AAPT UNITED STATES PHYSICS TEAM
AIP 2020

2020 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. The only scratch paper you may use is scratch paper provided by the proctor. You may not use your own.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones cannot be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, the answer sheet, and all scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 1, 2020.**

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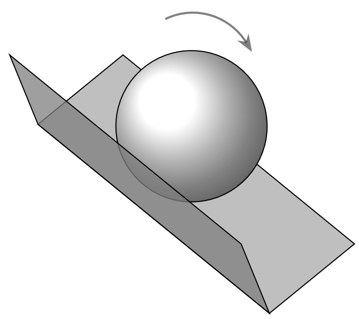
We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Ariel Amir, JiaJia Dong, Mark Eichenlaub, Abijith Krishnan, Daniel Longenecker, Kye Shi, Brian Skinner, Paul Stanley, Mike Winer, and Kevin Zhou.

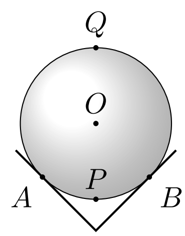
1. A ball is launched straight toward the ground from height h. When it bounces off the ground, it loses half of its kinetic energy. It reaches a maximum height of $2h$ before falling back to the ground again. What was the initial speed of the ball?

   (A) $\sqrt{gh}$
   (B) $\sqrt{2gh}$
   (C) $\sqrt{3gh}$
   (D) $\sqrt{4gh}$
   (E) $\sqrt{6gh}$

2. A rigid ball of radius R is rolling without slipping along the rib of a right-angle chute, as shown at left. A cross section of the ball, taken perpendicular to the ball's direction of travel, is shown at right. Which of the marked point(s) of the ball have the highest speed?



side (3d) view



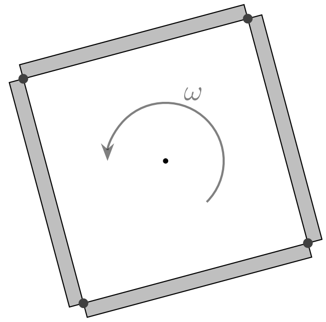
cross-section (2d) view

   (A) All the marked points have the same speed.
   (B) The contact points A and B.
   (C) The center O.
   (D) The point P.
   (E) The point Q.

3. When an axe is swung with kinetic energy E directly at a piece of wood, the edge of the axe is buried a depth L into the wood. If the axe is swung with kinetic energy $2E$, how deep will it be buried into the wood? Assume that the axe is wedge-shaped with a constant angle and that the force per unit contact area between the axe and the wood during the impact is proportional to the depth.

   (A) $2^{1/4}L$
   (B) $2^{1/3}L$
   (C) $\sqrt{2}L$
   (D) $2L$
   (E) $4L$

4. Four identical rods, each of mass m and length $2d$, are joined together to form a square. The square is then spun around its center, as shown in the figure, at an angular frequency of ω. What is the magnitude of the force that the joints between the rods (at the corners of the square) must bear?



(A) $m\omega^2 d/2$

(B) $m\omega^2 d/\sqrt{2}$

(C) $m\omega^2 d$

(D) $\sqrt{2}m\omega^2 d$

(E) $2m\omega^2 d$

5. A pendulum of length L oscillates inside a box. A person picks up the box and gently shakes it horizontally with frequency ω and a fixed amplitude for a fixed time. The final amplitude can be maximized if ω satisfies

(A) $\omega = \sqrt{g/L}$

(B) $\omega = 2\sqrt{g/L}$

(C) $\omega = (1/2)\sqrt{g/L}$

(D) There will be no effect on the amplitude for any value of ω.

(E) None of the above

6. A planet is orbiting a star in a circular orbit of radius r_0. Over a very long period of time, much greater than the period of the orbit, the star slowly and steadily loses 1% of its mass. Throughout the process, the planet's orbit remains approximately circular. The final orbit radius is closest to

(A) $1.02r_0$

(B) $1.01r_0$

(C) r_0

(D) $0.99r_0$

(E) $0.98r_0$

7. An astronaut standing on the exterior of the international space station wants to dispose of three pieces of trash. They face the station's direction of travel with the Earth to their left. From the astronaut's perspective, the three pieces are thrown (I) left, (II) right, and (III) up. To the astronaut's frustration, some of the pieces of trash return to the space station after several hours. They are

(A) II only

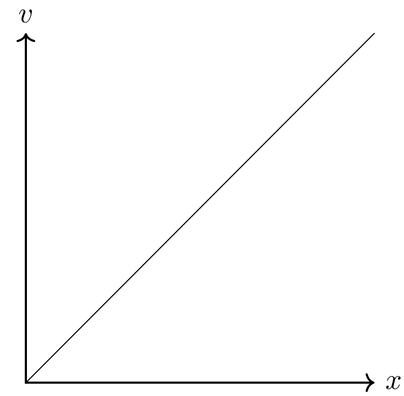
(B) III only

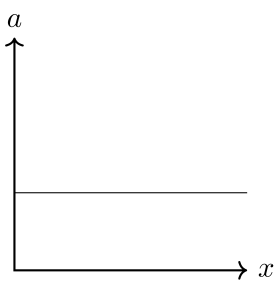
(C) I and II

(D) II and III

(E) I, II, and III

8. The velocity versus position plot of a particle is shown below. Which following choices is the correct acceleration vs. position plot of the particle?



(A)


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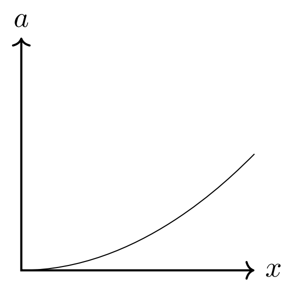

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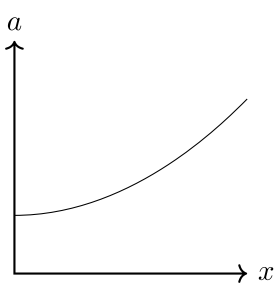

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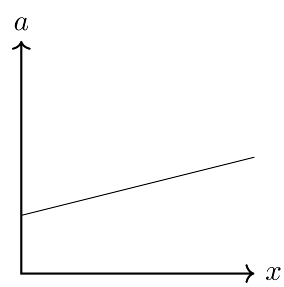

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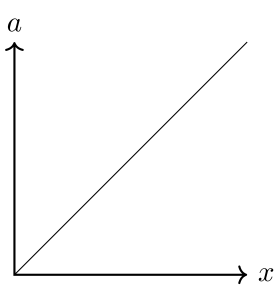
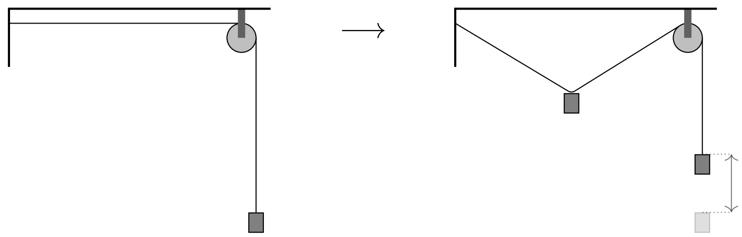


The following information is relevant to problems 9 and 10.

9. A block of mass m is attached to a massless string. The string is passed over a massless pulley and the end of the string is fixed in place. The horizontal part of the string has length L. Now a small mass m is hung from the horizontal part of the string, and the system comes to equilibrium. (Diagram not necessarily to scale.)



Neglecting friction everywhere, the tension at the end of the string is

(A) $mg/2$
(B) mg
(C) $3mg/2$
(D) $2mg$
(E) $3mg$

10. During this process, the block has been raised by approximately a height

(A) $0.15L$
(B) $0.23L$
(C) $0.31L$
(D) $0.37L$
(E) $0.40L$

11. The maximal tension per area a material can sustain without failure is called its *tensile strength*. Plain steel has a tensile strength of 415 MPa. What is the maximal mass one can hang on a vertical steel rod of negligible mass and a diameter of 2 cm?

(A) 1300 kg
(B) 5200 kg
(C) 13 000 kg
(D) 52 000 kg
(E) The answer depends on the length of the steel rod.

12. A point mass m is glued inside a massless hollow rod of length L at an unknown location. When the rod is pivoted at one end, the period of small oscillations is T. When the rod is pivoted at the other end, the period of small oscillations is $2T$. How far is the mass from the center?

(A) $L/8$
(B) $L/6$
(C) $L/4$
(D) $3L/10$
(E) $2L/5$

13. A ballerina with moment of inertia I is quickly twirling with angular velocity ω. In her hand she has a pen of mass m at a radius R from her axis of rotation. The ballerina releases the pen. Afterward, what happens to the vertical component of the angular momentum of the system consisting of the ballerina and the pen? You may ignore all friction, but not gravity or normal forces.

(A) It decreases until the pen hits the floor.

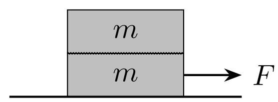
(B) It increases until the pen hits the floor.

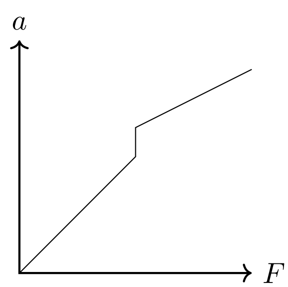
(C) It always stays the same.

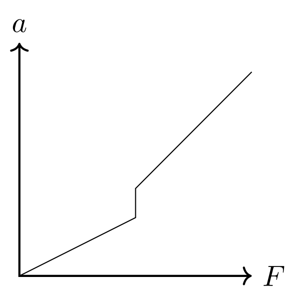
(D) It initially stays the same, but decreases when the pen hits the floor.

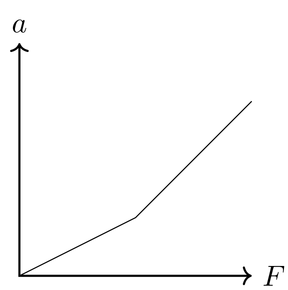
(E) It initially stays the same, but increases when the pen hits the floor.

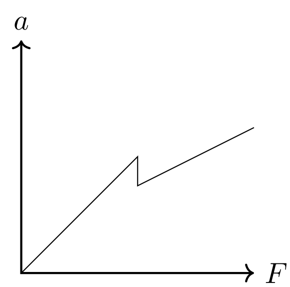
14. Two blocks of mass m are placed on top of each other, and the bottom block is placed on the ground. The ground is frictionless. The static and kinetic coefficients of friction between the two blocks are μ_s and μ_k, with $\mu_s < \mu_k$. The blocks are at rest initially. When a constant horizontal force F is then applied to the bottom block, which of the following graphs could show its acceleration as a function of F?

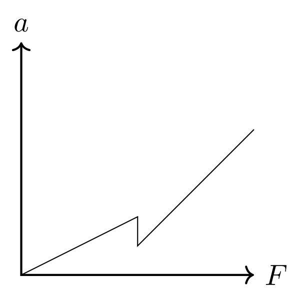


(A)


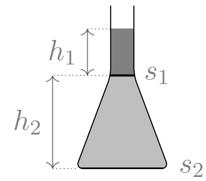
(B)


(C)


(D)


(E)


15. As shown in the figure, a vessel contains two types of liquid: the liquid with density ρ_1 on top and ρ_2 on the bottom. The depth of the top liquid is h_1, and the interface area between the top and the bottom liquid is s_1. The bottom liquid has a depth of h_2. The area of the bottom of the vessel is s_2. What is the gauge pressure (i.e. pressure in excess of atmospheric pressure) at the bottom of the vessel?



(A) $(\rho_1 h_1 + \rho_2 h_2)g$

(B) $\dfrac{\rho_1 s_1 h_1 + \rho_2 s_2 h_2}{s_2} g$

(C) $\dfrac{1}{2}(\rho_1 + \rho_2)(h_1 + h_2)g$

(D) $(\rho_1 + \rho_2)(h_1 + h_2)g$

(E) $\rho_2(h_1 + h_2)g$

16. Liquid droplets store a given amount of potential energy per unit surface area, due to their surface tension. When two identical, nearly spherical liquid droplets coalesce on a certain type of surface, part of this energy can be converted into upward kinetic energy, causing the coalesced droplet to jump. Assuming the conversion is 100% efficient, how does the maximum height h depend on the radius r of the initial droplets?

(A) $h \propto r$

(B) $h \propto r^{1/2}$

(C) $h \propto r^{-1/2}$

(D) $h \propto r^{-1}$

(E) $h \propto r^{-2}$

17. Paul the Giant stands outside on a force-meter calibrated in Newtons, which reads 5000 N. Paul is wearing a large cowboy hat, which has horizontal cross-sectional area $A = 1\,\text{m}^2$ and completely covers both him and the scale when seen from directly above. At time $t = 0$, rain begins to fall vertically downward on Paul, and any rain that hits his hat is collected in the hat's brim. The raindrops have a constant downward speed of 1 m/s, and the rain accumulates on the ground at a rate of 1 mm/s. What is the reading (in N) on the scale as a function of the time $t > 0$ (in s)? The density of water is $1000\,\text{kg/m}^3$.

(A) $5001 + 11t$

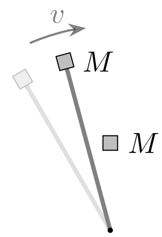
(B) $5001 + 10t$

(C) $5000 + 11t$

(D) $5001 + 1.1t$

(E) $5001 + t$

18. A massless rigid rod is pivoted at one end, and a mass M is at the other end. Originally, the rod rotates frictionlessly about the pivot with a uniform angular velocity such that the mass M has speed v. The rotating rod collides with another mass M at its midpoint, which then sticks to the rod. After the collision, what is the kinetic energy of the system?



(A) $\frac{1}{4}Mv^2$

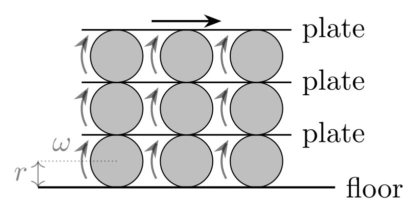
(B) $\frac{1}{3}Mv^2$

(C) $\frac{7}{18}Mv^2$

(D) $\frac{2}{5}Mv^2$

(E) $\frac{1}{2}Mv^2$

19. A system of cylinders and plates is set up as shown. The cylinders all have radius r, and roll without slipping to the right with angular velocity ω. What is the speed of the top plate?



(A) ωr

(B) $2\omega r$

(C) $3\omega r$

(D) $4\omega r$

(E) $6\omega r$

20. A car is driving against the wind at a constant speed v_0 relative to the ground. The wind direction is always opposite to the car's velocity, but its speed fluctuates about an average speed of v relative to the ground. The air drag force is Av_{rel}^2, where A is a constant and v_{rel} is the relative speed between the car and the wind. What is the average rate $\overline{P}$ of energy dissipation due to the air resistance?

(A) $\overline{P} = Av_0(v_0 + v)^2$

(B) $\overline{P} > Av_0(v_0 + v)^2$

(C) $\overline{P} < Av_0(v_0 + v)^2$

(D) Both (B) and (C) are possible depending on how v fluctuates.

(E) Both (A) and (C) are possible depending on how v fluctuates.

21. A circular table has radius R and $N > 2$ equally spaced legs of length h attached to its perimeter. Suppose the table has a uniform mass density with total mass m, and neglect the mass of the legs. Assuming the table does not slip, the minimum horizontal force needed to tip over the table is

(A) $\dfrac{mgR}{h}$

(B) $\dfrac{mgR}{h}\sin\left(\dfrac{N-2}{2N}\,\pi\right)$

(C) $\dfrac{mgR}{h}\cos\left(\dfrac{\pi}{N}\right)$

(D) $\dfrac{mgR}{h}\tan\left(\dfrac{N-2}{2N}\,\pi\right)$

(E) $\dfrac{mgR}{h}\sin\left(\dfrac{\pi}{2N}\right)$

22. A collision occurs between two masses. In each inertial reference frame, one can compute the change in total momentum $\Delta\mathbf{P}$ and the change in total kinetic energy ΔK due to the collision. Which of the following is true?

(A) $\Delta\mathbf{P}$ and ΔK do not depend on the frame.

(B) $\Delta\mathbf{P}$ and ΔK do not depend on the frame for perfectly elastic collisions, but $\Delta\mathbf{P}$ may depend on the frame for inelastic collisions.

(C) $\Delta\mathbf{P}$ and ΔK do not depend on the frame for perfectly elastic collisions, but ΔK may depend on the frame for inelastic collisions.

(D) $\Delta\mathbf{P}$ and ΔK do not depend on the frame for perfectly elastic collisions, but both may depend on the frame for inelastic collisions.

(E) $\Delta\mathbf{P}$ and ΔK may both depend on the frame, for both perfectly elastic and inelastic collisions.

23. Steve determines the spring constant k of a spring by applying a force F to it and measuring the change in length Δx. The tools he uses to measure F and Δx both have a constant absolute uncertainty, leading to an uncertainty in k of δk_S. If Tiffany measures the same spring constant with the same tools but by using a force that is five times larger, what will her uncertainty in k be in terms of δk_S?

(A) $\delta k_T = 0.04\,\delta k_S$

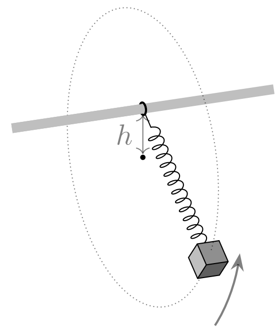
(B) $\delta k_T = 0.08\,\delta k_S$

(C) $\delta k_T = 0.2\,\delta k_S$

(D) $\delta k_T = 0.4\,\delta k_S$

(E) $\delta k_T = 0.5\,\delta k_S$

24. A mass m is connected to one end of a zero-length spring with spring constant k. The other end of the spring is connected to a frictionless bearing mounted around a horizontal pole so that the mass can swing in a vertical circle of radius R around the pole. The setup is shown in the figure below. What is the vertical distance h between the center of the circular orbit and the axis of the pole? Assume that both the diameter of the pole and the rest length of the spring are negligible compared to R.



(A) $\sqrt{mgR/k}$

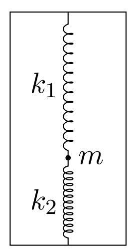
(B) $R\sqrt{(R + mg/k)/(R - mg/k)}$

(C) $R - mg/k$

(D) mg/k

(E) $\sqrt{R^2 - (mg/k)^2}$

25. A ball of negligible radius and mass m is connected to two ideal springs. Each spring has rest length ℓ_0. The springs are connected to the ball inside a box of height $2\ell_0$, and the ball is allowed to come to equilibrium, as shown. Under what condition is this equilibrium point stable with respect to small horizontal displacements?



(A) $k_1 > k_2$

(B) $k_2 > k_1$

(C) $k_1 - k_2 > mg/\ell_0$

(D) $k_1k_2/(k_1 + k_2) > mg/\ell_0$

(E) $k_1k_2/(k_1 - k_2) > mg/\ell_0$