AAPT | UNITED STATES PHYSICS TEAM
AIP | 2023

2023 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 25, 2023.**

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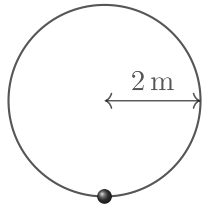
We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Kellan Colburn, Samuel Gebretsadkan, Abi Krishnan, Natalie LeBaron, Kye Shi, Mike Winer, and Kevin Zhou

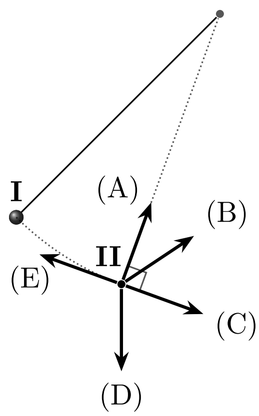
1. A bead on a circular hoop with radius 2 m travels counterclockwise for 10 s and completes 2.25 rotations, at which point it reaches the position shown.



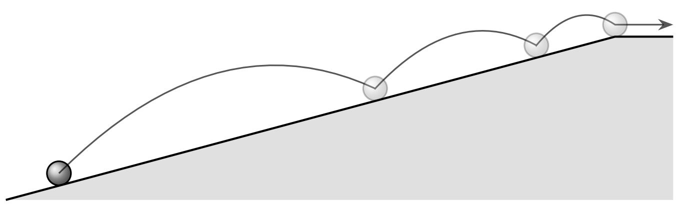
In the past 10 s, what were its average speed and the direction of its average velocity?

(A) $\frac{\sqrt{2}}{5}\,\frac{\text{m}}{\text{s}}, \swarrow$ (B) $\frac{2\pi}{5}\,\frac{\text{m}}{\text{s}}, \swarrow$ (C) $\frac{9\pi}{10}\,\frac{\text{m}}{\text{s}}, \swarrow$ (D) $\frac{2\pi}{5}\,\frac{\text{m}}{\text{s}}, \searrow$ (E) $\frac{9\pi}{10}\,\frac{\text{m}}{\text{s}}, \searrow$

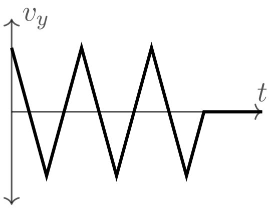
2. A mass on an ideal pendulum is released from rest at point I. When it reaches point II, which of the following shows the direction of its acceleration?



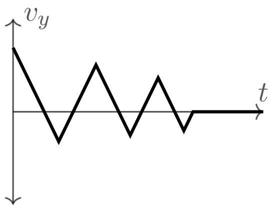
3. A soccer ball is kicked up a hill with a flat top, as shown. The ball bounces twice on the hill, at the points shown, then lands on the top and begins rolling horizontally. Which of the following shows the vertical component of its velocity as a function of time?



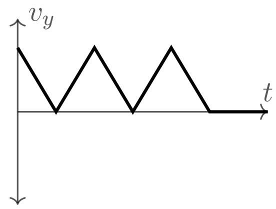
(A)



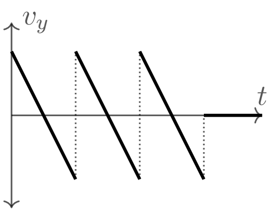
(B)



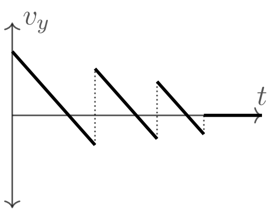
(C)



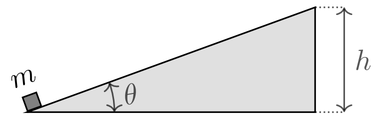
(D)



(E)



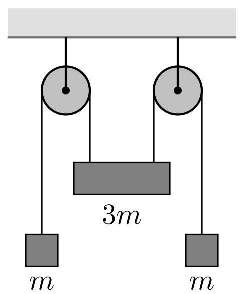
4. A box of mass m is at the bottom of an inclined plane with angle θ to the horizontal, and height h.



A person drags the box very slowly up the plane, by applying a force parallel to the plane. The coefficient of kinetic friction between the box and plane is μ_k. When the box reaches the top of the plane, how much work has the person done?

(A) $mgh\,(1 + \mu_k \sin\theta)$ (B) $mgh\,(1 + \mu_k \cos\theta)$ (C) $mgh\,(1 + \mu_k \tan\theta)$
(D) $mgh\,(1 + \mu_k \csc\theta)$ (E) $mgh\,(1 + \mu_k \cot\theta)$

5. Two blocks of mass m and a block of mass $3m$ are attached to a system of massless fixed pulleys and massless string, as shown.



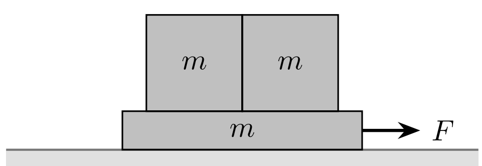
Assume all surfaces are frictionless. What is the acceleration of each mass m?

(A) $\frac{g}{8}$ (B) $\frac{g}{5}$ (C) $\frac{g}{4}$ (D) $\frac{g}{3}$ (E) $\frac{2g}{3}$

6. A ball at the end of a rope of length 0.5 m is swung in a horizontal circle, with a speed of 15 m/s. The other end of the rope is fixed in place. What is the height difference between the ends of the rope?

(A) 1.1 cm (B) 2.2 cm (C) 3.8 cm (D) 4.9 cm (E) 7.5 cm

7. Two boxes are stacked side-by-side on top of a larger box as shown.



All three boxes have mass m, the coefficient of static friction between the left box and the bottom box is μ_s, and all other surfaces are frictionless. A rightward force F is applied to the bottom box. What is the minimum value of μ_s so that the upper boxes don't slide?

(A) $\frac{2F}{mg}$ (B) $\frac{3F}{mg}$ (C) $\frac{F}{2mg}$ (D) $\frac{2F}{3mg}$ (E) $\frac{F}{3mg}$

8. Two stars α and β, with masses satisfying $m_\alpha/m_\beta = 10$, are in circular orbits around each other. In the rest frame of this system, find the ratio of the speeds v_α/v_β.

(A) $\frac{1}{11}$ (B) $\frac{1}{10}$ (C) $\frac{1}{9}$ (D) 9 (E) 10

9. A helium balloon is released from the floor in a room at rest, then slowly rises and comes to rest touching the ceiling. During this process, the gravitational potential energy of the balloon has increased. Since energy is conserved, the energy of something else must have decreased during this process. Which of the following is the main contribution to this decrease?

(A) The kinetic energy of the balloon decreased.
(B) The elastic potential energy of the balloon decreased.
(C) The thermal energy of the air in the balloon decreased.
(D) The thermal energy of the air in the room decreased.
(E) The gravitational potential energy of the air in the room decreased.

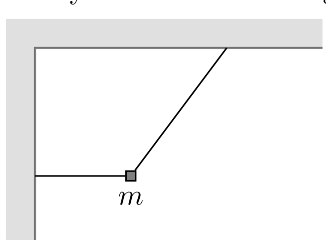
10. An archer takes aim at a target that is 100 m away. Assuming she holds the bow at the same height as the center of the target and shoots an arrow with velocity $v = 100\,\mathrm{m/s}$, at what angle above the horizontal should she aim the bow so that the arrow hits the center of the target?

(A) $\frac{\arccos(1/5)}{2}$ (B) $\frac{\arcsin(1/5)}{2}$ (C) $\frac{\arccos(1/10)}{2}$ (D) $\frac{\arcsin(1/10)}{2}$ (E) $\frac{\arctan(2/5)}{2}$

11. A projectile is thrown from a horizontal surface, and reaches a maximum height h and also lands a distance h from the launch point. Neglecting air resistance, what is the maximum height for a projectile thrown directly upward with the same initial speed?

(A) $\frac{17h}{16}$ (B) $\frac{13h}{12}$ (C) $\frac{9h}{8}$ (D) $\frac{5h}{4}$ (E) $2h$

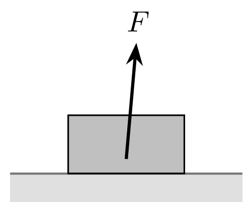
12. A block of mass m is initially held in place by two massless strings, as shown.



The tension in the diagonal string is T_1. Next, the horizontal string is cut, and immediately afterward the tension in the diagonal string is T_2. Which of the following is true?

(A) $T_1 < mg < T_2$ (B) $T_2 < mg < T_1$ (C) $T_1 < T_2 < mg$ (D) $mg < T_2 < T_1$ (E) $T_1 = T_2 < mg$

13. A uniform box with mass m is at rest on a horizontal surface, and the coefficient of static friction between them is μ_s. A force directed at an angle of 85° above the horizontal is applied to the center of the box, with a linearly increasing magnitude $F = \beta t$.



The box will eventually slide or lift off the ground. Which of the following is correct?

(A) If $\mu_s < \tan 85°$, the box will lift off the ground first.

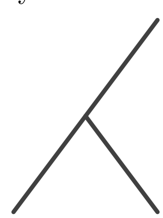
(B) If $\mu_s < \tan 85°$, the box will slide first.

(C) For any value of μ_s, the box will lift off the ground first.

(D) For any value of μ_s, the box will slide first.

(E) The answer depends on the values of β, g, and m.

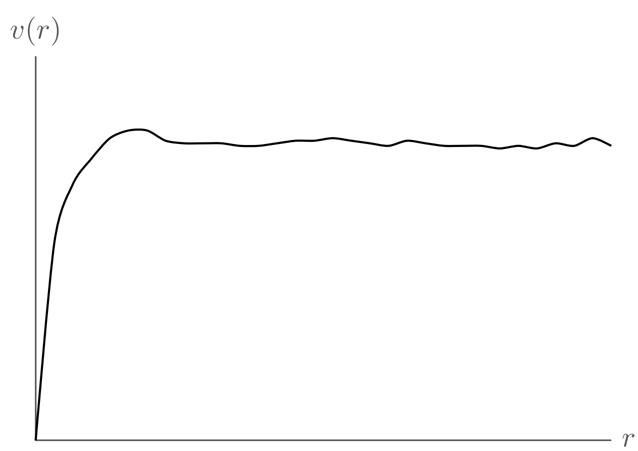
14. The object shown below is made of three rigidly connected, identical rods with uniform density.



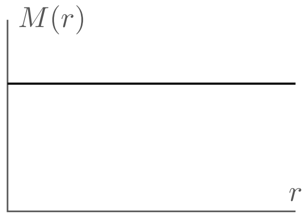
When it stands upright on a horizontal table, what fraction of its weight rests on the left leg?

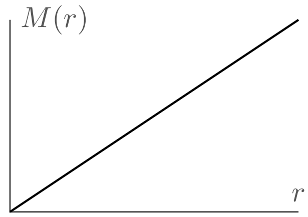
(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{5}{12}$

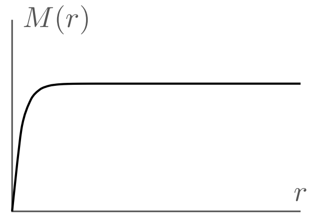
15. The following plot shows the speed $v(r)$ at which stars orbit about the center of a galaxy, as a function of their distance r from the center.

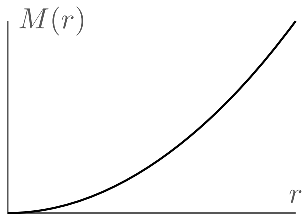


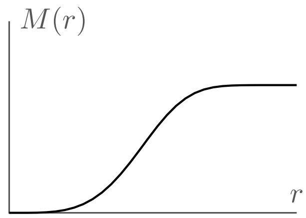
Assuming the galaxy has a spherically symmetric mass distribution, which plot best shows the mass of the galaxy $M(r)$ enclosed within radius r?

(A) 

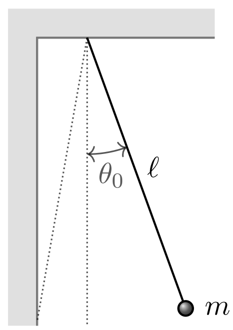
(B) 

(C) 

(D) 

(E) 

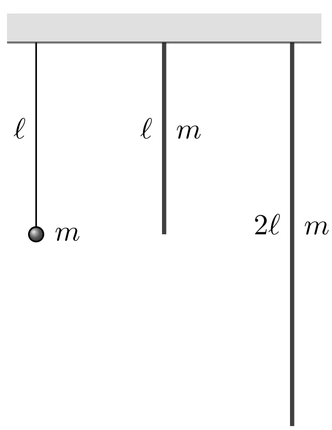
16. A bead attached to a string of length $\ell = 10\,\text{m}$ is released from a very small angle θ_0 to the vertical. A wall is placed in the path of the bead such that the bead collides elastically with the wall when the string is at an angle $\theta_0/2$ to the vertical, as shown.



What is the time interval between the bead's collisions with the wall?

(A) $\dfrac{2\pi}{3}$ s (B) $\dfrac{3\pi}{4}$ s (C) $\dfrac{4\pi}{3}$ s (D) $\dfrac{3\pi}{2}$ s (E) 2π s

17. Three physical pendulums are built as shown. The first is a typical pendulum with a massless rope, and the second and third are made of uniform rods.



What is the correct ranking of the moments of inertia I_1, I_2, and I_3 about the pivot points?

(A) $I_1 > I_2 > I_3$ (B) $I_3 > I_2 > I_1$ (C) $I_1 = I_3 > I_2$ (D) $I_2 > I_3 > I_1$ (E) $I_3 > I_1 > I_2$

18. Alice, Bob, and Carol are each given identical airtight bags containing identical rocks, and a large tub of water with a scale sitting on the bottom. Each of them measures the weight of their bag and rock by putting the bag on the scale, using three slightly different procedures.

- Alice closes the bag carefully, so that there is no air inside.
- Bob fills the rest of the bag with water before closing it.
- Carol closes the bag loosely, so that it contains some air.

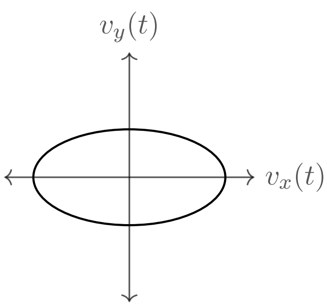
What is the correct ranking of their measured weights W_A, W_B, and W_C?

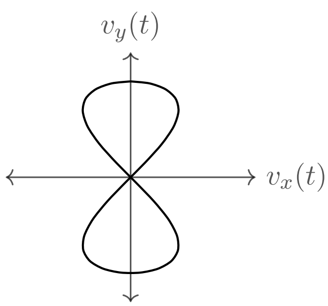
(A) $W_C < W_A < W_B$ (B) $W_A < W_C < W_B$ (C) $W_A < W_C = W_B$
(D) $W_A = W_C < W_B$ (E) $W_C < W_A = W_B$

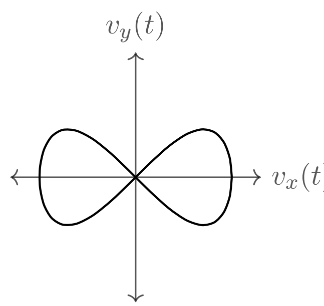
19. A student sets up a simple pendulum, measures its length to be $(0.50 \pm 0.01)\,\mathrm{m}$, and observes a period of oscillation of $(1.4 \pm 0.1)\,\mathrm{s}$. Using this data, the student computes $g = 10.1\,\mathrm{m/s^2}$. What is the uncertainty of this measurement?

(A) $0.7\,\mathrm{m/s^2}$ (B) $1.2\,\mathrm{m/s^2}$ (C) $1.4\,\mathrm{m/s^2}$ (D) $1.9\,\mathrm{m/s^2}$ (E) $2.7\,\mathrm{m/s^2}$

20. A mass attached to the end of a string oscillates like a pendulum with small amplitude. The mass has horizontal velocity $v_x(t)$ and vertical velocity $v_y(t)$. Which of the following could be a graph of the curve $(v_x(t), v_y(t))$ over a complete oscillation?

(A)


(B)


(C)


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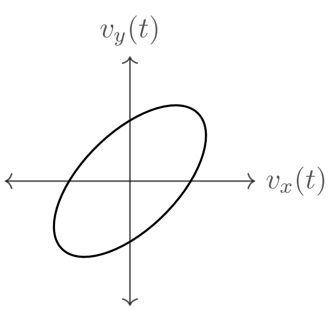
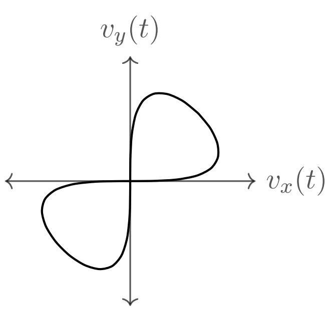
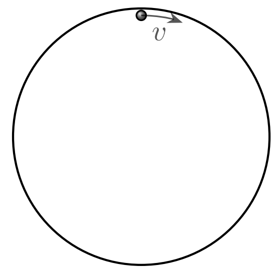


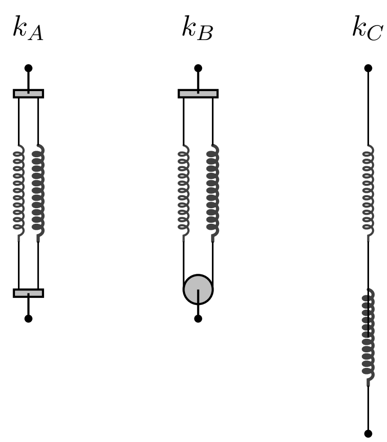
(E)


21. A smooth ring of radius R and mass m lies on a frictionless surface. A point mass, also of mass m, is placed just inside the ring and given a speed v tangent to the inner surface of the ring. How long does it take for the point mass to return to its initial position relative to the ring?



(A) $\frac{\pi R}{v}$ (B) $\frac{\sqrt{2}\pi R}{v}$ (C) $\frac{2\pi R}{v}$ (D) $\frac{2\sqrt{2}\pi R}{v}$ (E) $\frac{4\pi R}{v}$

22. Two springs with different spring constants are connected in three ways, as shown.

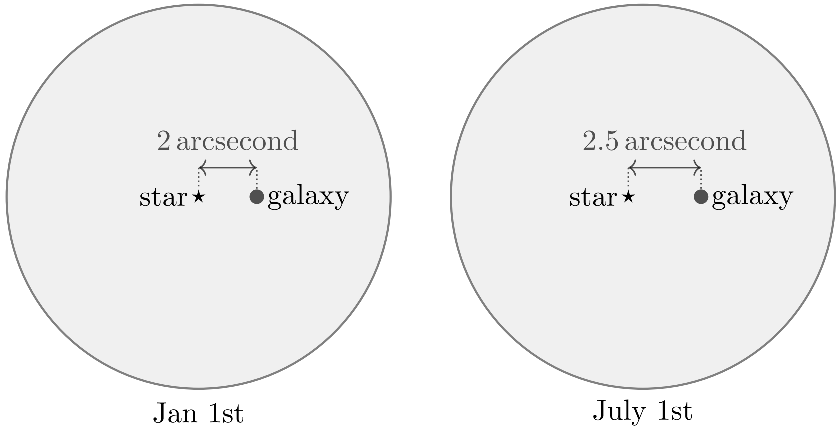


In the second case, the springs are connected to opposite ends of a string, which runs under a massless frictionless pulley. In each case, the two springs act like a single spring with an effective spring constant k_A, k_B, or k_C. Which of the following is correct?

(A) $k_A > k_B > k_C$ (B) $k_A > k_C > k_B$ (C) $k_C > k_B > k_A$ (D) $k_C > k_A > k_B$ (E) $k_B > k_A > k_C$

23. An astronomer on Earth, which is a distance $L_\odot$ from the Sun, observes a star and galaxy. The star is a distance $L_s \gg L_\odot$ away, and the galaxy is much further away than the star. Throughout the year, the angular distance between the star and galaxy appears to vary, reaching a minimum of 2 arcseconds on January 1st and a maximum of 2.5 arcseconds on July 1st. (One degree is equal to 3600 arcseconds.) Assume the Sun, star, and galaxy do not move relative to each other, and that the Earth's orbit lies

within their plane. What is the ratio $L_s/L_\odot$?



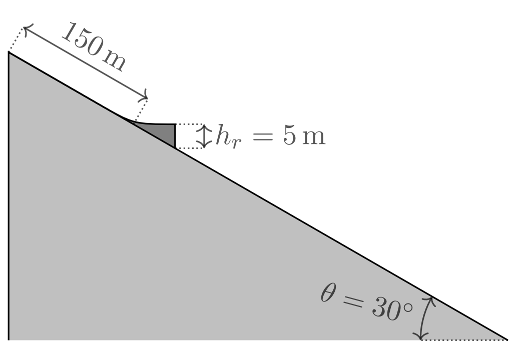
(astronomer's view through the telescope)

(A) 1.4×10^4 (B) 7×10^4 (C) 4×10^5 (D) 8×10^5 (E) 4×10^6

24. A very heavy plate continuously moves up and down with a small speed v, switching directions after each time t. At a random time, a ball is dropped from rest far above the plate, then bounces elastically off of it. How does the ball's speed change during this collision?

(A) The ball always slows down.
(B) The ball is more likely to slow down than to speed up.
(C) The ball is equally likely to slow down or speed up.
(D) The ball is more likely to speed up than to slow down.
(E) The ball always speeds up.

25. A skier slides from rest along a frictionless slope with incline angle $\theta = 30°$ for 150 m, then smoothly transitions into a horizontal jumping ramp of height $h_r = 5$ m. Neglecting air resistance, after the skier leaves the ramp, about how far will they travel *horizontally* before landing?



(A) 164 m (B) 173 m (C) 181 m (D) 200 m (E) 210 m