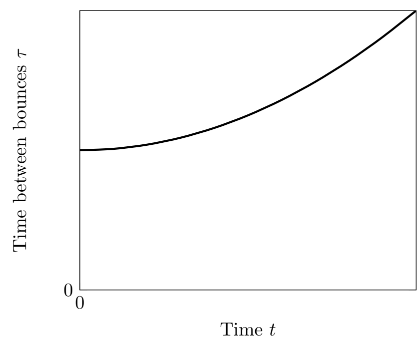
(B) ← **CORRECT**



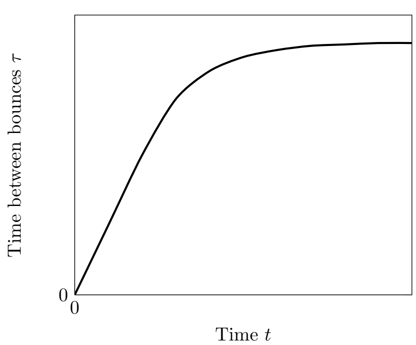
(C)



(D)



(E)



Solution

The time between bounces is given by

$$\tau_n = 2v_n/g$$

where

$$v_n = \alpha^n v_i$$

so

$$\tau_n = \tau_0 \alpha^n$$

where $\tau_0 = 2v_i/g$.

The total time to the nth bounce is then

$$t_n = \tau_0 \sum \alpha^n = \tau_0 \frac{1-\alpha^n}{1-\alpha}$$

$$t_n = \tau_0 \sum_{k=0}^{n-1} \alpha^n = \tau_0 \left(\frac{1-\alpha^n}{1-\alpha}\right) = \frac{\tau_0 - \tau_n}{1-\alpha},$$

so

$$\tau_n = \tau_0 - (1-\alpha)t_n.$$

13. A juggler juggles N identical balls, catching and tossing one ball at a time. Assuming that the juggler requires a minimum time T between ball tosses, the minimum possible power required for the juggler to continue juggling is proportional to

    (A) N^0
    (B) N^1
    (C) N^2 ← **CORRECT**
    (D) N^3
    (E) N^4

Solution

One can check that the optimal strategy, in terms of power, is to toss each ball so it takes time NT to return. This is the minimum time spacing that allows the juggler to keep up. Tossing the balls higher will allow a greater time spacing between tosses, but will also cost more energy, so the net power is higher.

Since the acceleration due to gravity is uniform, the initial velocity of each ball is hence proportional to N, so the initial energy of each ball is proportional to N^2, so the power is proportional to $N^2/T \propto N^2$.

14. A man standing at 30° latitude fires a bullet northward at a speed of 200 m/s. The radius of the Earth is 6371 km. What is the sideway deflection of the bullet after traveling 100 m?

(A) 3.1 mm west
(B) 1.8 mm west
(C) 0 mm
(D) 1.8 mm east ← **CORRECT**
(E) 3.1 mm east

Solution

The Coriolis acceleration is given by

$$\vec{a} = -2\vec{\omega} \times \vec{v}, \tag{3}$$

where $\vec{\omega}$ is the angular velocity of the Earth and $\vec{v}$ is the velocity of the bullet. Evaluating this cross product gives an acceleration toward the east with magnitude $a = 2\omega v \sin\theta = v\omega$, where $\theta = 30°$ and $\omega = 2\pi/\text{day}$.

The deflection of the bullet $d = (1/2)at^2$, where $t = L/v$ and $L = 100\,\text{m}$ is the distance traveled by the bullet. So

$$d = \frac{\omega L^2}{2v} \approx 1.8\,\text{mm}. \tag{4}$$

15. An upright rod of length ℓ is launched into the air with vertical velocity v_y. It is given enough angular momentum so that the rod rotates by an angle of 2π before landing. Find the initial horizontal velocity of the bottom of the rod.

(A) $\frac{2\ell g}{v_y^2}$
(B) $\frac{\pi \ell g}{2v_y}$ ← **CORRECT**
(C) $\sqrt{\ell g}$
(D) $\frac{2v_y}{\ell g}$
(E) $\frac{2v_y^2}{\pi\sqrt{\ell g}}$

Solution

The time t for the rod to return to the same height at which it was launched is given by

$$t = 2v_y/g. \tag{5}$$

The time required for the rod to make one full rotation is

$$t = 2\pi/\omega, \tag{6}$$

where ω is the angular velocity of the rod, which is related to the velocity at end of the rod by $v_x = \omega\ell/2$.

Equating the two expressions for t gives

$$v_x = \frac{\pi \ell g}{2v_y}. \tag{7}$$

16. The depth of a well, d, is measured by dropping a stone into it and measuring the time t until the splash is heard at the bottom. What is the smallest value of d for which ignoring the time for the sound to travel gives less than a 5% error in the depth measurement? The speed of sound in air is 330 m/s.

    (A) 3.5 m
    (B) 7 m
    (C) 14 m ← **CORRECT**
    (D) 54 m
    (E) 330 m

Solution

The time t for the rock to reach the bottom of the well is related to the depth d by

$$d = \frac{1}{2} g t^2. \tag{8}$$

The time Δt that it takes for the sound of the splash to reach the top of the well is $\Delta t = d/v_s$, where v_s is the speed of sound.

If one uses the time $t + \Delta t$ to estimate the well depth, then one arrives at an estimate

$$d + \Delta d = \frac{1}{2} g (t + \Delta t)^2, \tag{9}$$

so that

$$\Delta d = (d + \Delta d) - \frac{1}{2} g t^2 \approx g t \Delta t. \tag{10}$$

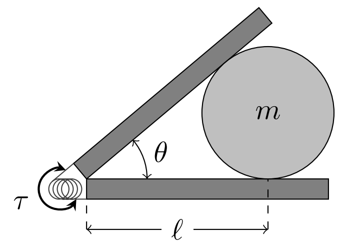
The ratio $(\Delta d)/d = 2(\Delta t)/t$, and we know that $(\Delta d)/d = 0.05$. We know an expression for Δt in terms of d, and we can arrive at an expression for t in terms of d by solving the first equation. We can then solve the equation $2(\Delta t)/t = 0.05$ in terms of d, which gives

$$d = \left(\frac{\Delta d}{d}\right)^2 \frac{v_s^2}{2g} \approx 14\,\mathrm{m}. \tag{11}$$

The following information applies to questions 17 and 18.

A launcher is designed to shoot objects horizontally across an ice rink. It consists of two boards of negligible mass connected via a spring-loaded hinge, which exerts a constant torque τ on each board to keep them together. For both problems, neglect friction with either the ice or the launch boards.

17. A hard disc is pushed into the launcher between the boards until the boards make contact with it a distance ℓ from the hinge and are open to an angle θ, as shown in the figure. What is the minimum force necessary to hold the disc in this position?



(A) $2\tau \sin(\theta/2)/\ell$ ← **CORRECT**

(B) $2\tau \cos(\theta/2)/\ell$

(C) $\tau \cos(\theta)/\ell$

(D) $\tau \tan(\theta)/\ell$

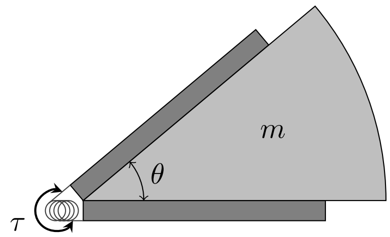
(E) $2\tau \tan(\theta)/\ell$

Solution

Each board exerts a force with magnitude $F = \tau/\ell$, directed toward the center of the disc. We can decompose these forces into a component $F_{\|} = (\tau/\ell)\sin(\theta/2)$ that is parallel to the symmetry axis of the launcher, and a perpendicular component $F_{\perp}$. The perpendicular components from the two forces cancel, while the parallel components add together. The minimal force F_{tot} required to keep the disc in place must equal the sum of the two parallel components, which gives

$$F_{\text{tot}} = 2F_{\|} = \frac{2\tau}{\ell}\sin(\theta/2). \quad (12)$$

18. The disc is removed and replaced with a pie-shaped wedge of the same mass m, so that the hinge is still initially held open at an angle θ, as shown in the following figure. If the wedge is released from rest, what is its speed after it exits the launcher?



(A) $\sqrt{\frac{\tau}{2m\theta}}$

(B) $\sqrt{\frac{4\tau\theta}{m}}$

(C) $\sqrt{\frac{\tau\theta^2}{2m}}$

(D) $\sqrt{\frac{2\tau\theta}{m}}$ ← **CORRECT**

(E) $\sqrt{\frac{2\tau}{m\theta}}$

Solution

Since the launcher's torque is constant, the energy stored in the hinge is $\tau\theta$. All of this energy goes into the kinetic energy $\frac{1}{2}mv^2$ of the wedge. Equating these two and solving for v gives

$$v = \sqrt{\frac{2\tau\theta}{m}}. \qquad (13)$$

19. A small rock is tied to a massless string of length 5 m. The density of the rock is twice the density of the water. The rock is lowered into the water, while the other end of the string is attached to a pivot. Neglect any resistive forces from the water. The rock oscillates like a pendulum with angular frequency of which of the following?

    (A) 1 rad/s ← **CORRECT**

    (B) 0.7 rad/s

    (C) 0.5 rad/s

    (D) 1.4 rad/s

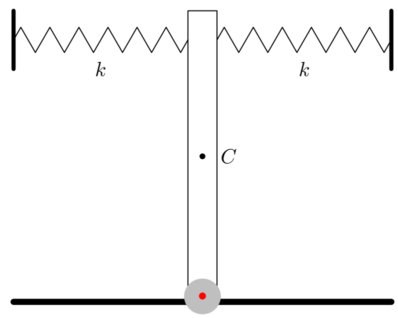
    (E) 2 rad/s

Solution

The submerged rock experiences an upward buoyant force equal to half its weight, since it displaces a volume of water that weighs half as much as the rock itself. Thus, if the rock were free-falling in water it would accelerate downward with an acceleration $g_{\text{eff}} = g/2$. The angular frequency of a normal pendulum is given by $\omega = \sqrt{g/\ell}$, which in this case becomes

$$\omega = \sqrt{\frac{g_{\text{eff}}}{\ell}} = \sqrt{\frac{g}{2\ell}} = 1\,\text{rad/s} \tag{14}$$

20. A uniform rod of mass M and length L is hinged on a horizontal surface at the bottom. Its top end is connected to two springs, both with spring constant k. What relation must M, k, and L satisfy such that the position shown in the figure is a stable equilibrium?



(A) $Mg < 4kL$ ← **CORRECT**

(B) $Mg < 2kL$

(C) $2kL < Mg < 4kL$

(D) $kL/2 < Mg < kL$

(E) $Mg < kL$

Solution

When the rod is given an infinitesimal angular displacement $\delta\theta$, the two springs are compressed or stretched by an amount $\delta x = L \sin\theta \approx L\delta\theta$, and they each provide a restoring torque $\approx k\delta x \times L$. So the total restoring torque is $2kL^2\delta\theta$.

On the other hand, the displacement causes the gravitational force to have a component $Mg \sin\theta \approx Mg\delta\theta$ in the azimuthal direction, and this gives a destabilizing torque $Mg\delta\theta \times L/2$. For stability, the restoring torque must be larger than the destabilizing torque. This gives $2kL^2\delta\theta > MgL\delta\theta/2$, or $Mg < 4kL$.

21. A spherical cloud of dust has uniform mass density ρ and radius R. Satellite A of negligible mass is orbiting the cloud at its edge, in a circular orbit of radius R, and satellite B is orbiting the cloud just inside the cloud, in a circular orbit of radius r, with $r < R$. If v_i is the speed of satellite i and T_i is the period of satellite i, which of the following is true? Neglect any drag forces from the dust.

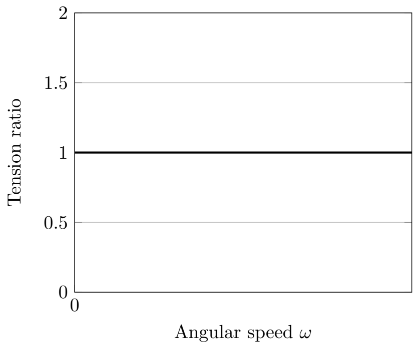
    (A) $T_A > T_B$ and $v_A > v_B$
    (B) $T_A > T_B$ and $v_A < v_B$
    (C) $T_A < T_B$ and $v_A > v_B$
    (D) $T_A < T_B$ and $v_A < v_B$
    (E) $T_A = T_B$ and $v_A > v_B$ ← **CORRECT**

Solution

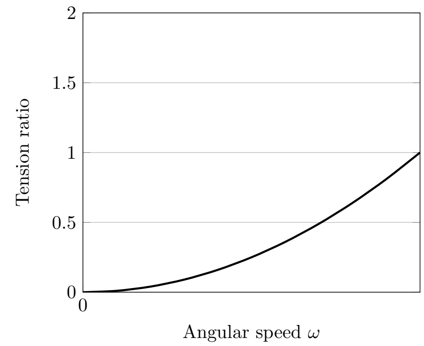
From Kepler's Laws, we have $T^2 \propto R^3/(MG)$. Then $T^2 \propto 1/(\rho G)$. Therefore $T_A = T_B$ because the density of matter enclosed is the same for both satellites. Note that if the periods are the same, then the angular velocities are the same and so the speeds are proportional to radii. Therefore, $v_A > v_B$.

22. A vertical pole has two massless strings, both of length L, attached a distance L apart. The other ends of the strings are attached to a mass M. The mass is rotated around the pole with an angular speed ω. Which of the following graphs best gives the ratio of the tension in the bottom string to the tension in the top string as a function of ω?

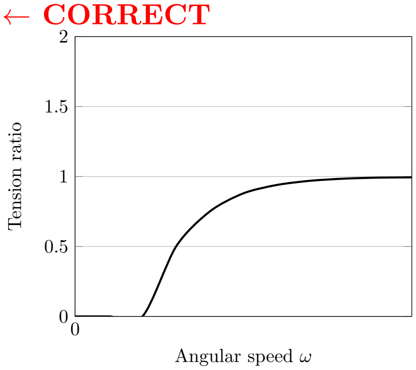
(A)



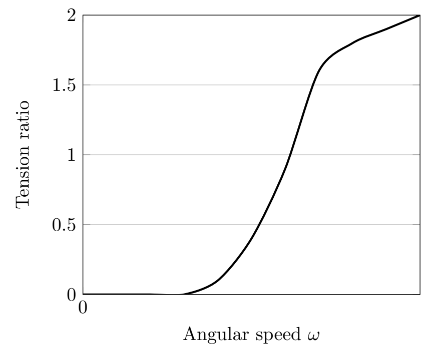
(B)



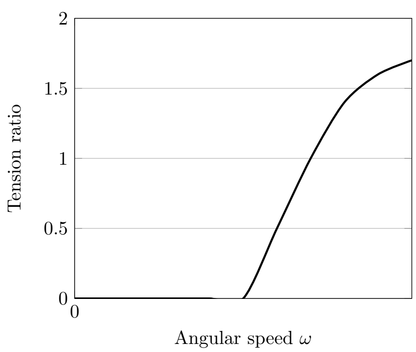
(C) ← **CORRECT**



(D)



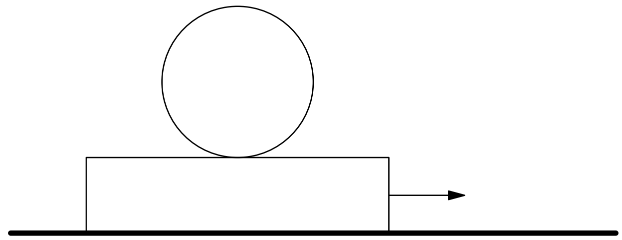
(E)



Solution

The bottom string won't even have a tension until some minimum angular speed is reached. If the ball is spinning fast enough, gravity doesn't matter, so the ratio must be one. Only (C) works.

23. A rectangular slab sits on a frictionless surface. A sphere sits on the slab. There is sufficient friction between the sphere and the slab such that the sphere will not slip relative to the slab. A force to the right is applied to the slab, with both the slab and the sphere initially at rest.



The sphere will then:

(A) begin spinning clockwise while its center of mass accelerates to the right.

(B) begin spinning counterclockwise while its center of mass accelerates to the left.

(C) begin spinning clockwise while its center of mass accelerates to the left.

(D) begin spinning counterclockwise while its center of mass accelerates to the right. ← **CORRECT**

(E) not spin, while its center of mass accelerates to the right.

Solution

There is a friction force between the sphere and the slab. The center of mass of the sphere accelerates in the same direction as the friction force on the sphere because the friction force is the only horizontal force on the sphere. The friction force also provides the only torque on the sphere about the center of the sphere. So if the friction force on the sphere points right, the sphere rotates counterclockwise. If the friction force point left, the sphere rotates clockwise. Suppose we pull the slab to the right some fixed distance with some fixed force. Then the energy delivered to the slab-sphere system is also fixed. That energy is divided between the slab and the sphere, so the slab must accelerate less in this scenario than it would if the sphere did not exist. This means the friction force on the slab must be to the left, so the friction force on the sphere is to the right. Its center of mass accelerates right and it spins counterclockwise.

24. A particle moves in the xy plane with the potential energy

$$U(x, y) = 9kx^2 + 16ky^2.$$

The particle can perform several different types of periodic motion. The ratio between the maximum and minimum possible periods is

(A) $2/\sqrt{3}$

(B) $4/3$

(C) $\sqrt{5}$

(D) 4 ← **CORRECT**

(E) 5

Solution

The particle performs simple harmonic oscillation in both the x and y directions independently. If the period of oscillations in the y direction is T, then the period of oscillations in the x direction is $4T/3$. The minimum possible period is T, while the maximum possible period is the lowest common multiplier of T and $4T/3$, i.e. $4T$. Then the desired ratio is 4.

25. A car is turning left along a circular track of radius r at a constant speed v. A cylindrical beaker is placed vertically inside the car. The beaker has a small hole on its right side. If the water's highest point in the beaker is a height h above the hole, at what instantaneous speed does water escape the hole, from a passenger's perspective?

    (A) $\sqrt{2gh}$ ← **CORRECT**
    (B) $\sqrt{(v^2/r)h}$
    (C) $\sqrt{gh}$
    (D) $\sqrt{h\sqrt{(v^2/r)^2 + g^2}}$
    (E) $\sqrt{2h\sqrt{(v^2/r)^2 + g^2}}$

Solution

The car has a centripetal acceleration v^2/r, so in the accelerated reference frame inside the car, the effective gravity points down and to the right, with the additional component to the right caused by "centrifugal" effective gravity:

$$\mathbf{g}_{\text{eff}} = \frac{v^2}{r}\hat{\mathbf{i}} + (-g)\hat{\mathbf{j}}$$

this vector points down and to the right at an angle θ to the vertical, where $\tan\theta = \frac{v^2/r}{g} = v^2/(gr)$, so that

$$g = \|\mathbf{g}_{\text{eff}}\| \cos\theta,$$
$$v^2/r = \|\mathbf{g}_{\text{eff}}\| \sin\theta.$$

Thus the normal of the water surface is also at an angle θ to the vertical, and thus the highest point on the water surface occurs at the right edge of the beaker, so the "effective" height of the water surface above the hole is

$$h' = h\cos\theta.$$

Then the (gauge) water pressure at the hole is

$$\begin{aligned}\rho\|\mathbf{g}_{\text{eff}}\|h' &\\ &= \rho\|\mathbf{g}_{\text{eff}}\|h\cos\theta \\ &= \rho(\|\mathbf{g}_{\text{eff}}\|\cos\theta)h \\ &= \rho g h,\end{aligned}$$

and by Bernoulli's equation,

$$\rho g h = \frac{1}{2}\rho u^2$$

(where u is the speed at which water exits the hole) we get

$$u = \sqrt{2gh}.$$