AAPT UNITED STATES PHYSICS TEAM
AIP 2023

2023 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

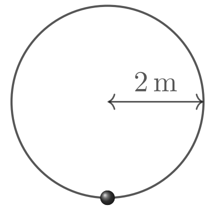
- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 25, 2023.**

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Kellan Colburn, Samuel Gebretsadkan, Abi Krishnan, Natalie LeBaron, Kye Shi, Mike Winer, and Kevin Zhou

1. A bead on a circular hoop with radius 2 m travels counterclockwise for 10 s and completes 2.25 rotations, at which point it reaches the position shown.

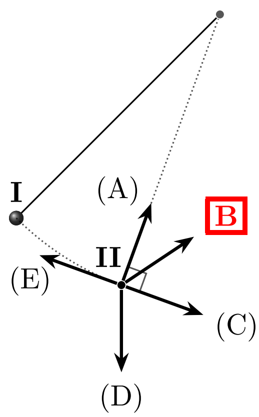


In the past 10 s, what were its average speed and the direction of its average velocity?

(A) $\frac{\sqrt{2}}{5}\,\frac{\text{m}}{\text{s}}, \swarrow$ (B) $\frac{2\pi}{5}\,\frac{\text{m}}{\text{s}}, \swarrow$ (C) $\frac{9\pi}{10}\,\frac{\text{m}}{\text{s}}, \swarrow$ (D) $\frac{2\pi}{5}\,\frac{\text{m}}{\text{s}}, \searrow$ **E** $\frac{9\pi}{10}\,\frac{\text{m}}{\text{s}}, \searrow$

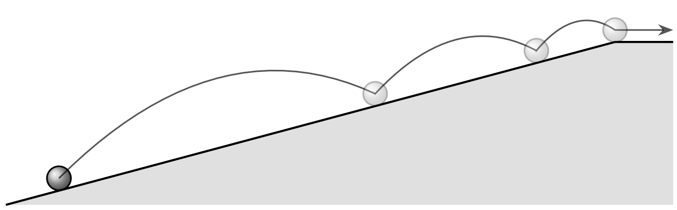
The total distance traveled is $(2.25)(2\pi \cdot 2\,\text{m}) = 9\pi\,\text{m}$, and the average speed is the distance per time, $(9\pi/10)\,\text{m/s}$. This rules out all choices but (C) and (E). To find the direction we note that 10 seconds ago, the bead was initially at the left side of the hoop, so the direction of the average velocity is $\searrow$.

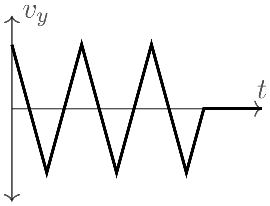
2. A mass on an ideal pendulum is released from rest at point I. When it reaches point II, which of the following shows the direction of its acceleration?

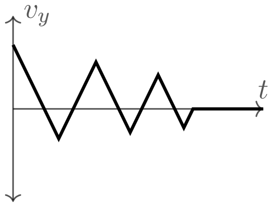


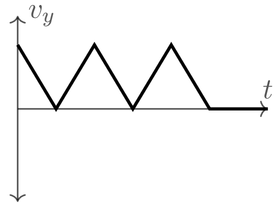
At this point, this is both an inward centripetal force because the mass is moving in a circle, and a tangential force because the mass is speeding up. Thus, the answer has to be a positive sum of vectors directed along (A) and (C), so it must be (B).

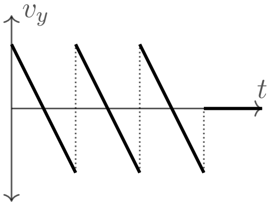
3. A soccer ball is kicked up a hill with a flat top, as shown. The ball bounces twice on the hill, at the points shown, then lands on the top and begins rolling horizontally. Which of the following shows the vertical component of its velocity as a function of time?

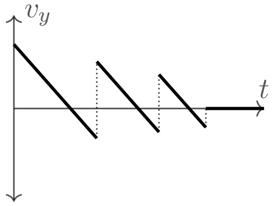


(A) 

(B) 

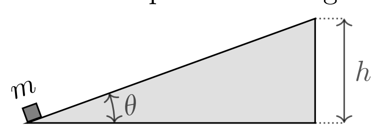
(C) 

(D) 

E 

Since gravity provides a uniform acceleration downward, the graph of v_y should be decreasing with constant slope everywhere except at the collisions, ruling out all but choices (D) and (E). The collisions provide the discontinuities. To decide between the two, we note that the typical vertical velocities shown in the figure get smaller as the ball nears the top, so the correct choice is (E).

4. A box of mass m is at the bottom of an inclined plane with angle θ to the horizontal, and height h.

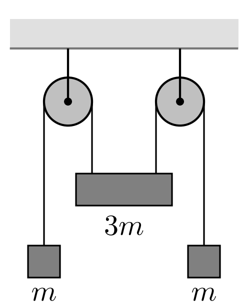


A person drags the box very slowly up the plane, by applying a force parallel to the plane. The coefficient of kinetic friction between the box and plane is μ_k. When the box reaches the top of the plane, how much work has the person done?

(A) $mgh\,(1 + \mu_k \sin\theta)$
(B) $mgh\,(1 + \mu_k \cos\theta)$
(C) $mgh\,(1 + \mu_k \tan\theta)$
(D) $mgh\,(1 + \mu_k \csc\theta)$
E $mgh\,(1 + \mu_k \cot\theta)$

When the box is moving up the ramp, there is a gravitational force $mg\sin\theta$ down the ramp, and a friction force $\mu_k mg\cos\theta$ down the ramp. The box must be dragged a total distance $h/\sin\theta$ along the ramp, so the work done is $mgh(\sin\theta + \mu_k\cos\theta)/\sin\theta = mgh(1 + \mu_k\cot\theta)$.

5. Two blocks of mass m and a block of mass $3m$ are attached to a system of massless fixed pulleys and massless string, as shown.



Assume all surfaces are frictionless. What is the acceleration of each mass m?

(A) $\frac{g}{8}$ **B** $\frac{g}{5}$ (C) $\frac{g}{4}$ (D) $\frac{g}{3}$ (E) $\frac{2g}{3}$

The system is equivalent to two disconnected Atwood's machines with masses m and $3m/2$, by splitting the large block down the middle. Hence the acceleration is $(3m/2 - m)g/(3m/2 + m) = g/5$.

6. A ball at the end of a rope of length 0.5 m is swung in a horizontal circle, with a speed of 15 m/s. The other end of the rope is fixed in place. What is the height difference between the ends of the rope?

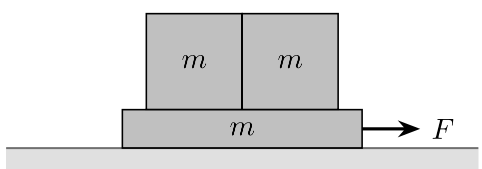
A 1.1 cm (B) 2.2 cm (C) 3.8 cm (D) 4.9 cm (E) 7.5 cm

For all of the answer choices, the rope is close to horizontal, so we can use the small angle approximation. The angle θ the rope makes with the horizontal must obey

$$\theta \approx \tan\theta = \frac{mg}{mv^2/r}$$

where the radius of the horizontal circle is $r = \ell\cos\theta \approx \ell$. Thus, the height difference is $\ell\sin\theta \approx \ell\theta = g\ell^2/v^2$, and plugging in the numbers gives the answer.

7. Two boxes are stacked side-by-side on top of a larger box as shown.



All three boxes have mass m, the coefficient of static friction between the left box and the bottom box is μ_s, and all other surfaces are frictionless. A rightward force F is applied to the bottom box. What is the minimum value of μ_s so that the upper boxes don't slide?

(A) $\frac{2F}{mg}$ (B) $\frac{3F}{mg}$ (C) $\frac{F}{2mg}$ **D** $\frac{2F}{3mg}$ (E) $\frac{F}{3mg}$

The acceleration of the entire system is $F/3m$, so the total horizontal force needed to accelerate the top two blocks is $2F/3$. This must be completely supplied by the friction force μmg on the left box, so we conclude $\mu \geq 2F/3mg$.

8. Two stars α and β, with masses satisfying $m_\alpha/m_\beta = 10$, are in circular orbits around each other. In the rest frame of this system, find the ratio of the speeds v_α/v_β.

(A) $\frac{1}{11}$ **B** $\frac{1}{10}$ (C) $\frac{1}{9}$ (D) 9 (E) 10

Because the total momentum is zero, $m_\alpha v_\alpha = m_\beta v_\beta$, so $v_\alpha/v_\beta = m_\beta/m_\alpha = 1/10$.

9. A helium balloon is released from the floor in a room at rest, then slowly rises and comes to rest touching the ceiling. During this process, the gravitational potential energy of the balloon has increased. Since energy is conserved, the energy of something else must have decreased during this process. Which of the following is the main contribution to this decrease?

(A) The kinetic energy of the balloon decreased.
(B) The elastic potential energy of the balloon decreased.
(C) The thermal energy of the air in the balloon decreased.
(D) The thermal energy of the air in the room decreased.
E The gravitational potential energy of the air in the room decreased.

The main contribution is choice (E). In terms of energy, objects that are lighter than air float upward because the air can move downward to where the object originally was, thereby reducing its gravitational potential energy by more than the gravitational potential energy of the object went up.

10. An archer takes aim at a target that is 100 m away. Assuming she holds the bow at the same height as the center of the target and shoots an arrow with velocity $v = 100\,\text{m/s}$, at what angle above the horizontal should she aim the bow so that the arrow hits the center of the target?

(A) $\dfrac{\arccos(1/5)}{2}$ (B) $\dfrac{\arcsin(1/5)}{2}$ (C) $\dfrac{\arccos(1/10)}{2}$ **D** $\dfrac{\arcsin(1/10)}{2}$ (E) $\dfrac{\arctan(2/5)}{2}$

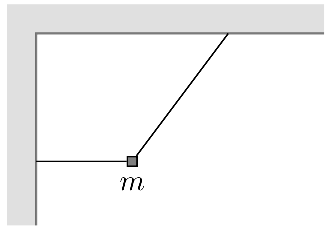
Since the initial and final heights are the same, we can apply the range equation $R = v^2 \sin(2\theta)/g$. Setting $R = 100\,\text{m}$ and $v = 100\,\text{m/s}$ and solving for θ gives the answer. Of course, an analysis using the usual kinematic equations would also give the same answer, after some more effort.

11. A projectile is thrown from a horizontal surface, and reaches a maximum height h and also lands a distance h from the launch point. Neglecting air resistance, what is the maximum height for a projectile thrown directly upward with the same initial speed?

A $\dfrac{17h}{16}$ (B) $\dfrac{13h}{12}$ (C) $\dfrac{9h}{8}$ (D) $\dfrac{5h}{4}$ (E) $2h$

From the maximum height, we know that $h = v_y^2/2g$, and from the range, we know that $h = v_x \Delta t = 2v_x v_y/g$. Combining these equations gives $v_y = 4v_x$. The maximum height if the projectile is thrown upward is $v^2/2g = (v_x^2 + v_y^2)/2g = (17/16)(v_x^2/2g) = 17h/16$.

12. A block of mass m is initially held in place by two massless strings, as shown.

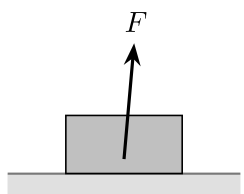


The tension in the diagonal string is T_1. Next, the horizontal string is cut, and immediately afterward the tension in the diagonal string is T_2. Which of the following is true?

(A) $T_1 < mg < T_2$ **B** $T_2 < mg < T_1$ (C) $T_1 < T_2 < mg$ (D) $mg < T_2 < T_1$ (E) $T_1 = T_2 < mg$

Let the angle from the vertical be θ. Before the string is cut, balancing forces gives $T_1 = mg/\cos\theta$. Right after the string is cut, balancing forces along the direction of the string gives $T_2 = mg\cos\theta$. Thus, $T_2 < mg < T_1$.

13. A uniform box with mass m is at rest on a horizontal surface, and the coefficient of static friction between them is μ_s. A force directed at an angle of 85° above the horizontal is applied to the center of the box, with a linearly increasing magnitude $F = \beta t$.



The box will eventually slide or lift off the ground. Which of the following is correct?

(A) If $\mu_s < \tan 85°$, the box will lift off the ground first.

(B) If $\mu_s < \tan 85°$, the box will slide first.

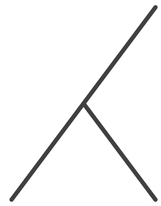
(C) For any value of μ_s, the box will lift off the ground first.

D For any value of μ_s, the box will slide first.

(E) The answer depends on the values of β, g, and m.

The box will slide first, no matter what the other parameters are. The box will only lift off the ground once the normal force goes to zero, which occurs when the vertical component of the applied force exceeds the weight of the box. Right before that moment, the normal force will be very small, so the maximal possible friction force is also very small, and cannot balance the horizontal component of the applied force.

14. The object shown below is made of three rigidly connected, identical rods with uniform density.



When it stands upright on a horizontal table, what fraction of its weight rests on the left leg?

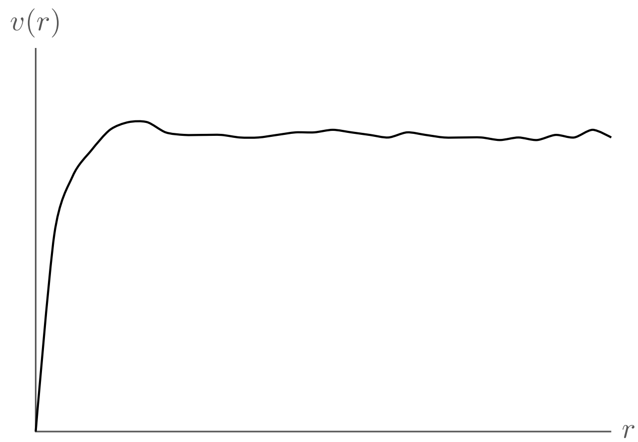
(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ **E** $\frac{5}{12}$

Let each rod have weight $W/3$, let the normal force on the left leg be N, and let the distance between the legs be ℓ. If we balance torques about bottom of the right leg, then the only contributions are from the weights of the rods and the normal force N, giving

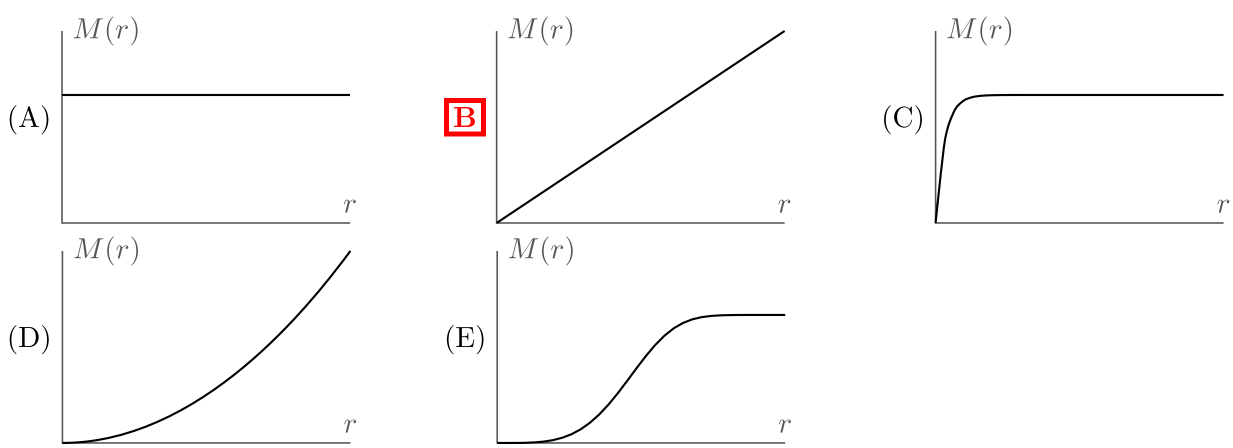
$$N\ell = \frac{W}{3}\left(\frac{\ell}{4} + \frac{\ell}{4} + \frac{3\ell}{4}\right)$$

which gives $N = 5W/12$.

15. The following plot shows the speed $v(r)$ at which stars orbit about the center of a galaxy, as a function of their distance r from the center.



Assuming the galaxy has a spherically symmetric mass distribution, which plot best shows the mass of the galaxy $M(r)$ enclosed within radius r?

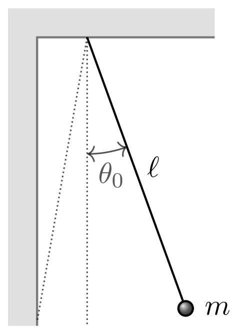


The graph shows that rotational speed becomes roughly constant once a certain distance from the center of the galaxy, but the rotational speed of a mass m is governed by

$$\frac{GM(r)m}{r^2} = \frac{mv(r)^2}{r}.$$

Then we must have $M(r) \propto r$ to keep $v(r)$ constant. (In addition, $M(r)$ should be smaller at very large r, to account for the region where $v(r)$ is small; this is not shown in the schematic answer choices.)

16. A bead attached to a string of length $\ell = 10\,\text{m}$ is released from a very small angle θ_0 to the vertical. A wall is placed in the path of the bead such that the bead collides elastically with the wall when the string is at an angle $\theta_0/2$ to the vertical, as shown.

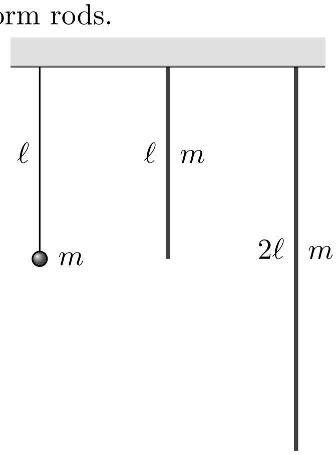


What is the time interval between the bead's collisions with the wall?

(A) $\frac{2\pi}{3}$ s (B) $\frac{3\pi}{4}$ s **C** $\frac{4\pi}{3}$ s (D) $\frac{3\pi}{2}$ s (E) 2π s

If the wall was not there, then the angle of the pendulum would be $\theta(t) = \theta_0 \cos(t\sqrt{g/L})$, which corresponds to period 2π seconds. When the bead collides with the wall, it effectively skips from a phase angle 120° in its oscillation to a phase angle 240°, which removes 1/3 of the oscillation cycle. Thus, the period is $(2/3)(2\pi) = 4\pi/3$ seconds.

17. Three physical pendulums are built as shown. The first is a typical pendulum with a massless rope, and the second and third are made of uniform rods.



What is the correct ranking of the moments of inertia I_1, I_2, and I_3 about the pivot points?

(A) $I_1 > I_2 > I_3$ (B) $I_3 > I_2 > I_1$ (C) $I_1 = I_3 > I_2$ (D) $I_2 > I_3 > I_1$ **E** $I_3 > I_1 > I_2$

This is a straightforward application of the standard moment of inertia formulas,

$$I_1 = m\ell^2, \qquad I_2 = \frac{1}{3}m\ell^2, \qquad I_3 = \frac{4}{3}m\ell^2$$

which gives $I_3 > I_1 > I_2$.

However, if you don't remember these formulas, you can get to the same result by noting that moment of inertia grows with distance from the pivot, so we must have $I_1 > I_2$ since the point mass is at least as far from the pivot as every point on the rod. We also have $I_3 > I_1$ by the parallel axis theorem, as the third pendulum has a nonzero moment of inertia about its center of mass, while the first pendulum doesn't.

18. Alice, Bob, and Carol are each given identical airtight bags containing identical rocks, and a large tub of water with a scale sitting on the bottom. Each of them measures the weight of their bag and rock by putting the bag on the scale, using three slightly different procedures.

- Alice closes the bag carefully, so that there is no air inside.
- Bob fills the rest of the bag with water before closing it.
- Carol closes the bag loosely, so that it contains some air.

What is the correct ranking of their measured weights W_A, W_B, and W_C?

(A) $W_C < W_A < W_B$ (B) $W_A < W_C < W_B$ (C) $W_A < W_C = W_B$
(D) $W_A = W_C < W_B$ **E** $W_C < W_A = W_B$

The scale's measurement is the weight of the object not supported by the buoyant force. In other words, the scale measures $mg - \rho V g$, where m is the mass of the bag, ρ is the density of water, and V is the volume of water. For Alice,

$$W_A = m_A g - \rho V_A g.$$

If Bob adds volume ΔV of water, he adds mass $\rho \Delta V$ as well, so

$$W_B = (m_A + \rho\Delta V)g - \rho(V_A + \Delta V)g = W_A.$$

Carol adds a volume ΔV while adding a negligible mass from the air, so

$$W_C = W_A - \rho \Delta V g.$$

Therefore, the answer is (E).

19. A student sets up a simple pendulum, measures its length to be (0.50 ± 0.01) m, and observes a period of oscillation of (1.4 ± 0.1) s. Using this data, the student computes $g = 10.1\,\mathrm{m/s^2}$. What is the uncertainty of this measurement?

(A) $0.7\,\mathrm{m/s^2}$ (B) $1.2\,\mathrm{m/s^2}$ **C** $1.4\,\mathrm{m/s^2}$ (D) $1.9\,\mathrm{m/s^2}$ (E) $2.7\,\mathrm{m/s^2}$

We have $g = 4\pi^2 L/T^2$, and we can use the standard rules for propagation of relative uncertainties: the relative uncertainty of a product (or ratio) of two quantities adds in quadrature, and the relative uncertainty of a squared quantity is doubled. Thus,

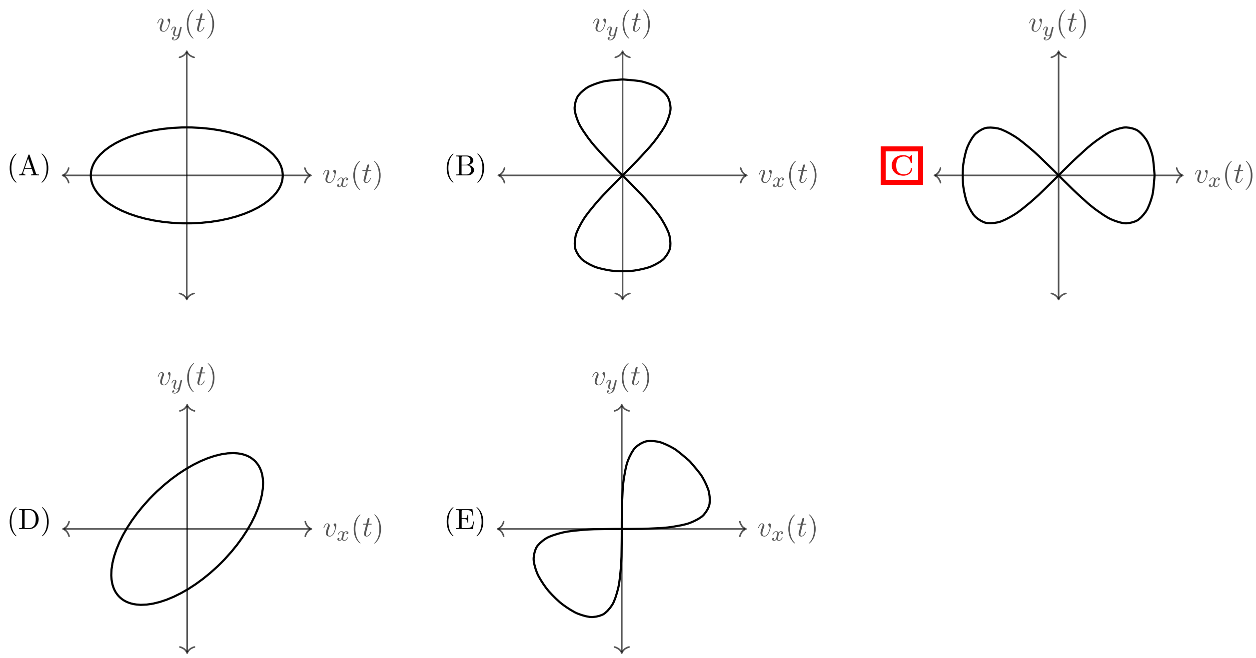
$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{2\Delta T}{T}\right)^2}$$

which gives $\Delta g = 1.4\,\mathrm{m/s^2}$. (At the level of precision needed here, you could also note that $\Delta L/L$ is negligible, so we just have $\Delta g = g(2\Delta T/T)$, which would give the same answer.) If one instead used

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

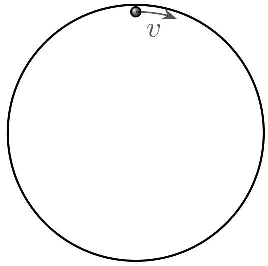
then the answer would instead be $\Delta g = 1.6\,\mathrm{m/s^2}$, which is not an answer choice. However, this is incorrect because independent relative uncertainties add in quadrature, rather than directly. For further details, see the solution to problem 12 of the 2018 $F = ma$ exam A.

20. A mass attached to the end of a string oscillates like a pendulum with small amplitude. The mass has horizontal velocity $v_x(t)$ and vertical velocity $v_y(t)$. Which of the following could be a graph of the curve $(v_x(t), v_y(t))$ over a complete oscillation?



The answer is choice (C), as one can show by plotting several points. For example, one simple way to rule out the other answers is to note that on a pendulum, $v_y(t)$ hits zero four times per cycle, which only choice (C) satisfies.

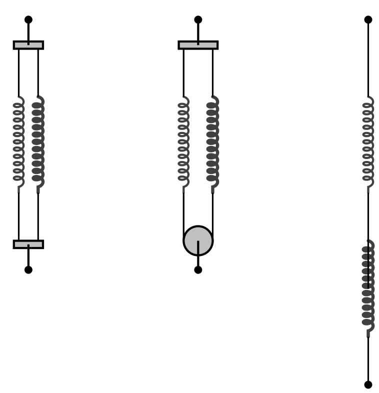
21. A smooth ring of radius R and mass m lies on a frictionless surface. A point mass, also of mass m, is placed just inside the ring and given a speed v tangent to the inner surface of the ring. How long does it take for the point mass to return to its initial position relative to the ring?



(A) $\frac{\pi R}{v}$ (B) $\frac{\sqrt{2}\pi R}{v}$ **C** $\frac{2\pi R}{v}$ (D) $\frac{2\sqrt{2}\pi R}{v}$ (E) $\frac{4\pi R}{v}$

The masses are undergoing small elastic collisions. During an elastic collision, the magnitude of the relative velocities between the masses are conserved, so the tangential velocity of the mass m, relative to the ring, is always v. Thus, the time is $2\pi R/v$.

22. Two springs with different spring constants are connected in three ways, as shown.



In the second case, the springs are connected to opposite ends of a string, which runs under a massless frictionless pulley. In each case, the two springs act like a single spring with an effective spring constant k_A, k_B, or k_C. Which of the following is correct?

A $k_A > k_B > k_C$ (B) $k_A > k_C > k_B$ (C) $k_C > k_B > k_A$ (D) $k_C > k_A > k_B$ (E) $k_B > k_A > k_C$

Let the springs have spring constants k_1 and k_2. In the first case, the forces of the springs simply add, so $k_A = k_1 + k_2$. In the third case, the springs adjust so they exert an equal force. If the total displacement is ℓ, then $\ell = \ell_1 + \ell_2$ and $F = k_1\ell_1 = k_2\ell_2$. Solving for the force gives an effective spring constant

$$k_C = \frac{F}{\ell} = \frac{k_1 k_2}{k_1 + k_2}.$$

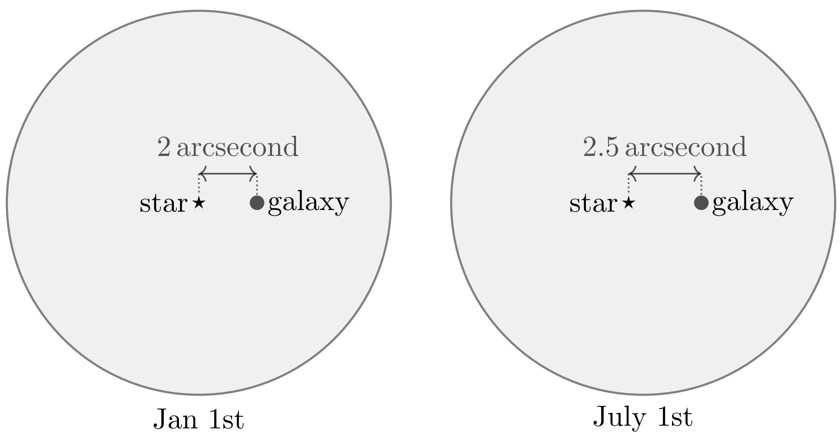
The second case is a bit less familiar, but it is actually similar to the third case. Again, we have the constraint that the forces exerted by the springs are equal, but the difference is that (1) pulling a distance

ℓ now increases the length of the spring system by 2ℓ, and (2) the forces from each spring add together. Each doubles the resulting force, so

$$k_B = \frac{4k_1k_2}{k_1 + k_2}$$

and so clearly $k_B > k_C$. We also have $k_A > k_B$ since the arithmetic mean is always greater than the geometric mean.

23. An astronomer on Earth, which is a distance $L_\odot$ from the Sun, observes a star and galaxy. The star is a distance $L_s \gg L_\odot$ away, and the galaxy is much further away than the star. Throughout the year, the angular distance between the star and galaxy appears to vary, reaching a minimum of 2 arcseconds on January 1st and a maximum of 2.5 arcseconds on July 1st. (One degree is equal to 3600 arcseconds.) Assume the Sun, star, and galaxy do not move relative to each other, and that the Earth's orbit lies within their plane. What is the ratio $L_s/L_\odot$?



(astronomer's view through the telescope)

(A) 1.4×10^4 (B) 7×10^4 (C) 4×10^5 (D) 8×10^5 (E) 4×10^6

The change in apparent position of the star is due to parallax. Using the small angle approximation,

$$\Delta\theta = \frac{2L_\odot}{L_s}$$

where $\Delta\theta = 0.5$ arcseconds $= 1°/7200 = 2.4 \times 10^{-6}$ rad, and there is a factor of 2 since the change in position of the Earth across half a orbit is $2L_s$. Solving for the ratio gives $L_s/L_\odot = 8 \times 10^5$.

24. A very heavy plate continuously moves up and down with a small speed v, switching directions after each time t. At a random time, a ball is dropped from rest far above the plate, then bounces elastically off of it. How does the ball's speed change during this collision?

(A) The ball always slows down.

(B) The ball is more likely to slow down than to speed up.

(C) The ball is equally likely to slow down or speed up.

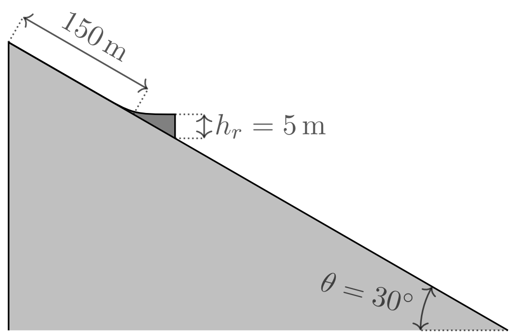
D The ball is more likely to speed up than to slow down.

(E) The ball always speeds up.

When the lowest point of the ball enters the collision region, the plates move up or down with equal probability. if the plate moves up, then the speed of the ball will increase. If the plate moves down, it still has time to turn around and collide with the ball as it will move up.

This is a simple model for how a gas heats up in a container with hot walls, or more exotically, the Fermi acceleration mechanism for cosmic rays.

25. A skier slides from rest along a frictionless slope with incline angle $\theta = 30^\circ$ for 150 m, then smoothly transitions into a horizontal jumping ramp of height $h_r = 5\,\text{m}$. Neglecting air resistance, after the skier leaves the ramp, about how far will they travel *horizontally* before landing?



(A) 164 m (B) 173 m **C** 181 m (D) 200 m (E) 210 m

Upon leaving the ramp, the skier has speed $v = \sqrt{2gd\sin\theta} = \sqrt{gd}$. From there the skier travels a horizontal distance $x = vt$, where t is the time of flight. To find t, shift into the rotated coordinate frame (x' along the slope, y' perpendicular/normal to the slope); in this frame, the skier is launched with initial "vertical" velocity $v'_y = v\sin\theta = v/2 = \sqrt{gd}/2$ from height $h'_r = h_r\cos\theta = h_r\sqrt{3}/2$ and accelerates downward at a constant rate $g' = g\cos\theta = g\sqrt{3}/2$. Then the time of flight is

$$t = \frac{v'_y + \sqrt{(v'_y)^2 + 2g'h'}}{g'} = \frac{\sqrt{gd}/2 + \sqrt{gd/4 + 3gh_r/2}}{g\sqrt{3}/2} = \frac{\sqrt{d} + \sqrt{d + 6h_r}}{\sqrt{3g}},$$

and so the horizontal distance traveled is

$$x = vt = \sqrt{gd}t = \frac{d + \sqrt{d(d + 6h_r)}}{\sqrt{3}} = 181\,\text{m}.$$